Florida Interim Assessment
Item Bank and Test Platform

Item Specifications

Mathematics
Grades K–2
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I. Introduction

In July 2010 the Florida Department of Education (FDOE) approved the adoption of the Common Core State Standards (CCSS) for Mathematics to support its pursuit of improved outcomes for all Florida mathematics students and participation in national educational initiatives, such as Race to the Top. The U.S. Department of Education awarded a Race to the Top grant to Florida in August 2010. An important component of this grant focused on the development of high-quality assessment items and balanced assessments for use by districts, schools, and teachers. The assessment items will be stored in the Florida Interim Assessment Item Bank and Test Platform (IBTP), a statewide secure system that allows Florida educators to search the item bank, export test items, and generate customized high-quality assessments for computer-based delivery or paper-and-pencil delivery. The IBTP allows Florida educators to determine what students know and are able to do relative to instruction based on Florida’s Next Generation Sunshine State Standards and the Common Core State Standards.

A. Purpose of the Item Specifications

The Item Specifications define the expectations for content, standards alignment, and format of assessment items for the Item Bank and Test Platform. The Item Specifications are intended for use by item writers and reviewers in the development of high-quality assessment items.

B. Scope

The Item Specifications provide general and grade-specific guidelines for the development of all Mathematics assessment items available in the Florida Interim Assessment Item Bank.

C. Standards Alignment

Items developed for the Florida Interim Assessment Item Bank and Test Platform will align to the Common Core State Standards for Mathematics. The Common Core State Standards for Mathematics are structured into three levels of specificity: Domains, Clusters, and Standards. These define what mathematics students should know and be able to do at every grade level/course, kindergarten through high school.

II. Criteria for Item Development

Mathematics item writers for the Florida Interim Assessment Item Bank must have a comprehensive knowledge of mathematics curriculum based on the Common Core State Standards and an understanding of the range of cognitive abilities of the target student population. Item writers should understand and consistently apply the guidelines established in this document. Item writers are expected to use their best judgment in writing items that measure the Mathematics standards of the CCSS without introducing extraneous elements that reflect bias for or against a group of students.

A. Overall Considerations for Item Development

These guidelines are provided to ensure the development of high-quality assessment items for the Florida Interim Assessment Item Bank.
1. Each item should be written to measure primarily one Common Core State Standard; however, other standards may also be addressed for some item types. In addition to the content standard alignment, each item should align to at least one Mathematical Practice Standard.

2. Items should be appropriate for students in terms of grade-level/course instruction, experience and difficulty, cognitive development, and reading level. The reading level of the test items should be on grade level.

3. Items should be written at or above the cognitive level (DOK) of the standard unless otherwise noted in the Individual Standard Specifications sections.

4. Each item should be written clearly and unambiguously to elicit the desired response.

5. Items should not disadvantage or exhibit disrespect to anyone in regard to age, gender, race, ethnicity, language, religion, socioeconomic status, disability, occupation, or geographic region.

6. At grades kindergarten through 5, items should be able to be answered without using a calculator. For grades 6 through 7, a four-function calculator may be used. For grade 8, a scientific calculator may be used. For Algebra 1, Geometry, and Algebra 2, both a scientific calculator and a graphing calculator (with functionalities similar to that of a TI-84) may be used. For all grades, calculators should not be used for items where computational skills or fluency are being assessed.

B. Item Contexts

The context in which an item is presented is called the item context or scenario. These guidelines are provided to assist item writers with development of items within an appropriate context.

1. The item context should be designed to interest students at the targeted level. Scenarios should be appropriate for students in terms of grade-level experience and difficulty, cognitive development, and reading level.

2. The context should be directly related to the question asked. The context should lead the student cognitively to the question. Every effort should be made to keep items as concise as possible without losing cognitive flow or missing the overall idea or concept.

3. Item contexts should include subject areas other than mathematics. Specifically, topics from grade-level/course Next Generation Sunshine State Standards for Science and Social Studies, and Common Core State Standards for English Language Arts may be used where appropriate.

4. Items including specific information or data must be accurate and verified against reliable sources. Source documentation must accompany these types of items.

5. Mathematics item stimuli should include written text and/or visual material, such as graphs, tables, diagrams, maps, models, and/or other illustrations.

6. All item scenarios, graphics, diagrams, and illustrations must be age-, grade-, and experience-appropriate.
7. All graphs used in item stems or answer options must be complete with title, scale, and labeled axes, except when these components are to be completed by the student.

8. Any graphics in items should be uncluttered and should clearly depict the necessary information. Graphics should contain relevant details that contribute to the students' understanding of the item or that support the context of the item. Graphics should not introduce bias to the item.

9. Item content should be timely but not likely to become dated too quickly.

C. Use of Media

Media can be used to provide either necessary or supplemental information—that is, some media contain information that is necessary for answering the question, while other media support the context of the question. Items may include diagrams, illustrations, charts, tables, audio files, or video files unless otherwise noted in the Individual Standard Specifications. Some standards require a heavier use of graphics than others. Geometry, for example, relies heavily on graphics to convey information.

1. Items should not begin with media. Media in items are always preceded by text.

2. All visual media (tables, charts, graphs, photographs, etc.) should be titled. Titles should be in all caps, boldfaced, and centered, and may be placed above or below the visual media.

D. Item Style and Format

This section presents stylistic guidelines and formatting directions that should be followed while developing items.

1. Items should be clear and concise, and they should use vocabulary and sentence structure appropriate for the assessed grade level.

2. The words most likely or best should be used only when appropriate to the question.

3. Items using the word not should emphasize the word not using all uppercase letters (e.g., Which of the following is NOT an example of . . . ). The word not should be used sparingly.

4. For items that refer to an estimate (noun), lowercase letters should be used.

5. As appropriate, boldface type should be used to emphasize key words in the item (e.g., least, most, greatest, percent, mode, median, mean, range).

6. Masculine pronouns should NOT be used to refer to both sexes. Plural forms should be used whenever possible to avoid gender-specific pronouns (e.g., instead of “The student will make changes so that he . . . ,” use “The students will make changes so that they . . . ”).

7. An equal balance of male and female names should be used, including names representing different ethnic groups appropriate for Florida.

8. For clarity, operation symbols, equality signs, and ordinates should be preceded and followed by one space.
9. Decimal numbers between –1 and 1 (including currency) should have a leading zero.

