#### Grade 5 Fraction Unit of Instruction

This is a progressive unit of instruction using the <u>Concrete-Representational-Abstract (CRA) Instructional Model</u>. CRA is a three-part instructional model that begins by using concrete materials, then progresses to representational pictures, and finally abstract notation. This unit is not intended to replace your district's curriculum, but rather it serves to support the teaching and learning of the grade five fraction standards. In this unit, students will begin by investigating the standards while using manipulatives to explore the concepts. Then, students will represent their learning through pictures, visuals and drawings. Finally, students will demonstrate their understanding through abstract notation and algorithms. This unit of study will cover the fifth grade fraction standards MAFS.5.NF.1.1, MAFS.5.NF.1.2, MAFS.5.NF.2.3, MAFS.5.NF.2.4, MAFS.5.NF.2.5, MAFS.5.NF.2.6, and MAFS.5.NF.2.7.

The unit begins with a list of review lessons and tools to assist in teaching fractions to fifth grade students. Then, each of the seven fifth grade fraction standards is listed along with aligned instructional resources and formative assessments. The component of CRA is identified for each of the resources and formative assessments. The resources presented in this document may only cover portions of the aligned standard and represent only a small sample of those available on <a href="CPALMS">CPALMS</a>.

The Mathematical Practices are habits of mind that describe varieties of expertise that mathematics educators at all levels should seek to develop in their students. The Mathematical Practices should be infused during the course and will be assessed throughout the Grade 5 Mathematics FSA. More information about each Mathematical Practice can be found by clicking on the links below.

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MAFS.K12.MP.1.1 Make sense of problems and persevere in solving them.
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MAFS.K12.MP.2.1 Reason abstractly and quantitatively.

MAFS.K12.MP.3.1 Construct viable arguments and critique the reasoning of others.

MAFS.K12.MP.4.1 Model with mathematics.

MAFS.K12.MP.5.1 Use appropriate tools strategically.

MAFS.K12.MP.6.1 Attend to precision.

MAFS.K12.MP.7.1 Look for and make use of structure.

MAFS.K12.MP.8.1 Look for and express regularity in repeated reasoning.

# Number and Operations- Fractions

A bibliography of children's literature with a focus on	1. Cut Down to Size at High Noon, Scott Sundby
fractions is provided. These books and articles can be	2. Fractions, Decimals, and Percents, David Adler
integrated into the fraction lessons to connect	3. Funny and Fabulous Fraction Stories, Dan Greenberg
mathematics and literature.	4. Give Me Half, Stuart J. Murphy
	5. Multiplying Menace, Pam Calvert
	6. Piece = Part = Portion, Scott Gifford
	7. Pizza Counting, Christina Dobson
	8. Remainder of One, Elinor J. Pinczes
5th Grade Mathematics Course Description	Course descriptions provide an overview for a course and designate which
	standards are in that course. The course description includes resources for all 40
	standards within the 5th grade mathematics course.
Fun with Fractions- Review	In this five lesson unit, students will explore relationships among fractions
Lesson Plan	through work with pattern blocks as concrete representations. This early work
	with fraction relationships helps students make sense of basic fraction concepts.
Concrete-Representational-Abstract	The lessons in this unit incorporate the use of physical and virtual manipulatives.
<u>Test Item Specifications</u>	The Test Item Specifications indicate the alignment of items with the Florida
	Standards. Assessment limits are included in the specifications, which define the
	range of content knowledge in the assessment items for the standard. Sample
	items for each standard are also included in the specifications document.
Test Design Summary and Blueprint	The Test Design Summary and Blueprint shows the reporting categories with a
	corresponding weight for the 5th Grade Mathematics FSA.
<u>Florida Students</u>	Resources specifically designed with students in mind are available on Florida
	Students. Florida Students is an interactive site that provides educational
	resources aligned to the Florida Standards.
5th Grade Mathematics Parent Guide	The parent guide will support parents and families with children in Grade 5

MAFS.5.NF.1.1 Add and subtract fractions with unlike denominators (including mixed numbers) by replacing given fractions with equivalent fractions in such a way as to produce an equivalent sum or difference of fractions with like denominators. For example, 2/3 + 5/4 = 8/12 + 15/12 = 23/12. (In general, a/b + c/d = (ad + bc)/bd.)

