Florida’s B.E.S.T. Standards for K-5 Mathematics

July 21, 2020
Pop Quiz

- What strands are within the B.E.S.T. Standards for Mathematics?
  - Respond at PollEv.com/cstarling825
  - Text CSTARLING825 to 22333 once to join, then text your message
Objectives

• Gain an understanding of the Florida’s B.E.S.T. Standards for K-5 Mathematics
  • Benchmark language, examples and clarifications
  • Horizontal and vertical progressions
  • Appendices
Current Instructional Design

Standard → Content → Instruction
B.E.S.T. Instructional Design
B.E.S.T. Standards for K-5 Mathematics: Benchmark Language, Examples & Clarifications
# Intentional Benchmark Language

<table>
<thead>
<tr>
<th>Current Language</th>
<th>B.E.S.T. Language</th>
</tr>
</thead>
<tbody>
<tr>
<td>“the” standard algorithm</td>
<td>“a” standard algorithm</td>
</tr>
<tr>
<td>“shapes”, “objects”, “figures”</td>
<td>two- and three-dimensional figures</td>
</tr>
<tr>
<td>picture graph</td>
<td>pictograph</td>
</tr>
<tr>
<td>half circle</td>
<td>semi-circle</td>
</tr>
<tr>
<td>“count to 100” [K only]</td>
<td>“recite number names to 100” [K only]</td>
</tr>
</tbody>
</table>
Benchmark Examples

- Written as example questions that could be used as tasks within the classroom or ways to solve a problem that could be used to understand intention of benchmark

Benchmark Clarifications

- Written to support instruction, including the extent of which benchmark should be taught
Examples

Combine two-dimensional figures to form a given composite figure.

MA.K.GR.1.5  Figures used to form a composite shape are limited to triangles, rectangles and squares.

Example: Two triangles can be used to form a given rectangle.

Benchmark Clarifications:
Clarification 1: This benchmark is intended to develop the understanding of spatial relationships.

Find the value of combinations of pennies, nickels and dimes up to one dollar, and the value of combinations of one, five and ten dollar bills up to $100. Use the ¢ and $ symbols appropriately.

Benchmark Clarifications:
Clarification 1: Instruction includes the identification of a one, five and ten-dollar bill and the computation of the value of combinations of pennies, nickels and dimes or one, five and ten dollar bills.
Clarification 2: Instruction focuses on the connection to place value and skip counting.
Clarification 3: Within this benchmark, the expectation is not to use decimal values or to find the value of a combination of coins and dollars.
Examples

MA.2.FR.1.1 Partition circles and rectangles into two, three or four equal-sized parts. Name the parts using appropriate language, and describe the whole as two halves, three thirds or four fourths.

Benchmark Clarifications:
Clarification 1: Within this benchmark, the expectation is not to write the equal-sized parts as a fraction with a numerator and denominator.
Clarification 2: Problems include mathematical and real-world context.

MA.3.AR.1.2 Solve one- and two-step real-world problems involving any of four operations with whole numbers.

Example: A group of students are playing soccer during lunch. How many students are needed to form four teams with eleven players each and to have two referees?

Benchmark Clarifications:
Clarification 1: Instruction includes understanding the context of the problem, as well as the quantities within the problem.
Clarification 2: Multiplication is limited to factors within 12 and related division facts. Refer to Situations Involving Operations with Numbers (Appendix A).
MA.4.NSO.1.4  
Round whole numbers from 0 to 10,000 to the nearest 10, 100 or 1,000.

*Example:* The number 6,325 is rounded to 6,300 when rounded to the nearest 100.

*Example:* The number 2,550 is rounded to 3,000 when rounded to the nearest 1,000.

MA.5.DP.1.2  
Interpret numerical data, with whole-number values, represented with tables or line plots by determining the mean, mode, median or range.

*Example:* Rain was collected and measured daily to the nearest inch for the past week. The recorded amounts are 1, 0, 3, 1, 0, 0 and 1. The range is 3 inches, the modes are 0 and 1 inches and the mean value can be determined as \( \frac{1+0+3+1+0+0+1}{7} \) which is equivalent to \( \frac{6}{7} \) of an inch. This mean would be the same if it rained \( \frac{6}{7} \) of an inch each day.