10. Metric numbers should be expressed in a single unit when possible (e.g., 1.4 kilograms instead of 1 kilogram 400 grams).

11. Decimal notation should be used for numbers with metric units (e.g., 1.2 grams instead of 151 grams).

12. Commas should be used within numbers greater than or equal to 1,000. Commas may be omitted within an equation or expression.

13. Units of measure should be spelled out, except in graphics, where an abbreviation may be used (e.g., \(\text{ft}\) or \(\text{yd}\)). Abbreviations that also spell a word must be followed by a period to avoid confusion. For example, to avoid confusion with the preposition \(\text{in}\), the abbreviation \(\text{in.}\) should be used for the unit of measure \(\text{inches}\) and should include a period. If an abbreviation is used in a graphic, an explanation of the meaning of the abbreviation should be included in the stem.

14. In titles for tables and charts and in labels for axes, the units of measure should be included, preferably in lowercase letters and in parentheses, e.g., \(\text{height (in inches)}\).

15. Fractions should be typed with a horizontal fraction bar. The numerator and denominator should be centered with respect to each other. The bar should cover all portions (superscripts, parentheses, etc.) of the numerator and denominator. In a mixed number, a half space should appear between the whole number and the fraction. If a variable appears before or after a fraction bar, the variable should be centered with respect to the fraction bar. If a stimulus, stem, or set of responses contains a fraction in fractional notation, that portion of the item should be 1.5-spaced.

16. In general, numbers zero through nine should be presented as words and numbers 10 and above should be presented as numerals. In the item stem, any numbers needed to compute answers should be presented as numerals.

E. Item Types

This section presents guidelines for development of the following types of items:

1. Selected Response (SR) Items (1 point)
2. Gridded Response (GR) and Short Response (SHR) Items (1 point)
3. Constructed Response and Extended Response Items
   a. Constructed Response (CR) Items (2 points)
   b. Extended Response (ER) Items (4 points)
4. Essay Response (ESR) Items (6 points)
5. Performance Task (PT) Items (1–10 points)
1. **Selected Response (SR) Items (1 point)**

Selected response items require students to choose an answer from the choices given. Each item consists of a stem and either three or four answer options, depending on the grade level/course (see c below). One of the answer options is the correct answer, and the remaining options are called distractors. Selected response items may include a stimulus and/or passage.

a. SR items should take an average of 1 minute per item to solve.

b. SR items are worth 1 point each.

c. SR items in grades K, 1, and 2 should have three answer choices (A, B, and C). SR items for all other grades and courses should have four answer choices (A, B, C, and D).

d. Answer options that are single words should be arranged in alphabetical or reverse alphabetical order.

e. Answer options that are phrases or sentences should be arranged from shortest to longest or longest to shortest.

f. Numerical answer options should be arranged in ascending or descending order.

g. Numerical answer options that represent relative magnitude or size should be arranged as they are shown in the stem or some other logical order.

h. When the item requires the identification of a choice from the item stem, table, chart, or illustration, the options should be arranged as they are presented in the item stem, table, chart, or illustration.

i. If the answer options for an item are neither strictly numerical nor denominate numbers, the options should be arranged by the logic presented in the item, by alphabetical order, or by length.

j. Distractor rationales should represent computational or conceptual errors or misconceptions commonly made by students who have not mastered the assessed concepts.

k. Outliers (i.e., answer choices that are longer phrases or sentences than the other choices, or choices with significantly more/fewer digits than the other choices) should NOT be used.

l. Options such as none of the above, all of the above, not here, not enough information, or cannot be determined should not be used as answer options.
2. Gridded Response (GR) and Short Response (SHR) Items (1 point)
   a. Gridded response and short response items are worth 1 point.
   b. The GR format is designed for items that require a positive numeric solution (whole numbers, decimals, percents, or fractions).
   c. The bubble grids used with GR items should contain eight columns. Each column will contain the digits 0 through 9, decimal point (.), and fraction bar (/) enclosed in bubbles.
   d. Gridded response items should include instructions that specify the unit in which the answer is to be provided (e.g., inches). If several units of measure are in the item (e.g., in an item involving a conversion), the final unit needed for the answer should be written in boldface.
   e. The short response format is designed for items that result in a value or answer that cannot be answered in the gridded response format (negative numbers, expressions, etc.).

3. Constructed Response and Extended Response Items
   Mathematics constructed response and extended response items require students to produce a response in words, pictures, diagrams, and/or numbers. As such, these items are especially suited to assessing many of the more complex tasks and high-level thinking skills demanded by the Common Core State Standards for Mathematics. The Florida Interim Assessment Item Bank will include 2-point constructed response items (CR) and 4-point extended response items (ER).
   Overall characteristics for mathematics CRs and ERs are as follows:
   a. The item should measure understanding and insight of mathematical concepts rather than rote memory or factual recall.
   b. Real-world, factual stimulus material (charts, graphs, tables, etc.) must cite the source used.
   c. Items requiring students to produce responses as pictures, diagrams, graphs, tables, etc., should provide workspace and/or templates where appropriate.
a. Constructed Response (CR) Items (2 points)

Constructed response items usually include a scenario and instructions on how to respond. The recommended time allotment for a student to respond is 5 minutes. A complete answer is worth 2 points, and a partial answer is worth 1 point. The constructed response holistic rubric and exemplar specific to each item are used for scoring as follows:

<table>
<thead>
<tr>
<th>SCORING RUBRIC</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
</tr>
<tr>
<td>1</td>
</tr>
<tr>
<td>0</td>
</tr>
</tbody>
</table>

**Exemplars:** A specific exemplar should be developed for each constructed response item. Exemplars will be used as scoring guides and should be specific to the item, but not so specific as to discount multiple correct answers. Exemplars should include a clear and defensible description of the top score point, and contain straightforward language that is accurate, complete, and easy to interpret.
b. **Extended Response (ER) Items (4 points)**

Extended response items include a scenario and instructions on how to respond and are worth 4 points. However, ER items are usually more complex than SHR and 2-point CR items. The recommended time allotment for a student to respond is 10–15 minutes. The extended response holistic rubric and exemplar specific to each item are used for scoring as follows:

<table>
<thead>
<tr>
<th><strong>SCORING RUBRIC</strong></th>
</tr>
</thead>
<tbody>
<tr>
<td>4</td>
</tr>
<tr>
<td>Work demonstrates a <strong>clear and complete</strong> understanding of the mathematical concepts and/or procedures required by the task. Appropriate strategy is shown with clear and complete explanations and interpretations.</td>
</tr>
<tr>
<td>3</td>
</tr>
<tr>
<td>Work demonstrates a <strong>clear</strong> understanding of the mathematical concepts and/or procedures but is not complete. Appropriate strategy is shown, but explanation or interpretation has minor flaws. <strong>OR</strong> Response is incorrect because of calculation errors. Work and strategy indicate a <strong>clear</strong> demonstration of the problem.</td>
</tr>
<tr>
<td>2</td>
</tr>
<tr>
<td>Response demonstrates a <strong>partial</strong> understanding of the mathematical concepts and/or procedures. Appropriate strategy is shown, but explanation or interpretation has minor flaws.</td>
</tr>
<tr>
<td>1</td>
</tr>
<tr>
<td>Response shows <strong>minimal</strong> understanding of the mathematical concepts and/or procedures or provides no explanation or interpretation for the solution or shows major flaws.</td>
</tr>
<tr>
<td>0</td>
</tr>
<tr>
<td>Response is irrelevant, inappropriate, or not provided.</td>
</tr>
</tbody>
</table>

**Exemplars:** A specific exemplar should be developed for each extended response item. Exemplars will be used as scoring guides and should be specific to the item, but not so specific as to discount multiple correct answers. Exemplars should include a clear and defensible description of the top score point, and contain straightforward language that is accurate, complete, and easy to interpret.
4. Essay Response (ESR) Items (6 points)

The essay response item consists of asking a general question or providing a stimulus (such as an article or research paper on a relevant topic), and asking students to express their thoughts or provide facts about the topic using logic and reason. Essay response items encompass a higher level of thinking and a broader range of skills that includes CCSS literacy standards, which is critical to future success in higher education and the workforce.

In most cases, essay responses will go beyond a single paragraph in length, with a distinct introduction, body, and conclusion. An essay response will be worth a total of 6 points, with a rubric structure similar to that of the 4-point extended response. Students should be given about 20 to 30 minutes to complete each item.

**Exemplars:** A specific exemplar should be developed for each essay response item. Exemplars will be used as scoring guides and should be specific to the item, but not so specific as to discount multiple correct answers. Exemplars should include a clear and defensible description of the top score point, and contain straightforward language that is accurate, complete, and easy to interpret.

5. Performance Tasks (PT) (1–10 points)

Performance tasks are used to measure students’ ability to demonstrate knowledge and skills from one or more CCSS. Specifically, performance tasks may require students to create a product, demonstrate a process, or perform an activity that demonstrates proficiency in Mathematics. They are evaluated using customized scoring exemplars, and each task may be worth 1–10 points.

Performance tasks may have the following characteristics:

a. Performance tasks may cover a short time period or may cover an extended period.

b. Performance tasks must contain clear and explicit directions for understanding and completing the required component tasks and producing the objective output.

c. All tasks, skills, and/or behaviors required by the performance tasks must be objective, observable, and measurable.

d. All necessary equipment, materials, and resources should be referenced within the text of the performance task.

e. Performance tasks should elicit a range of score points.

f. Performance tasks generally require students to organize, apply, analyze, synthesize, and/or evaluate concepts.

g. Performance tasks may measure performance in authentic situations and outside the classroom, where appropriate and practical.

h. Typical response formats include demonstrations, laboratory performance, oral presentations, exhibits, or other products.
i. Every performance task requires a companion exemplar to be used for scoring purposes. Exemplars should meet the following criteria.

   i The exemplars and performance tasks should be developed in tandem to ensure compatibility.

   ii Exemplars must be specific to the individual requirements of each performance task; generic rubrics are not acceptable.

   iii The exemplar must allow for efficient and consistent scoring.

   iv Each part of the performance task must have a clearly stated score point in the exemplar and when a part of the task is divided into sections or requirements, each of those must have a maximum score indicated.

   v The exemplar descriptors consist of an ideal response exemplar and should allow for all foreseeable methods of correctly and thoroughly completing all requirements of the performance task.

F. Readability

Items must be written with readability in mind. In addition, vocabulary must be appropriate for the grade level being tested. The following sources provide information about the reading level of individual words:


G. Cognitive Complexity

1. Overview


2. Levels of Depth of Knowledge for Mathematics

   Level 1 (Recall) includes the recall of information such as a fact, definition, term, or a simple procedure, as well as performing a simple algorithm or applying a formula. That is, in mathematics a one-step, well-defined, or straight algorithmic procedure should be included at this lowest level.

   Some examples that represent but do not constitute all of Level 1 performance are:

   • Count to 100 by ones and by tens.

   • Fluently multiply and divide within 100, using strategies such as the relationship between multiplication and division (e.g., knowing that $8 \times 5 = 40$, one knows $40 \div 5 = 8$).
• Write simple expressions that record calculations with numbers, and interpret numerical expressions without evaluating them. For example, express the calculation “add 8 and 7 and then multiply by 2” as $2 \times (8 + 7)$. Recognize that $3 \times (18932 + 921)$ is three times as large as $18932 + 921$ without having to calculate the indicated sum or product.

• Enter measurement data into a data table.

• Identify the variables indicated in a two-dimensional graph.

**Level 2 (Basic Application of Concepts & Skills)** includes the engagement of some mental processing beyond a habitual response. A Level 2 standard or assessment item requires students to make some decisions as to how to approach the problem or activity, whereas Level 1 requires students to demonstrate a rote response, perform a well-known algorithm, follow a set procedure (like a recipe), or perform a clearly defined series of steps. For example, to compare data requires first identifying characteristics of the objects or phenomenon and then grouping or ordering the objects. Interpreting information from a simple graph, requiring reading information from the graph, also is a Level 2. Interpreting information from a complex graph that requires some decisions on what features of the graph need to be considered and how information from the graph can be aggregated is a Level 3. Caution is warranted in interpreting Level 2 as only skills because some reviewers will interpret skills very narrowly as primarily numerical skills, and such interpretation excludes from this level other skills such as visualization skills and probability skills, which may be more complex simply because they are less common and require more mental processing.