<u>Discovering Common Denominators</u>	Students use pattern blocks to represent fractions with unlike denominators.
Lesson Plan	Students discover that they need to convert all the pattern blocks to the same
	shape in order to add them. Therefore, they find and use common denominators
Concrete	for the addition of fractions.
Using Models to Add Fractions with Unlike Denominators	This lesson is specific to adding fractions with unlike denominators. It requires
Lesson Plan	students to already have a working knowledge of adding fractions with common
	denominators, and equivalent fractions.
Concrete-Representational	
Adding and Subtracting Fractions	Kahn Academy video tutorial on how to add and subtract fractions with like and
Tutorial	unlike denominators.
Representational-Abstract	
Egyptian Fractions	One goal of this task is to help students develop comfort and ease with adding
Problem-Solving Task	fractions with unlike denominators. Another goal is to help them develop fraction
	number sense by having students decompose fractions.
Representational-Abstract	
Adding and Subtracting Mixed Numbers	This lesson helps fifth graders combine their understanding of adding and
Lesson Plan	subtracting fractions with unlike denominators, finding equivalent fractions, and
	adding and subtracting mixed numbers with like denominators to move on to
Concrete-Representational-Abstract	adding and subtracting mixed numbers with unlike denominators.
Finding Common Denominators to Subtract	The purpose of this task to help students realize that there are multiple common
Problem-Solving Task	denominators they could choose to add and subtract fractions. Students can draw
	a picture if they want, but this subtraction problem is easier to do symbolically,
Abstract	which helps students appreciate the power of symbolic notation.

Jog-a-Thon	The purpose of this task is to present students with a situation where it is natural
Problem-Solving Task	to add fractions with unlike denominators. Teachers should anticipate two types
	of solutions: one where students calculate the distance Alex ran to determine an
	answer, and one where students compare the two parts of his run to benchmark
Abstract	fractions.
Making S'mores	The purpose of this instructional task is to motivate a discussion about adding
Problem-Solving Task	fractions and the meaning of the common denominator. The different parts of
	the task have students moving back and forth between the abstract
Abstract	representation of the fractions and the meaning of the fractions in the context.
Mixed Numbers with Unlike Denominators	The purpose of this task is to help students realize there are different ways to add
Problem-Solving Task	mixed numbers. The two primary ways one can expect students to add are
	converting the mixed numbers to fractions greater than 1 or adding the whole
Abstract	numbers and fractional parts separately.

Adding Fractions with Unlike Denominators	Students are asked to add two pairs of fractions with unlike denominators.
Abstract	
Adding More Fractions with Unlike Denominators	Students are asked to add pairs of fractions with unlike denominators.
Abstract	
Subtracting Fractions	Students are asked to subtract fractions with unlike denominators.
Abstract	
Subtracting More Fractions	Students are asked to subtract improper fractions and mixed numbers with unlike
	denominators.
Abstract	

MAFS.5.NF.1.2 Solve word problems involving addition and subtraction of fractions referring to the same whole, including cases of unlike denominators, e.g., by using visual fraction models or equations to represent the problem. Use benchmark fractions and number sense of fractions to estimate mentally and assess the reasonableness of answers. For example, recognize an incorrect result 2/5 + 1/2 = 3/7, by observing that 3/7 < 1/2.