**Benchmark Clarifications:**

*Clarification 1:* Instruction includes interpreting the mean in real-world problems as a leveling out, a balance point or an equal share.
Horizontal & Vertical Progressions
Horizontal Progression

- Each grade level or course indicates the areas of emphasis
- Intentional progression of content within the strands and across the strands

In grade 2, instructional time will emphasize four areas:

1. extending understanding of place value in three-digit numbers;
2. building fluency and algebraic reasoning with addition and subtraction;
3. extending understanding of measurement of objects, time and the perimeter of geometric figures and
4. developing spatial reasoning with number representations and two-dimensional figures.
# Horizontal Progression – Example

**MA.4.GR.1 Draw, classify and measure angles.**

<table>
<thead>
<tr>
<th>MA.4.GR.1.1</th>
<th>MA.4.GR.1.2</th>
<th>MA.4.GR.1.3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Informally explore angles as an attribute of two-dimensional figures. Identify and classify angles as acute, right, obtuse, straight or reflex.</td>
<td>Estimate angle measures. Using a protractor, measure angles in whole-number degrees and draw angles of specified measure in whole-number degrees. Demonstrate that angle measure is additive.</td>
<td>Solve real-world and mathematical problems involving unknown whole-number angle measures. Write an equation to represent the unknown.</td>
</tr>
</tbody>
</table>
Horizontal Progression – Example

MA.2.NSO.2.3
Add two whole numbers with sums up to 100 with procedural reliability. Subtract a whole number from a whole number, each no larger than 100, with procedural reliability.

MA.2.AR.2.2
Determine the unknown whole number in an addition or subtraction equation, relating three or four whole numbers, with the unknown in any position.

MA.2.M.1.2
Measure the lengths of two objects using the same unit and determine the difference between their measurements.

MA.2.GR.2.2
Find the perimeter of a polygon with whole-number side lengths. Polygons are limited to triangles, rectangles, squares and pentagons.

MA.2.DP.1.2
Interpret data represented with tally marks, tables, pictographs or bar graphs including solving addition and subtraction problems.
In thinking about the prior example, what are some similarities within the benchmarks?

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Responses

What is one similarity within the benchmarks?
Discussion

- In thinking about the prior example, what other benchmarks, within the grade level, would you consider part of the horizontal progression?
  - Respond at PollEv.com/cstarling825
  - Text CSTARLING825 to 22333 once to join, then text your message
Responses

- Benchmarks involving operations
  - MA.2.NSO.2.1
  - MA.2.NSO.2.3
  - MA.2.M.1.3 (Sarasota)
  - MA.2.M.1.3 and MA.2.M.2.2 (Duval All Stars)
  - MA.2.FR.1.1
  - MA.2.AR.1.1
- Benchmarks involving data analysis
Vertical Progression

• Intentional progression of content from one year to the next or across multiple grade levels
  • Progression within the same strand and across strands
• Intentional progression of content among the strands from one grade level to the next
### Vertical Progression – Example

<table>
<thead>
<tr>
<th>MA.K.DP.1</th>
<th>MA.1.DP.1</th>
<th>MA.2.DP.1</th>
<th>MA.3.DP.1</th>
<th>MA.4.DP.1</th>
<th>MA.5.DP.1</th>
</tr>
</thead>
<tbody>
<tr>
<td>Develop an understanding for collecting, representing and comparing data.</td>
<td>Collect, represent and interpret data using pictographs and tally marks.</td>
<td>Collect, categorize, represent and interpret data using appropriate titles, labels and units.</td>
<td>Collect, represent and interpret numerical and categorical data.</td>
<td>Collect, represent and interpret data and find the mode, median and range of a data set.</td>
<td>Collect, represent and interpret data and find the mean, mode, median or range of a data set.</td>
</tr>
</tbody>
</table>
### Vertical Progression – Content Example