Some examples that represent but do not constitute all of Level 2 performance are:

• Understand that the last number name said tells the number of objects counted. The number of objects is the same regardless of their arrangement or the order in which they were counted.

• Express the length of an object as a whole number of length units by laying multiple copies of a shorter object (the length unit) end to end.

• Measure and estimate liquid volumes and masses of objects using standard units of grams (g), kilograms (kg), and liters (l).

• Apply properties of operations as strategies to add and subtract rational numbers.

• Measure and record data and produce graphs of relevant variables.

• Graph proportional relationships, interpreting the unit rate as the slope of the graph.

**Level 3 (Strategic Thinking & Complex Reasoning)** requires reasoning, planning, using evidence, and a higher level of thinking than the previous two levels. In most instances, requiring students to explain their thinking is a Level 3. Activities that require students to make conjectures are also at this level. The cognitive demands at Level 3 are complex and abstract. The complexity does not result from the fact that there are multiple answers, a possibility for
both levels 1 and 2, but because the task requires more demanding reasoning. However, an activity that has more than one possible answer and requires students to justify the response they give would most likely be a Level 3.

Some examples that represent but do not constitute all of Level 3 performance are:

- Explain why addition and subtraction strategies work, using place value and the properties of operations.
- Interpret the rate of change and initial value of a linear function in terms of the situation it models, and in terms of its graph or a table of values.
- Given a real-world situation, formulate a problem.
- Organize, represent, and interpret data obtained through experiments or observations.
- Formulate a mathematical model to describe a complex phenomenon.
- Justify a solution to a problem.
- Analyze a deductive argument.

**Level 4 (Extended Thinking & Complex Reasoning)** in mathematics involves the application of Level 3 processes and skills over an extended period. This is likely to incorporate demands from other content areas (e.g., English language arts, science) in the development and support of mathematical arguments that describe some real-world phenomenon or situation.

Some examples that represent but do not constitute all of Level 4 performance are:

- Derive a mathematical model to explain a complex phenomenon or make a prediction.
- Complete a project requiring the formulation of questions, devising a plan, collecting data, analyzing the data, and preparing a written report describing the justification of the conclusions reached.

**H. Item Difficulty**

Item writers will not be expected to make a prediction of difficulty for each item created. However, item writers should develop items that reflect a range of difficulty.
I. Universal Design

The application of universal design principles helps develop assessments that are usable to the greatest number of students, including students with disabilities and nonnative speakers of English. To support the goal of providing access to all students, the items in the Florida Interim Assessment Item Bank maximize readability, legibility, and compatibility with accommodations, and item development includes a review for potential bias and sensitivity issues.

Items must allow for the widest possible range of student participation. Item writers must attend to the best practices suggested by universal design, including, but not limited to,

1. reduction in wordiness
2. avoidance of ambiguity
3. selection of reader-friendly construction and terminology
4. consistently applied concept names and graphic conventions

Universal design principles also inform decisions about item layout and design, including, but not limited to, type size, line length, spacing, and graphics.

J. Sample Items

Appendix A of this document contains a selection of sample items. The sample items represent a range of cognitive complexities and item types.
III. Review Procedures for Florida Interim Assessment Item Bank Items

Prior to being included in the Florida Interim Assessment Item Bank, all mathematics items must pass several levels of review as part of the item development process.

A. Review for Item Quality

Assessment items developed for the Florida Interim Assessment Item Bank will be reviewed by Florida educators, the FDOE, and the contractors to ensure the quality of the items, including grade-level/course appropriateness, alignment to the standard, accuracy, and other criteria for overall item quality.

B. Review for Bias and Sensitivity

Items are reviewed by groups of Florida educators generally representative of Florida’s geographic regions and culturally diverse population. Items are reviewed for the following kinds of bias: gender, racial, ethnic, linguistic, religious, geographic, and socioeconomic. Item reviews also include consideration of issues related to individuals with disabilities.

This review is to ensure that the primary purpose of assessing student achievement is not undermined by inadvertently including in the item bank any material that students, parents, or other stakeholders may deem inappropriate. Reviewers are asked to consider the variety of cultural, regional, philosophical, political, and religious backgrounds throughout Florida and to determine whether the subject matter will be acceptable to Florida students, their parents, and other members of Florida communities.

IV. Guide to the Individual Standard Specifications

A. CCSS Mathematics Standards Classification System

The graphic below demonstrates the coding schema for the Common Core State Standards for Mathematics.

Using this schema:

Subject Code MACC: Mathematics Common Core
Grade: Kindergarten
Domain CC: Counting and Cardinality
Cluster 1: Know number names and the count sequence.
Standard 1: Count to 100 by ones and by tens.
Using the schema, the bottom row refers to:

Subject Code MACC: Mathematics Common Core
Grade: High school Grades 9–12
Category A: Algebra
Domain APR: Arithmetic with Polynomials and Rational Expressions
Cluster 1: Perform arithmetic operations on polynomials.
Standard 1: Understand that polynomials form a system analogous to the integers, namely, they are closed under the operations of addition, subtraction, and multiplication; add, subtract, and multiply polynomials.

B. Definitions of Cluster and Standard Specifications

The Item Specifications identify how the standards in the CCSS are assessed by items in the Florida Interim Assessment Item Bank. For each assessed standard, the following information is provided in the Individual Standards Specifications section.

<table>
<thead>
<tr>
<th>Domain</th>
<th>refers to larger groups of related standards. Standards from different domains may sometimes be closely related.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cluster</td>
<td>refers to groups of related standards. Note that standards from different clusters may sometimes be closely related because mathematics is a connected subject.</td>
</tr>
<tr>
<td>Standards</td>
<td>define what students should understand and be able to do.</td>
</tr>
<tr>
<td>Standards Clarifications/Content Limits</td>
<td>Standards clarifications, when needed as an explanation for some of the standards listed above, explain the type of behavior that the student should exhibit for mastery of the standard. The clarification statements explain what students are expected to do when responding to the question. Content limits define the range of content knowledge and degree of difficulty that should be assessed in the items for the standard. Content limits may be used to identify content beyond the scope of the targeted standard if the content is more appropriately assessed by another standard. These statements help to provide validity by ensuring the test items are clearly aligned to the targeted standard.</td>
</tr>
</tbody>
</table>
V. Individual Standards Specifications for Florida Interim Assessment Item Bank Mathematics Items

This section of the Item Specifications provides standard-specific guidance for assessment item development for the Florida Interim Assessment Item Bank based on the Common Core State Standards.