Using Models to Subtract Fractions	This lesson is specific to subtracting fractions with unlike denominators. It
Lesson Plan	requires students to already have a working knowledge of subtracting fractions
	with common denominators, and equivalent fractions.
Concrete-Representational	
Aaron and Anya's Discovery	In this situational story, Aaron and Anya find several pieces of ribbon/cord of
Lesson Plan	varying fractional lengths. They decide to choose 3 pieces and make a belt. All of
	the fractions have different denominators; students have to determine common
Representational-Abstract	denominators in order to add the fractional pieces.
Fractions Make the Real World Problems Go Round	In this lesson students will use a graphic organizer to solve addition and
Lesson Plan	subtraction word problems. Students will create their own word problems in
	PowerPoint, by using pen and paper, or dry erase boards to help them
Representational-Abstract	understand the structure of word problems.
Estimating Fractions Using Benchmark Fractions	In this lesson, students use models (fractions tiles or number lines) to estimate
Lesson Plan	fractions using benchmark fractions of 0, 1/2, or 1.
Concrete-Representational-Abstract	
Let's Have a Fraction Party!	In this lesson, students will use addition and subtraction of fractions with unlike
Lesson Plan	denominators to solve word problems involving situations that arise with the
	children who were invited to a party. They will use fraction strips as number
Concrete-Representational-Abstract	models and connect the algorithm with these real-life word problems.

Baking Cakes	Students are asked to estimate the sum of two mixed numbers and then calculate
	the sum.
Abstract	
Sarah's Hike	Students are asked to estimate the difference between two fractional lengths and
	then calculate the difference.
Abstract	
Just Run	Students are given a word problem involving subtraction of fractions with unlike
	denominators. Students are asked to determine if a given answer is reasonable,
Abstract	explain their reasoning, and calculate the answer.
Maria Has a Party	Students are given a word problem involving fractions with unlike denominators
	and are asked to estimate the sum, explain their reasoning, and then determine
Abstract	the sum.

MAFS.5.NF.2.3 Interpret a fraction as division of the numerator by the denominator ( $a/b = a \div b$ ). Solve word problems involving division of whole numbers leading to answers in the form of fractions or mixed numbers, e.g., by using visual fraction models or equations to represent the problem. For example, interpret 3/4 as the result of dividing 3 by 4, noting that 3/4 multiplied by 4 equals 3, and that when 3 wholes are shared equally among 4 people each person has a share of size 3/4. If 9 people want to share a 50-pound sack of rice equally by weight, how many pounds of rice should each person get? Between what two whole numbers does your answer lie?

Picture This! Fractions as Division	In this lesson the student will apply and extend previous understandings of
Lesson Plan	division to represent division as a fraction. This includes representations and
	word problems where the answer is a fraction.
Representational-Abstract	·
Fraction Frenzy!	Students will draw models to solve real-life word problems and show the
Lesson Plan	relationship between division and fractions. By the end of this lesson, they should
	be able to create their own word problems and explain if their answer will be a
Representational-Abstract	mixed number or fractional part.
Sharing Fairly	The students will connect fractions with division. They will solve word problems
Lesson Plan	involving dividing whole numbers by using the strategy of drawing a model
	and/or equations with a fraction or mixed number for the answer.
Representational-Abstract	
How Much Pie?	The purpose of this task is to help students see the connection between a÷b and
Problem-Solving Task	ab in a particular example. The relationship between the division problem 3÷8
	and the fraction 3/8 is actually very subtle.
Representational-Abstract	
What is 23 ÷ 5?	When a division problem involving whole numbers does not result in a whole
Problem-Solving Task	number quotient, it is important for students to be able to decide whether the
	context requires the result to be reported as a whole number with remainder or a
Representational-Abstract	mixed number/decimal.
Those Pesky Remainders	This is a lesson to help students understand how to interpret the remainder in a
Lesson Plan	division problem. Real world problems are presented in a PowerPoint so students
	may visualize situations and discover the four treatments of a remainder.
Concrete-Representational-Abstract	

Sharing Pizzas	Students are asked to draw a visual fraction model to solve a division word
	problem.
Representational-Abstract	
Sharing Brownies	Students are asked to draw a visual fraction model to solve a division word
	problem.
Representational-Abstract	
Five Thirds	Students are asked to interpret an improper fraction and then write a word
	problem to match the context of the fraction.
Abstract	
Two Thirds	Students are asked to interpret a fraction and write a word problem to match the
	context of the fraction.
Abstract	

MAFS.5.NF.2.4 Apply and extend previous understandings of multiplication to multiply a fraction or whole number by a fraction.