<table>
<thead>
<tr>
<th>MA.1.AR.1.2</th>
<th>MA.2.DP.1.2</th>
<th>MA.3.GR.2.4</th>
<th>MA.4.AR.2.2</th>
<th>MA.5.AR.2.1</th>
</tr>
</thead>
<tbody>
<tr>
<td>Solve addition and subtraction real-world problems using objects, drawings or equations to represent the problem.</td>
<td>Interpret data represented with tally marks, tables, pictographs or bar graphs including solving addition and subtraction problems.</td>
<td>Solve mathematical and real-world problems involving the perimeter and area of composite figures composed of non-overlapping rectangles with whole-number side lengths.</td>
<td>Given a mathematical or real-world context, write an equation involving multiplication or division to determine the unknown whole number with the unknown in any position.</td>
<td>Translate written real-world and mathematical descriptions into numerical expressions and numerical expressions into written mathematical descriptions.</td>
</tr>
</tbody>
</table>
Discussion

- In thinking about the prior example, what is one similarity and one difference within this vertical progression?
  
  - Respond at PollEv.com/cstarling825
  - Text CSTARLING825 to 22333 once to join, then text your message
## Responses

<table>
<thead>
<tr>
<th>Similarities</th>
<th>Differences</th>
</tr>
</thead>
<tbody>
<tr>
<td>• Whole numbers</td>
<td>• Specific skills</td>
</tr>
<tr>
<td>• Real world/Problem solving</td>
<td>• Different strands</td>
</tr>
<tr>
<td>• Beginning with operations and then moving into applying the concept</td>
<td>• Complexity</td>
</tr>
<tr>
<td>• Utilizing various representations to model the situations</td>
<td>• Utilizing various representations to model the situations</td>
</tr>
</tbody>
</table>
Horizontal and Vertical Progression

- Within the Number Sense and Operations (NSO) strand progressions developed with a hierarchy in mind consisting of three stages: exploration, procedural reliability and procedural fluency.
- Interwoven into this hierarchy is the development of direct recall of basic arithmetic facts.
Fluency with Arithmetic Operations

- Stage 1: Exploration
  - Develop understanding through the use of manipulatives, visual models, discussions, estimation and drawings

MA.K.NSO.3.1 Explore addition of two whole numbers from 0 to 10, and related subtraction facts.

Benchmark Clarifications:
Clarity 1: Instruction includes objects, fingers, drawings, number lines and equations.
Clarity 2: Instruction focuses on the connection that addition is “putting together” or “counting on” and that subtraction is “taking apart” or “taking from.” Refer to Situations Involving Operations with Numbers (Appendix A).
Clarity 3: Within this benchmark, it is the expectation that one problem can be represented in multiple ways and understanding how the different representations are related to each other.
Fluency with Arithmetic Operations

- **Stage 2: Procedural reliability**
  - Utilize skills from the exploration stage to develop an accurate, reliable method that aligns with the student’s understanding and learning style.

**MA.2.NSO.2.3**

Add two whole numbers with sums up to 100 with procedural reliability. Subtract a whole number from a whole number, each no larger than 100, with procedural reliability.

*Example:* The sum $41 + 23$ can be found by using a number line and “jumping up” by two tens and then by three ones to “land” at 64.

*Example:* The difference $87 - 25$ can be found by subtracting 20 from 80 to get 60 and then 5 from 7 to get 2. Then add 60 and 2 to obtain 62.

**Benchmark Clarifications:**

*Clarification 1:* Instruction focuses on helping a student choose a method they can use reliably.
Stage 3: Procedural fluency

- Utilize skills from the procedural reliability stage to become fluent with an efficient and accurate procedure, including a standard algorithm.

- **MA.5.NSO.2.1** Multiply multi-digit whole numbers including using a standard algorithm with procedural fluency.

- **MA.5.NSO.2.3** Add and subtract multi-digit numbers with decimals to the thousandths, including using a standard algorithm with procedural fluency.
Automaticity with Basic Arithmetic Facts

- Embedded within Stage 1 – Stage 3: Automaticity
  - Directly recall basic arithmetic facts from memory.