Each item developed for the Florida Interim Assessment Item Bank and Test Platform should assess one or more of the Mathematical Practice Standards listed in Appendix B.

A. Kindergarten Item Specifications

<table>
<thead>
<tr>
<th>Domain</th>
<th>COUNTING AND CARDINALITY</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cluster</td>
<td>Know number names and the count sequence.</td>
</tr>
<tr>
<td>Standards</td>
<td>MACC.K.CC.1.1—Count to 100 by ones and by tens.</td>
</tr>
<tr>
<td></td>
<td>MACC.K.CC.1.2—Count forward beginning from a given number within the known sequence (instead of having to begin at 1).</td>
</tr>
<tr>
<td></td>
<td>MACC.K.CC.1.3—Write numbers from 0 to 20. Represent a number of objects with a written numeral 0–20 (with 0 representing a count of no objects).</td>
</tr>
<tr>
<td>Standards Clarifications/Content Limits</td>
<td>As an explanation for some of the standards listed above, in Grade K, students will</td>
</tr>
<tr>
<td></td>
<td>• supply missing numbers in or extend counting sequences counting by ones</td>
</tr>
<tr>
<td></td>
<td>• extend counting sequences to 20 when a number of objects are pictured</td>
</tr>
<tr>
<td></td>
<td>• write the numerals in order from 0 to 20, beginning at any number</td>
</tr>
<tr>
<td>Domain</td>
<td>COUNTING AND CARDINALITY</td>
</tr>
<tr>
<td>----------------------</td>
<td>--------------------------</td>
</tr>
<tr>
<td>Cluster</td>
<td>Count to tell the number of objects.</td>
</tr>
<tr>
<td>Standards</td>
<td></td>
</tr>
<tr>
<td></td>
<td>MACC.K.CC.2.4—Understand the relationship between numbers and quantities; connect counting to cardinality.</td>
</tr>
<tr>
<td></td>
<td>MACC.K.CC.2.4.a—When counting objects, say the number names in the standard order, pairing each object with one and only one number name and each number name with one and only one object.</td>
</tr>
<tr>
<td></td>
<td>MACC.K.CC.2.4.b—Understand that the last number name said tells the number of objects counted. The number of objects is the same regardless of their arrangement or the order in which they were counted.</td>
</tr>
<tr>
<td></td>
<td>MACC.K.CC.2.4.c—Understand that each successive number name refers to a quantity that is one larger.</td>
</tr>
<tr>
<td></td>
<td>MACC.K.CC.2.5—Count to answer “how many?” questions about as many as 20 things arranged in a line, a rectangular array, or a circle, or as many as 10 things in a scattered configuration; given a number from 1–20, count out that many objects.</td>
</tr>
<tr>
<td>Standards Clarifications/Content Limits</td>
<td>As an explanation for some of the standards listed above, in Grade K, students will</td>
</tr>
<tr>
<td></td>
<td>• for standard MACC.K.CC.2.5, count or identify 10 or fewer scattered objects</td>
</tr>
<tr>
<td></td>
<td>• for standard MACC.K.CC.2.5, count or identify 1 through 20 objects arranged in a line, array, or circle</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Domain</th>
<th>COUNTING AND CARDINALITY</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cluster</td>
<td>Compare numbers.</td>
</tr>
<tr>
<td>Standards</td>
<td></td>
</tr>
<tr>
<td></td>
<td>MACC.K.CC.3.6—Identify whether the number of objects in one group is greater than, less than, or equal to the number of objects in another group, e.g., by using matching and counting strategies.</td>
</tr>
<tr>
<td></td>
<td>MACC.K.CC.3.7—Compare two numbers between 1 and 10 presented as written numerals.</td>
</tr>
<tr>
<td>Standards Clarifications/Content Limits</td>
<td>As an explanation for some of the standards listed above, in Grade K, students will</td>
</tr>
<tr>
<td></td>
<td>• use numerals and pictures of objects to compare</td>
</tr>
<tr>
<td>Domain</td>
<td>OPERATIONS AND ALGEBRAIC THINKING</td>
</tr>
<tr>
<td>------------------------</td>
<td>-----------------------------------</td>
</tr>
<tr>
<td>Cluster</td>
<td>Understand addition as putting together and adding to, and understand subtraction as taking apart and taking from.</td>
</tr>
</tbody>
</table>
| Standards              | MACC.K.OA.1.1—Represent addition and subtraction with objects, fingers, mental images, drawings, sounds (e.g., claps), acting out situations, verbal explanations, expressions, or equations.  
MACC.K.OA.1.2—Solve addition and subtraction word problems, and add and subtract within 10, e.g., by using objects or drawings to represent the problem.  
MACC.K.OA.1.3—Decompose numbers less than or equal to 10 into pairs in more than one way, e.g., by using objects or drawings, and record each decomposition by a drawing or equation (e.g., \(5 = 2 + 3\) and \(5 = 4 + 1\)).  
MACC.K.OA.1.4—For any number from 1 to 9, find the number that makes 10 when added to the given number, e.g., by using objects or drawings, and record the answer with a drawing or equation.  
MACC.K.OA.1.5—Fluently add and subtract within 5. |
| Standards Clarifications/Content Limits | As an explanation for some of the standards listed above, in Grade K, students will  
• fluently add and subtract within 5  
• understand and apply addition and subtraction through 10  
• add and subtract for understanding and/or problem solving through 10 |

<table>
<thead>
<tr>
<th>Domain</th>
<th>NUMBER AND OPERATIONS IN BASE TEN</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cluster</td>
<td>Work with numbers 11–19 to gain foundations for place value.</td>
</tr>
<tr>
<td>Standards</td>
<td>MACC.K.NBT.1.1—Compose and decompose numbers from 11 to 19 into ten ones and some further ones, e.g., by using objects or drawings, and record each composition or decomposition by a drawing or equation (e.g., (18 = 10 + 8)); understand that these numbers are composed of ten ones and one, two, three, four, five, six, seven, eight, or nine ones.</td>
</tr>
</tbody>
</table>
| Standards Clarifications/Content Limits | In Grade K, students will  
• compose and decompose numbers from 11 through 19  
• use numbers 11 through 19 only |
<table>
<thead>
<tr>
<th>Domain</th>
<th>MEASUREMENT AND DATA</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cluster</td>
<td>Describe and compare measurable attributes.</td>
</tr>
</tbody>
</table>