- a. Interpret the product  $(a/b) \times q$  as a parts of a partition of q into b equal parts; equivalently, as the result of a sequence of operations  $a \times q \div b$ . For example, use a visual fraction model to show  $(2/3) \times 4 = 8/3$ , and create a story context for this equation. Do the same with  $(2/3) \times (4/5) = 8/15$ . (In general,  $(a/b) \times (c/d) = ac/bd$ .)
- b. Find the area of a rectangle with fractional side lengths by tiling it with unit squares of the appropriate unit fraction side lengths, and show that the area is the same as would be found by multiplying the side lengths. Multiply fractional side lengths to find areas of rectangles, and represent fraction products as rectangular areas.

Folding Strips of Paper	The purpose of this task is to provide students with a concrete experience they
Problem-Solving Task	can relate to fraction multiplication. Perhaps more importantly, the task also
	purposefully relates length and locations of points on a number line, a common
Concrete-Representational	trouble spot for students.
Modeling Fraction Multiplication	This lesson involves students modeling fraction multiplication with rectangular
Lesson Plan	arrays in order to discover the rule for multiplication of fractions.
Representational-Abstract	
Area Models: Multiplying Fractions	In this lesson students will use area models to develop understanding of the
Lesson Plan	concept of multiplying fractions as well as to find the product of two common
	fractions. The teacher will use GeoGebra to provide students with a visual
Representational-Abstract	representation of how area models can be used for multiplying fractions.
Multiplying Fractions with GeoGebra Using Area Models	In this lesson, students will derive an algorithm for multiplying fractions by using
Lesson Plan	area models. They will use a GeoGebra applet to visualize fraction multiplication.
	They will also translate between pictorial and symbolic representations of
Representational-Abstract	fraction multiplication.
Multiplying a Fraction by a Fraction	Students will find the fraction of a fraction. They will also know how to find a
Lesson Plan	fraction of a mixed number. The students will section off a square through rows
	and columns that will prove the strategy of multiplying numerators and then
Representational-Abstract	denominators.

Connor and Makayla Discuss Multiplication	The purpose of this task is to have students think about the meaning of
Problem-Solving Task	multiplying a number by a fraction, and use this understanding of fraction
	multiplication to make sense of the commutative property of multiplication in the
Representational-Abstract	case of fractions.
Painting a Wall	The purpose of this task is for students to find the answer to a question in context
Problem-Solving Task	that can be represented by fraction multiplication. This task is appropriate for
	either instruction or assessment depending on how it is used and where students
Representational-Abstract	are in their understanding of fraction multiplication.
Garden Variety Fractions	Students explore the multiplication of a fraction times a fraction through story
Lesson Plan	problems about a garden using models on Geoboards and pictorial
	representations on grid paper. Students make a connection between their models
Concrete-Representational-Abstract	and the numerical representation of the equation.

Multiplying Fractions by Fractions	Students are asked to consider an equation involving multiplication of fractions, then create a visual fraction model, and write a story context to match.
Representational-Abstract	
Multiplying Fractions by Whole Numbers	Students are asked to consider an equation involving multiplication of a fraction by a whole number and create a visual fraction model. Additionally, the student is
Representational-Abstract	asked to interpret multiplying the number of parts by the whole number.
<u>Using Visual Fraction Models</u>	Students interpret a visual fraction model showing multiplication of two fractions less than one.
Representational-Abstract	
The Rectangle	Students determine the area of a rectangle with given fractional dimensions by multiplying. Students are then asked to draw a model to find the area of the same
Representational-Abstract	rectangle.