<table>
<thead>
<tr>
<th>Standard</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>MA.1.NSO.2.1</td>
<td>Recall addition facts with sums to 10 and related subtraction facts with automaticity.</td>
</tr>
<tr>
<td>MA.2.NSO.2.1</td>
<td>Recall addition facts with sums to 20 and related subtraction facts with automaticity.</td>
</tr>
<tr>
<td>MA.4.NSO.2.1</td>
<td>Recall multiplication facts with factors up to 12 and related division facts with automaticity.</td>
</tr>
</tbody>
</table>
Activity

- Choose one benchmark within the NSO strand that has an expectation of “explore”, “procedural reliability”, “procedural fluency” or “recall”.
  - Determine at least 2 benchmarks from other strands that support the NSO benchmark.
Responses

- **MA.1.NSO.2.1**
  - MA.1.AR.1.1 & MA.1.AR.2.1
  - MA.1.AR.1.1 & MA.1.AR.2.2

- **MA.1.NSO.2.4**
  - MA.1.AR.2.3 & MA.1.M.2.3 (Volusia)

- **MA.2.NSO.2.3**
  - MA.2.M.2.2 & MA.2.AR.1.1

- **MA.3.NSO.2.1**
  - MA.3.AR.1.2
Responses

- MA.3.NSO.2.2
  - MA.3.AR.1.1 & MA.3.M.1.2 (Flagler)
  - MA.3.AR.1.1, MA.3.M.1.2 & MA.3.GR.2.2
  - MA.3.AR.3.2 & MA.3.M.2.2

- MA.3.NSO.2.4
  - MA.3.AR.2.1

- MA.4.NSO.2.3
  - MA.4.AR.1.1 & MA.4.GR.2.1

- MA.4.NSO.2.5
  - MA.4.AR.1.1, MA.4.M.1.2 & MA.4.FR.2.4 (Palm Beach)
Appendices

A. Situations Involving Operations
B. Fluency and Automaticity Chart
C. K-12 Glossary
D. Properties of Operations, Equality and Inequality
E. K-12 Formulas
Discussion

What is one thing within one of the appendices you feel will be useful for K-5 instruction for all students and why?

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Responses

- Situations Involving Operations
  - Allows students to make sense of problem and explore different types of word problems (Sarasota)
  - Foundation for understanding
  - Emphasizes real life situations, making connections

- Fluency Chart
  - Progression clear for teachers, parents and students
  - Includes conceptual understanding which is essential (Pasco Litmatics)
  - Visual for progressions/connections through grade levels (FAMU DRS)
Responses

- K-12 Glossary
  - Common language for learning & clear understanding for teachers, parents and students
  - Images & examples
  - Consistency of math language and definitions
  - Streamlining for common materials and assessments
  - Help to reduce misinterpretations (FAMU DRS)
  - Single definition of a trapezoid #LeeMath

- Properties
  - Teacher knowledge
Using your copy of Florida’s B.E.S.T. Standards for Mathematics, find one thing in each appendix you feel will be useful for K-5 instruction for all students.

A. Situations Involving Operations
B. Fluency and Automaticity Chart
C. K-12 Glossary
D. Properties of Operations, Equality and Inequality
E. K-12 Formulas
Appendix A: Situations Involving Addition & Subtraction

- **Add to**
  - Result unknown, Change unknown, start unknown

- **Take from**
  - Result unknown, Change unknown, start unknown

- **Put together**
  - Total unknown, Addend unknown, Both addends unknown

- **Compare**
  - Difference unknown, bigger unknown, smaller unknown
### Appendix A: Situations Involving Addition & Subtraction

<table>
<thead>
<tr>
<th>Add To</th>
<th>Result Unknown</th>
<th>Change Unknown</th>
<th>Start Unknown</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Three birds sat on a wire. Two more birds landed next to them. How many birds are on the wire now?</td>
<td>Three birds sat on a wire. Some more birds landed next to them. Then there were five birds on the wire. How many birds landed on the wire next to the first three?</td>
<td>Some birds were sitting on a wire. Two more birds landed there. Then there were five birds. How many birds were on the wire to start?</td>
</tr>
<tr>
<td></td>
<td>$3 + 2 = ?$</td>
<td>$3 + ? = 5$</td>
<td>$? + 2 = 5$</td>
</tr>
<tr>
<td>Take From</td>
<td>Five snacks were on the table. Three snacks were eaten. How many snacks are on the table now?</td>
<td>Five snacks were on the table. Some snacks were eaten. Then there were two snacks on the table. How many snacks were eaten?</td>
<td>Some snacks were on the table. Then three snacks were eaten. Now there are two snacks left on the table. How many snacks were on the table at the start?</td>
</tr>
<tr>
<td></td>
<td>$5 - 3 = ?$</td>
<td>$5 - ? = 2$</td>
<td>$? - 3 = 2$</td>
</tr>
</tbody>
</table>
Appendix A: Situations Involving Multiplication & Division

