### Algebraic Reasoning (AR)

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<tr>
<td>Solve addition problems with sums between 0 and 10 and subtraction problems using related facts.</td>
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<td>Represent and solve problems involving the four operations with whole numbers and fractions.</td>
<td>Solve one- and two-step real-world problems with related facts.</td>
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Develop an understanding of equality and subtraction.

Explain why addition or subtraction equations are true or false.

Determine the unknown whole number in an addition or subtraction equation, relating two or three whole numbers, with the unknown in any position.

Determine and explain whether an equation involving any of the four operations is true or false.

Evaluate multi-step numerical expressions.

Given a mathematical or real-world context, write an equation or written expressions that represent relationships between quantities from a graph, a written description of a linear function, and determine and interpret its solutions algebraically or graphically.

Write, solve and graph mathematical and real-world functions. Interpret key features and determine domain constraints in terms of the context.

Given an equation in the form of $x^2 = p$ and $x^3 = q$, where $p$ is a whole number and $q$ is an integer, determine solutions algebraically or graphically.

Write a linear two-variable equation for a given line and goes through a given point.

Write two-variable linear inequalities to represent relationships between quantities from a graph or a written description of a linear function, graph the solution set to a two-variable linear inequality.

Given a mathematical or real-world context, write and solve one-variable linear equations, one-variable linear inequalities, and two-step linear inequalities. Represent solutions algebraically or graphically.

Write and solve multi-step linear equations.

Write and solve one-step equations and inequalities in one variable.

Solve two-step linear inequalities involving rational number coefficients. Include solutions with ordered pairs that are integers.

Write and solve two-step linear inequalities in one variable.

Write and solve one-step equations and inequalities.

Develop an understanding for solving equations and inequalities. Write and solve one-step equations in one variable.

Given a table, equation or written expression involving any of the four operations and one variable, write an equation or inequality in one variable and represent solutions algebraically or graphically.

Given an equation or inequality in one variable, with rational number coefficients, include solutions with ordered pairs that are integers.

Write and solve multi-step linear inequalities.

Write and solve one-variable equation and inequalities.
| MA.2.AR.3 | 3.1 Write, solve and graph simple multiplication equations. Represent solutions algebraically or graphically.
| MA.5.AR.3 | 3.1 Use ratios to solve real-world problems involving the conversion of measurements within a single measurement system.
| MA.6.AR.3.1 | Use a coordinate plane to represent solutions to real-world and mathematical problems involving coordinates in one or two variables.
| MA.6.AR.3.2 | Represent and solve real-world problems involving the conversion of measurements within a single measurement system.
| MA.6.AR.3.3 | Use ratios to solve real-world problems involving the conversion of measurements within a single measurement system.
| MA.6.AR.3.4 | Use ratios to solve real-world problems involving the conversion of measurements within a single measurement system.
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| MA.7.AR.3 | Use patterns, relationships, and proportional reasoning to solve problems.
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| MA.912.AR.3.9 | Use patterns, relationships, and proportional reasoning to solve problems.
| MA.912.AR.3.10 | Use patterns, relationships, and proportional reasoning to solve problems.
Analyze and represent two-dimensional systems of equations.

Determine whether two quantities have a proportional relationship by examining a table, graph or written description of a proportional relationship.

Develop an understanding of proportional relationships.

Given a system of two linear equations and a specified set of possible solutions, determine which ordered pairs satisfy the system of linear equations.

Write, solve and graph absolute value equations, functions and inequalities in one and two variables.

Given a mathematical or real-world context, write and solve one-variable absolute value equations.

Given a system of two linear equations represented graphically, determine whether there is one solution, infinitely many solutions, or no solution.

Given a mathematical or real-world context, graph proportional relationships from a table, equation or a written description of a situation.

Given a mathematical or real-world context, write and solve absolute value inequalities. Represent a solution, no solution or infinitely many solutions.

Given any representation of a proportional relationship, translate the representation to a written description, table or equation.

Solve real-world problems involving proportional relationships.
MA.912.AR.5
Write, solve and graph exponential and logarithmic equations and functions in one and two variables.

MA.912.AR.5.1 Solve one-variable exponential equations using the properties of exponents.

MA.912.AR.5.2 Solve one-variable equations involving logarithms or exponential expressions.

MA.912.AR.5.3 Given a mathematical or real-world context, when suitable factorization is possible, solve one-variable polynomial equations and functions in one variable of degree 3 or higher over the real and complex number systems.