**Standards**
- MACC.K.MD.1.1—Describe measurable attributes of objects, such as length or weight. Describe several measurable attributes of a single object.
- MACC.K.MD.1.2—Directly compare two objects with a measurable attribute in common to see which object has “more of”/“less of” the attribute, and describe the difference. *For example, directly compare the heights of two children and describe one child as taller/shorter.*

**Standards Clarifications/Content Limits**
As an explanation for some of the standards listed above, in Grade K, students will
- understand measurable attributes of objects
- compare the measurable attributes of two objects

<table>
<thead>
<tr>
<th>Domain</th>
<th>MEASUREMENT AND DATA</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cluster</td>
<td>Classify objects and count the number of objects in each category.</td>
</tr>
</tbody>
</table>

**Standards**
- MACC.K.MD.2.3—Classify objects into given categories; count the numbers of objects in each category and sort the categories by count.

**Standards Clarifications/Content Limits**
In Grade K, students will
- sort objects, pictures of objects, and shapes into given categories
- count the number of objects in given sets
<table>
<thead>
<tr>
<th>Domain</th>
<th>GEOMETRY</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cluster</td>
<td>Identify and describe shapes (squares, circles, triangles, rectangles, hexagons, cubes, cones, cylinders, and spheres).</td>
</tr>
</tbody>
</table>
| Standards   | MACC.K.G.1.1—Describe objects in the environment using names of shapes, and describe the relative positions of these objects using terms such as above, below, beside, in front of, behind, and next to.  
MACC.K.G.1.2—Correctly name shapes regardless of their orientations or overall size.  
MACC.K.G.1.3—Identify shapes as two-dimensional (lying in a plane, “flat”) or three-dimensional (“solid”). |
| Standards Clarifications/Content Limits | As an explanation for some of the standards listed above, in Grade K, students will  
• name shapes and identify them as two-dimensional or three-dimensional |

<table>
<thead>
<tr>
<th>Domain</th>
<th>GEOMETRY</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cluster</td>
<td>Analyze, compare, create, and compose shapes.</td>
</tr>
</tbody>
</table>
| Standards   | MACC.K.G.2.4—Analyze and compare two- and three-dimensional shapes, in different sizes and orientations, using informal language to describe their similarities, differences, parts (e.g., number of sides and vertices/“corners”), and other attributes (e.g., having sides of equal length).  
MACC.K.G.2.5—Model shapes in the world by building shapes from components (e.g., sticks and clay balls) and drawing shapes.  
MACC.K.G.2.6—Compose simple shapes to form larger shapes. For example, “Can you join these two triangles with full sides touching to make a rectangle?” |
| Standards Clarifications/Content Limits | As an explanation for some of the standards listed above, in Grade K, students will  
• analyze and compare two-dimensional shapes (limited to squares, circles, triangles, rectangles, and hexagons)  
• analyze and compare three-dimensional shapes (limited to cubes, cones, cylinders, and spheres)  
• model shapes from real-world stimuli  
• manipulate two 2-dimensional shapes to form a larger shape |
## B. Grade 1 Item Specifications

<table>
<thead>
<tr>
<th>Domain</th>
<th>OPERATIONS AND ALGEBRAIC THINKING</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cluster</td>
<td>Represent and solve problems involving addition and subtraction.</td>
</tr>
</tbody>
</table>

### Standards

- **MACC.1.OA.1.1**—Use addition and subtraction within 20 to solve word problems involving situations of adding to, taking from, putting together, taking apart, and comparing with unknowns in all positions, e.g., by using objects, drawings, and equations with a symbol for the unknown number to represent the problem.
- **MACC.1.OA.1.2**—Solve word problems that call for addition of three whole numbers whose sum is less than or equal to 20, e.g., by using objects, drawings, and equations with a symbol for the unknown number to represent the problem.

### Standards Clarifications/Content Limits

As an explanation for some of the standards listed above, in Grade 1, students will

- identify, create models, and/or solve problems involving addition of two or three numbers with a sum less than or equal to 20
- identify, create models, and/or solve problems involving subtraction of two numbers through 20
- solve word problems by solving for the unknown using objects, drawings, and equations

For MACC.1.OA.1.1, refer to Table 1 in the CCSS-M glossary.
### Domain

<table>
<thead>
<tr>
<th>Cluster</th>
<th>OPERATIONS AND ALGEBRAIC THINKING</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Cluster</strong></td>
<td>Understand and apply properties of operations and the relationship between addition and subtraction.</td>
</tr>
</tbody>
</table>
| **Standards** | MACC.1.OA.2.3—Apply properties of operations as strategies to add and subtract. *Examples: If 8 + 3 = 11 is known, then 3 + 8 = 11 is also known. (Commutative property of addition.) To add 2 + 6 + 4, the second two numbers can be added to make a ten, so 2 + 6 + 4 = 2 + 10 = 12. (Associative property of addition.)*  
MACC.1.OA.2.4—Understand subtraction as an unknown-addend problem. *For example, subtract 10 − 8 by finding the number that makes 10 when added to 8.* |
| **Standards Clarifications/Content Limits** | As an explanation for some of the standards listed above, in Grade 1, students will  
• apply and/or demonstrate their knowledge of the Commutative and Associative properties of addition or the Additive Identity property of zero as strategies to add and subtract. This can involve models to represent properties.  
• understand and apply these properties but need not use the formal terms  
• understand the concept of subtraction through the relationship between addition and subtraction |

### Domain

<table>
<thead>
<tr>
<th>Cluster</th>
<th>OPERATIONS AND ALGEBRAIC THINKING</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Cluster</strong></td>
<td>Add and subtract within 20.</td>
</tr>
</tbody>
</table>
| **Standards** | MACC.1.OA.3.5—Relate counting to addition and subtraction (e.g., by counting on 2 to add 2).  
MACC.1.OA.3.6—Add and subtract within 20, demonstrating fluency for addition and subtraction within 10. Use strategies such as counting on; making ten (e.g., 8 + 6 = 8 + 2 + 4 = 10 + 4 = 14); decomposing a number leading to a ten (e.g., 13 − 4 = 13 − 3 − 1 = 10 − 1 = 9); using the relationship between addition and subtraction (e.g., knowing that 8 + 4 = 12, one knows 12 − 8 = 4); and creating equivalent but easier or known sums (e.g., adding 6 + 7 by creating the known equivalent 6 + 6 + 1 = 12 + 1 = 13). |
| **Standards Clarifications/Content Limits** | As an explanation for some of the standards listed above, in Grade 1, students will  
• relate some of the various counting strategies to add and subtract  
• solve problems with sums of no more than 20 |
## Domain: OPERATIONS AND ALGEBRAIC THINKING

### Cluster: Work with addition and subtraction equations.

**Standards**

MACC.1.OA.4.7—Understand the meaning of the equal sign, and determine if equations involving addition and subtraction are true or false. For example, which of the following equations are true and which are false? $6 = 6$, $7 = 8 - 1$, $5 + 2 = 2 + 5$, $4 + 1 = 5 + 2$.