- **Equal groups**
  - Unknown products, Partitive or Fair shares division, Quotative or Measurement division

- **Arrays**
  - Unknown products, Partitive or Fair shares division, Quotative or Measurement division

- **Multiplicative comparisons**
  - Unknown products, Partitive or Fair shares division, Quotative or Measurement division
# Appendix A: Situations Involving Multiplication & Division

<table>
<thead>
<tr>
<th>Equal Groups</th>
<th>Unknown Product</th>
<th>Group Size Unknown (Partitive or Fair Shares Division)</th>
<th>Number of Groups Unknown (Quotative or Measurement Division)</th>
</tr>
</thead>
<tbody>
<tr>
<td>There are 3 bags with 6 plums in each bag. How many plums are there in all?</td>
<td>$3 \times 6 = ?$</td>
<td>$3 \times ? = 18$  $18 \div 3 = ?$</td>
<td>$? \times 6 = 18$  $18 \div 6 = ?$</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Arrays</th>
<th>Unknown Product</th>
<th>Group Size Unknown (Partitive or Fair Shares Division)</th>
<th>Number of Groups Unknown (Quotative or Measurement Division)</th>
</tr>
</thead>
<tbody>
<tr>
<td>There are 3 rows of apples with 6 apples in each row. How many apples are there?</td>
<td></td>
<td>If 18 apples are arranged into 3 equal rows, how many apples will be in each row?</td>
<td>If 18 apples are arranged into equal rows of 6 apples, how many rows will there be?</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Multiplicative Comparisons</th>
<th>Unknown Product</th>
<th>Group Size Unknown (Partitive or Fair Shares Division)</th>
<th>Number of Groups Unknown (Quotative or Measurement Division)</th>
</tr>
</thead>
<tbody>
<tr>
<td>A blue hat costs $6. A red hat costs 3 times as much as the blue hat. How much does the red hat cost?</td>
<td></td>
<td>A red hat costs $18 and that is 3 times as much as a blue hat costs. How much does the blue hat cost?</td>
<td>A red hat costs $18 and a blue hat costs $6. How many times as much does the red hat cost as the blue hat?</td>
</tr>
</tbody>
</table>
Appendix B: Fluency and Automaticity Chart

- Table that provides educators with an overview of procedural fluencies and recall with automaticity within number sense and operations and measurement from Kindergarten to Grade 8

<table>
<thead>
<tr>
<th>Grade Level</th>
<th>Required Procedural Reliability, Procedural Fluency and Basic Fact Automaticity</th>
<th>Measurement</th>
</tr>
</thead>
<tbody>
<tr>
<td>3</td>
<td><strong>Number Sense: Counting and Place Value</strong>&lt;br&gt;Round whole numbers from 0 to 1,000 to the nearest 10 or 100&lt;br&gt;Plot, order and compare:&lt;br&gt;• whole numbers up to 10,000&lt;br&gt;• fractional numbers with the same numerator or the same denominator</td>
<td>Length of an object to the nearest centimeter and half or quarter inch&lt;br&gt;Volume of a liquid within a beaker to the nearest milliliter and half or quarter cup&lt;br&gt;Temperature to the nearest degree</td>
</tr>
<tr>
<td></td>
<td><strong>Operations: Addition and Subtraction</strong>&lt;br&gt;Procedural Fluency: Multi-digit whole numbers, including using a standard algorithm</td>
<td></td>
</tr>
<tr>
<td></td>
<td><strong>Operations: Multiplication and Division</strong>&lt;br&gt;Procedural Reliability: Multiplication of a one-digit whole number by a multiple of 10 up to 90 or a multiple of 100 up to 900&lt;br&gt;Procedural Reliability: Two whole numbers with factors from 0 to 12 and related division facts</td>
<td></td>
</tr>
</tbody>
</table>
Appendix C: K-12 Glossary