MA.912.AR.5.4 Solve one-variable equations involving logarithms or exponential expressions.

MA.912.AR.5.5 Given a mathematical or real-world context, classify an exponential function as representing growth or decay.

MA.912.AR.5.6 Given a table, equation or written description of an exponential function, graph that function and determine its key features.

MA.912.AR.5.7 Given a table, equation or written description of a logarithmic function, graph that function and determine its key features.

MA.912.AR.5.8 Solve and graph mathematical and real-world problems that are modeled with exponential functions. Interpret key features and determine constraints in terms of the context.

MA.912.AR.5.9 Solve and graph mathematical and real-world problems that are modeled with logarithmic functions. Interpret key features and determine constraints in terms of the context.

MA.912.AR.6
Solve and graph polynomial equations and functions in one and two variables.

MA.912.AR.6.1 Given a mathematical or real-world context, when suitable factorization is possible, solve one-variable polynomial equations and functions in one variable of degree 3 or higher over the real and complex number systems.

MA.912.AR.6.2 Explain and apply the Remainder Theorem to solve mathematical and real-world problems.

MA.912.AR.6.3 Explain and apply theorems for polynomials to solve mathematical and real-world problems.

MA.912.AR.6.4 Given a table, equation or written description of a polynomial function of degree 3 or higher, graph that function and determine its key features.

MA.912.AR.6.5 Sketch a rough graph of a polynomial function of degree 3 or higher using zeros, multiplicity and knowledge of end behavior.

MA.912.AR.6.6 Solve and graph mathematical and real-world problems that are modeled with polynomial functions of degree 3 or higher. Interpret key features and determine constraints in terms of the context.
Solve and graph radical equations and functions in one and two variables.

**MA.912.AR.7**

- Solve one-variable radical equations.
  - Interpret solutions as viable in terms of context and identify any extraneous solutions.
  - MA.912.AR.7.2
    - Given a table, equation or written description of a square root or cube root function, graph that function and determine its key features.
  - MA.912.AR.7.3
    - Solve and graph mathematical and real-world problems that are modeled with square root or cube root functions. Interpret key features and determine constraints in terms of the context.
  - MA.912.AR.7.4
    - Solve mathematical and real-world problems that are modeled with radical functions. Interpret key features and determine constraints in terms of the context.
  - MA.912.AR.7.5
    - Given a real-world context, represent constraints as systems of linear equations or inequalities. Interpret solutions to problems as viable or non-viable options.
  - MA.912.AR.7.6
    - Solve real-world problems involving linear programming in two variables.
  - MA.912.AR.7.7
    - Given a mathematical or real-world context, solve a system of three-variable linear equations algebraically.

Solve and graph rational equations and functions in one and two variables.

**MA.912.AR.8**

- Write and solve one-variable rational equations. Interpret solutions as viable in terms of the context and identify any extraneous solutions.
  - MA.912.AR.8.2
    - Given a mathematical or real-world context, solve a system consisting of a two-variable linear equation and a non-linear equation algebraically or graphically.
  - MA.912.AR.8.3
    - Given a mathematical or real-world context, solve a system consisting of two-variable linear or non-linear equations algebraically or graphically.
  - MA.912.AR.8.4
    - Graph the solution set of a system of two-variable linear inequalities.
  - MA.912.AR.8.5
    - Graph the solution set of a system of two-variable inequalities.
  - MA.912.AR.8.6
    - Given a real-world context, represent constraints as systems of linear equations or inequalities. Interpret solutions to problems as viable or non-viable options.
  - MA.912.AR.8.7
    - Given a real-world context, represent constraints as systems of linear and non-linear equations or inequalities. Interpret solutions to problems as viable or non-viable options.
  - MA.912.AR.8.8
    - Solve real-world problems involving linear programming in two variables.
Write and solve problems involving arithmetic sequences, series equations, functions and inequalities in one and two variables.

Recognize and apply the formula for the sum of a finite arithmetic series to solve mathematical and real-world problems.

Recognize and apply the formula for the sum of a finite or an infinite geometric series to solve mathematical and real-world problems.

Given a mathematical or real-world context, write a sequence using function notation, defined explicitly or recursively, to represent relationships between quantities from a written description.

Given a mathematical or real-world context, find the domain of a given sequence defined recursively or explicitly.