MACC.1.OA.4.8—Determine the unknown whole number in an addition or subtraction equation relating three whole numbers. For example, determine the unknown number that makes the equation true in each of the equations $8 + ? = 11$, $5 = [] - 3$, $6 + 6 = []$.

### Standards Clarifications/Content Limits

As an explanation for some of the standards listed above, in Grade 1, students will:

- demonstrate understanding of the meaning of the equal sign by finding equations that show the number/expression on each side of the equal sign to be the same value
- use word problems involving only addition when relating three whole numbers
- use equations involving a symbol representing the unknown, such as a box or question mark

## Domain: NUMBER AND OPERATIONS IN BASE TEN

### Cluster: Extend the counting sequence.

**Standards**

MACC.1.NBT.1.1—Count to 120, starting at any number less than 120. In this range, read and write numerals and represent a number of objects with a written numeral.

### Standards Clarifications/Content Limits

In Grade 1, students will:

- demonstrate counting to 120 and up to 120; represent the numbers in the range from 1 to 120 in written form, and identify certain numbers of objects
<table>
<thead>
<tr>
<th>Domain</th>
<th>NUMBER AND OPERATIONS IN BASE TEN</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cluster</td>
<td>Understand place value.</td>
</tr>
<tr>
<td>Standards</td>
<td>MACC.1.NBT.2.2—Understand that the two digits of</td>
</tr>
<tr>
<td></td>
<td>a two-digit number represent amounts of tens and</td>
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<tr>
<td></td>
<td>ones. Understand the following as special cases:</td>
</tr>
<tr>
<td></td>
<td>MACC.1.NBT.2.2.a—10 can be thought of as a</td>
</tr>
<tr>
<td></td>
<td>bundle of ten ones—called a “ten.”</td>
</tr>
<tr>
<td></td>
<td>MACC.1.NBT.2.2.b—The numbers from 11 to 19 are</td>
</tr>
<tr>
<td></td>
<td>composed of a ten and one, two, three, four, five,</td>
</tr>
<tr>
<td></td>
<td>six, seven, eight, or nine ones.</td>
</tr>
<tr>
<td></td>
<td>MACC.1.NBT.2.2.c—The numbers 10, 20, 30, 40, 50,</td>
</tr>
<tr>
<td></td>
<td>60, 70, 80, 90 refer to one, two, three, four,</td>
</tr>
<tr>
<td></td>
<td>five, six, seven, eight, or nine tens (and 0</td>
</tr>
<tr>
<td></td>
<td>ones).</td>
</tr>
<tr>
<td></td>
<td>MACC.1.NBT.2.3—Compare two two-digit numbers</td>
</tr>
<tr>
<td></td>
<td>based on meanings of the tens and ones digits,</td>
</tr>
<tr>
<td></td>
<td>recording the results of comparisons with the</td>
</tr>
<tr>
<td></td>
<td>symbols &gt;, =, and &lt;.</td>
</tr>
<tr>
<td>Standards Clarifications/</td>
<td></td>
</tr>
<tr>
<td>Content Limits</td>
<td>As an explanation for some of the standards listed</td>
</tr>
<tr>
<td></td>
<td>above, in Grade 1, students will</td>
</tr>
<tr>
<td></td>
<td>• understand the place value concept of two-digit</td>
</tr>
<tr>
<td></td>
<td>numbers as tens and ones. Base ten models will be</td>
</tr>
<tr>
<td></td>
<td>important in helping to build and compare two-digit</td>
</tr>
<tr>
<td></td>
<td>numbers.</td>
</tr>
<tr>
<td></td>
<td>• compare the magnitude of numbers by understanding</td>
</tr>
<tr>
<td></td>
<td>the value of the tens and ones digits</td>
</tr>
<tr>
<td></td>
<td>• understand the meaning of the symbols &gt;, =, and</td>
</tr>
<tr>
<td></td>
<td>&lt; and use them to compare two 2-digit numbers</td>
</tr>
<tr>
<td>Domain</td>
<td>NUMBER AND OPERATIONS IN BASE TEN</td>
</tr>
<tr>
<td>--------</td>
<td>----------------------------------</td>
</tr>
<tr>
<td>Cluster</td>
<td>Use place value understanding and properties of operations to add and subtract.</td>
</tr>
</tbody>
</table>
| Standards| MACC.1.NBT.3.4—Add within 100, including adding a two-digit number and a one-digit number, and adding a two-digit number and a multiple of 10, using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; relate the strategy to a written method and explain the reasoning used. Understand that in adding two-digit numbers, one adds tens and tens, ones and ones, and sometimes it is necessary to compose a ten.  
MACC.1.NBT.3.5—Given a two-digit number, mentally find 10 more or 10 less than the number, without having to count; explain the reasoning used.  
MACC.1.NBT.3.6—Subtract multiples of 10 in the range 10−90 from multiples of 10 in the range 10−90 (positive or zero differences), using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; relate the strategy to a written method and explain the reasoning used. |
| Standards Clarifications/Content Limits| As an explanation for some of the standards listed above, in Grade 1, students will  
• use their ability to know basic facts for addition and subtraction within 20 quickly and accurately to add two-digit numbers to a one-digit number or a multiple of 10  
• use place value understanding to add the same place values and compose 10 ones as one 10 appropriately  
• demonstrate their understanding of performing addition and subtraction and explain the strategy they used |
<table>
<thead>
<tr>
<th>Domain</th>
<th>MEASUREMENT AND DATA</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cluster</td>
<td>Measure lengths indirectly and by iterating length units.</td>
</tr>
</tbody>
</table>
| **Standards** | MACC.1.MD.1.1—Order three objects by length; compare the lengths of two objects indirectly by using a third object.  
MACC.1.MD.1.2—Express the length of an object as a whole number of length units by laying multiple copies of a shorter object (the length unit) end to end; understand that the length measurement of an object is the number of same-size length units that span it with no gaps or overlaps. *Limit to contexts where the object being measured is spanned by a whole number of length units with no gaps or overlaps.* |
| **Standards Clarifications/Content Limits** | As an explanation for some of the standards listed above, in Grade 1, students will  
• order and compare objects by length without the use of actual measurements/measurement tools  
• understand that a smaller object can be used to measure the length of a larger object by laying multiple copies of the smaller object end to end without any gaps or overlaps |
| Domain | MEASUREMENT AND DATA |
| Cluster | Tell and write time. |
| **Standards** | MACC.1.MD.2.3—Tell and write time in hours and half-hours using analog and digital clocks. |
| **Standards Clarifications/Content Limits** | In Grade 1, students will  
• use their knowledge of the hour and minute hands to identify the time on a clock  
• use their understanding of fractional halves and wholes when telling time on an analog clock  
• match the times on analog clocks to digital clocks and vice versa |
<table>
<thead>
<tr>
<th>Domain</th>
<th>MEASUREMENT AND DATA</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cluster</td>
<td>Represent and interpret data.</td>
</tr>
<tr>
<td>Standards</td>
<td>MACC.1.MD.3.4—Organize, represent, and interpret data with up to three categories; ask and answer questions about the total number of data points, how many in each category, and how many more or less are in one category than in another.</td>
</tr>
</tbody>
</table>
| Standards Clarifications/Content Limits | In Grade 1, students will  
• sort and represent/display data on appropriate tables or graphs, such as picture graphs and bar graphs  
• answer questions about which category has more or less than another category and find how many more or less are in one category than another  
• interpret the meaning of the total number in each category |
## Domain