#### MAFS.5.NF.2.5 Interpret multiplication as scaling (resizing), by:

- a. Comparing the size of a product to the size of one factor on the basis of the size of the other factor, without performing the indicated multiplication.
- b. Explaining why multiplying a given number by a fraction greater than 1 results in a product greater than the given number (recognizing multiplication by whole numbers greater than 1 as a familiar case); explaining why multiplying a given number by a fraction less than 1 results in a product smaller than the given number; and relating the principle of fraction equivalence a/b =  $(n \times a)/(n \times b)$  to the effect of multiplying a/b by 1.

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Multiplying a Fraction by a Fraction	In this lesson, students will solve problems related to training for a marathon to
Lesson Plan	apply and make sense of multiplying fractions. This lesson utilizes the models as a
	visual representation and moves towards the standard algorithm (a/b) $x$ (c/d) =
Representational-Abstract	ac/bd.
Running a Mile	The two solutions reflect different competencies described in 5.NF.5. The first
Problem-Solving Task	solution uses the idea that multiplying by a fraction less than 1 results in a smaller
	value. The second actually uses the meaning of multiplying to explain why
Representational-Abstract	multiplying by that fraction will result in a smaller value.
<u>Calculator Trouble</u>	This particular problem deals with multiplication. Even though students can solve
Problem-Solving Task	this problem by multiplying, it is unlikely they will. Here it is much easier to
	answer the question if you can think of multiplying a number by a factor as
Abstract	scaling the number.
Comparing a Number and a Product	The purpose of this task is for students to compare a number and its product with
Problem-Solving Task	other numbers that are greater than and less than one.
Abstract	
Reasoning about Multiplication	This is task is meant to work with kids to try to explain their thinking clearly and
Problem-Solving Task	precisely. Teachers should be willing to work with many different ways of
	explaining the relationship between the magnitude of the factors and the
Abstract	magnitude of the product.

More Than or Less Than Two Miles	Students are asked to reason about the size of the product of fractions and whole numbers presented in context.
Representational-Abstract	
Estimating Products	Students are given three products, each involving a whole number and a fraction, and are asked to estimate the size of the product and explain their reasoning.
Abstract	
Multiplying by a Fraction Greater Than One	Students are asked to describe the size of a product of a fraction greater than one and a whole number without multiplying.
Abstract	
Multiplying by a Fraction Less Than One	Students are asked to describe the size of a product of a fraction less than one and a whole number without multiplying.
Abstract	

<u>MAFS.5.NF.2.6</u> Solve real world problems involving multiplication of fractions and mixed numbers, e.g., by using visual fraction models or equations to represent the problem.

Real-World Fractions	This lesson focuses on providing students with real-world experiences where they
Lesson Plan	will be required to multiply fractions. A variety of situational problems involving
	scaling are provided for different levels of learners. Students will be required to
Representational-Abstract	use visual fraction models or equations to represent the problem.
Running to School	This task asks for students to subdivide a quantity into equal parts in order to find
Problem-Solving Task	a fraction of the quantity. Such solutions should be introduced if students do not
	generate them on their own. Students benefit from reasoning through the
	solution to a complex word problems before they are told how they can be
Representational-Abstract	solved.
<u>Drinking Juice</u>	This is task involves fraction multiplication that can be solved with pictures or
Problem-Solving Task	number lines. This task does require students to subdivide the unit fractions that
	comprise 1/2 in order to find 3/4 of 1/2.
Representational-Abstract	
Half of a Recipe	This problem involves fraction multiplication that can be solved with pictures or
Problem-Solving Task	number lines. This task requires subdivision and involves multiplying a fraction
	and a mixed number.
Representational-Abstract	
Making Cookies	This tasks lends itself very well to multiple solution methods. Students may learn
Problem-Solving Task	a lot by comparing different methods. Students who are already comfortable with
	fraction multiplication can go straight to the numeric solutions given below.
	Students who are still unsure of the meanings of these operations can draw
Abstract	pictures or diagrams.