- Reference list provided for teachers to support the expectations of the Florida’s B.E.S.T Standards for Mathematics

<table>
<thead>
<tr>
<th>Vocabulary</th>
<th>Definition</th>
<th>Example</th>
</tr>
</thead>
<tbody>
<tr>
<td>equilateral triangle</td>
<td>A triangle with three equal-length sides and three 60-degree interior angles. Also known as an equiangular triangle.</td>
<td><img src="triangle.png" alt="Image" /></td>
</tr>
<tr>
<td>expression</td>
<td>A mathematical statement containing numerals, operators, grouping symbols and symbols or variables for unknown values. An expression does not contain an equal sign or inequality symbol.</td>
<td>(4 \times 2), (\frac{9}{5} - \frac{1}{3})</td>
</tr>
<tr>
<td>factors (of positive whole numbers)</td>
<td>Whole numbers into which a positive whole number can be evenly divided.</td>
<td>1, 3, 5, and 15 are factors of 15. One is a factor of every whole number.</td>
</tr>
<tr>
<td>hexagon</td>
<td>A polygon containing exactly six sides and six vertices.</td>
<td><img src="hexagon.png" alt="Image" /></td>
</tr>
</tbody>
</table>
Appendix D: Properties of Operations, Equality and Inequality

<table>
<thead>
<tr>
<th>Property of Operation</th>
<th>Example</th>
</tr>
</thead>
<tbody>
<tr>
<td>Associative property of addition</td>
<td>((a + b) + c = a + (b + c))</td>
</tr>
<tr>
<td>Commutative property of addition</td>
<td>(a + b = b + a)</td>
</tr>
<tr>
<td>Additive identity property of zero</td>
<td>(a + 0 = a)</td>
</tr>
<tr>
<td></td>
<td>(0 + a = a)</td>
</tr>
<tr>
<td>Existence of additive inverses</td>
<td>For every (a) there exists (-a) so that (a + (-a) = 0) and ((-a) + a = 0).</td>
</tr>
<tr>
<td>Associative property of multiplication</td>
<td>((a \times b) \times c = a \times (b \times c))</td>
</tr>
<tr>
<td>Commutative property of multiplication</td>
<td>(a \times b = b \times a)</td>
</tr>
<tr>
<td>Multiplicative identity property of one</td>
<td>(a \times 1 = a)</td>
</tr>
<tr>
<td></td>
<td>(1 \times a = a)</td>
</tr>
<tr>
<td>Existence of multiplicative inverses</td>
<td>For every (a \neq 0) there exists (\frac{1}{a}) so that (a \times \frac{1}{a} = 1) and (\frac{1}{a} \times a = 1).</td>
</tr>
<tr>
<td>Distributive property of multiplication over addition</td>
<td>(a \times (b + c) = (a \times b) + (a \times c))</td>
</tr>
</tbody>
</table>
Appendix E: K-12 Formulas

- **Area of two-dimensional figures**

<table>
<thead>
<tr>
<th>Area of a two-dimensional figure</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Rectangle</strong></td>
</tr>
<tr>
<td>$A = lw$, where $l$ is the length and $w$ is the width</td>
</tr>
<tr>
<td>$A = bh$, where $b$ is the base and $h$ is the height</td>
</tr>
<tr>
<td><strong>Square</strong></td>
</tr>
<tr>
<td>$A = lw$, where $l$ is the length and $w$ is the width</td>
</tr>
<tr>
<td>$A = bh$, where $b$ is the base and $h$ is the height</td>
</tr>
<tr>
<td>$A = s^2$, where $s$ is the side length</td>
</tr>
</tbody>
</table>
Questions?

- Ashley Harvey, Elementary Mathematics Specialist
  - Email: Ashley.Harvey@fldoe.org
- Courtney Starling, Secondary Mathematics Specialist
  - Email: Courtney.Starling@fldoe.org
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