Pizza Party	Students are asked to solve a word problem by finding the product of two
	fractions.
Representational-Abstract	
Box Factory	Students are asked to solve a word problem by finding the product of two
	fractions.
Representational-Abstract	
Half of a Recipe	Students are asked to solve a word problem by finding the product of a fraction
	and a mixed number.
Abstract	
Candy at the Party	Students are asked to solve a word problem by finding the product of two mixed
	numbers.
Abstract	

MAFS.5.NF.2.7 Apply and extend previous understandings of division to divide unit fractions by whole numbers and whole numbers by unit fractions.

- a. Interpret division of a unit fraction by a non-zero whole number, and compute such quotients. For example, create a story context for  $(1/3) \div 4$ , and use a visual fraction model to show the quotient. Use the relationship between multiplication and division to explain that  $(1/3) \div 4 = 1/12$  because  $(1/12) \times 4 = 1/3$ .
- b. Interpret division of a whole number by a unit fraction, and compute such quotients. For example, create a story context for  $4 \div (1/5)$ , and use a visual fraction model to show the quotient. Use the relationship between multiplication and division to explain that  $4 \div (1/5) = 20$  because  $20 \times (1/5) = 4$ .
- c. Solve real world problems involving division of unit fractions by non-zero whole numbers and division of whole numbers by unit fractions, e.g., by using visual fraction models and equations to represent the problem. For example, how much chocolate will each person get if 3 people share 1/2 lb. of chocolate equally? How many 1/3-cup servings are in 2 cups of raisins?

Banana Pudding	The purpose of this task is to provide students with a situation they can model by
Problem-Solving Task	dividing a whole number by a unit fraction. For students who are just beginning to
	think about the meaning of division by a unit fraction (or students who have
	never cooked), the teacher can bring in a 1/4 cup measuring cup so that students
Concrete	can act it out.
It's My Party and I'll Make Dividing by Fractions Easier	During this lesson students will relate their understanding of whole number
Lesson Plan	division situations to help them interpret situations involving dividing by unit
	fractions. They will then develop models and strategies for representing the
Representational-Abstract	division of a whole number by a unit fraction.
Origami Stars	The purpose of this task is to present students with a situation in which they need
Problem-Solving Task	to divide a whole number by a unit fraction in order to find a solution. Calculating
	the number of origami stars that Avery and Megan can make illustrates students'
Representational-Abstract	understanding of the process of dividing a whole number by a unit fraction.
	Students use visual models to show their understanding.
Painting a Room	The purpose of this task is to provide students with a situation in which it is
Problem-Solving Task	natural for them to divide a unit fraction by a non-zero whole number.
	Determining the amount of paint that Kulani needs for each wall illustrates an
Representational-Abstract	understanding of dividing a unit fraction by a non-zero whole number.

Dividing by One-Half	This task requires students to recognize both "number of groups unknown" and
Problem-Solving Task	"group size unknown" division problems in the context of a whole number
	divided by a unit fraction. It also addresses a common misconception that
	students have where they confuse dividing by 2 or multiplying by 1/2 with
Abstract	dividing by 1/2.
How Many Servings of Oatmeal?	This task provides a context for performing division of a whole number by a unit
Problem-Solving Task	fraction. This problem is a "How many groups?" example of division: the "groups"
	in this case are the servings of oatmeal and the question is asking how many
Abstract	servings (or groups) there are in the package.

Fractions Divided by Whole Numbers	Students are given a division expression and asked to write a story context to match the expression and use a visual fraction model to solve the problem.
Representational-Abstract	
Whole Numbers Divided by Fractions	Students are given a division expression and asked to write a story context to match the expression and use a visual fraction model to solve the problem.
Representational-Abstract	
Bags of Fudge	Students are asked to solve a word problem involving division of a whole number by a fraction.
Abstract	
Relay Race	Students are asked to solve a word problem involving division of a fraction by a whole number.
Abstract	