<table>
<thead>
<tr>
<th>Cluster</th>
<th>GEOMETRY</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Standards</strong></td>
<td>Reason with shapes and their attributes.</td>
</tr>
</tbody>
</table>

| Standards | MACC.1.G.1.1—Distinguish between defining attributes (e.g., triangles are closed and three-sided) versus non-defining attributes (e.g., color, orientation, overall size); build and draw shapes to possess defining attributes.  
MACC.1.G.1.2—Compose two-dimensional shapes (rectangles, squares, trapezoids, triangles, half-circles, and quarter-circles) or three-dimensional shapes (cubes, right rectangular prisms, right circular cones, and right circular cylinders) to create a composite shape, and compose new shapes from the composite shape.  
MACC.1.G.1.3—Partition circles and rectangles into two and four equal shares, describe the shares using the words halves, fourths, and quarters, and use the phrases half of, fourth of, and quarter of. Describe the whole as two of, or four of the shares. Understand for these examples that decomposing into more equal shares creates smaller shares. |

| Standards Clarifications/Content Limits | As an explanation for some of the standards listed above, in Grade 1, students will  
• understand that defining attributes are features that are always true for classifying and identifying shapes and that non-defining attributes are features that may be present but are not always true for every shape  
• manipulate two-dimensional and three-dimensional shapes to create new shapes  
• sort, compare, and identify shapes based on their attributes  
• understand the concept of fractions through applying the idea of equal-sized parts or fair shares through context and models |
## C. Grade 2 Item Specifications

<table>
<thead>
<tr>
<th>Domain</th>
<th>OPERATIONS AND ALGEBRAIC THINKING</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cluster</td>
<td>Represent and solve problems involving addition and subtraction.</td>
</tr>
<tr>
<td><strong>Standards</strong></td>
<td>MACC.2.OA.1.1—Use addition and subtraction within 100 to solve one- and two-step word problems involving situations of adding to, taking from, putting together, taking apart, and comparing, with unknowns in all positions, e.g., by using drawings and equations with a symbol for the unknown number to represent the problem.</td>
</tr>
</tbody>
</table>
| **Standards Clarifications/Content Limits** | In Grade 2, students will  
• identify, create models, and/or solve problems involving addition and subtraction situations within 100  
• solve word problems by solving for the unknown using drawings and equations (see Table 1 in the CCSS-M glossary)  
• use equations involving a symbol representing the unknown, such as a box or question mark |

<table>
<thead>
<tr>
<th>Domain</th>
<th>OPERATIONS AND ALGEBRAIC THINKING</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cluster</td>
<td>Add and subtract within 20.</td>
</tr>
<tr>
<td><strong>Standards</strong></td>
<td>MACC.2.OA.2.2—Fluently add and subtract within 20 using mental strategies. By end of Grade 2, know from memory all sums of two one-digit numbers.</td>
</tr>
</tbody>
</table>
| **Standards Clarifications/Content Limits** | In Grade 2, students will  
• demonstrate solidified addition and subtraction problem-solving skills  
• relate some of the various strategies to add and subtract |
<table>
<thead>
<tr>
<th>Domain</th>
<th>OPERATIONS AND ALGEBRAIC THINKING</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cluster</td>
<td>Work with equal groups of objects to gain foundations for multiplication.</td>
</tr>
</tbody>
</table>
| Standards | MACC.2.OA.3.3—Determine whether a group of objects (up to 20) has an odd or even number of members, e.g., by pairing objects or counting them by 2s; write an equation to express an even number as a sum of two equal addends.  
MACC.2.OA.3.4—Use addition to find the total number of objects arranged in rectangular arrays with up to 5 rows and up to 5 columns; write an equation to express the total as a sum of equal addends. |
| Standards Clarifications/Content Limits | As an explanation for some of the standards listed above, in Grade 2, students will  
• identify, create models, and/or solve problems involving odd and even numbers up to 20  
• begin to identify and model the foundations of multiplication of arrays of objects through writing repeated addition equations |
<table>
<thead>
<tr>
<th>Domain</th>
<th>NUMBER AND OPERATIONS IN BASE TEN</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cluster</td>
<td>Understand place value.</td>
</tr>
<tr>
<td><strong>Standards</strong></td>
<td>MACC.2.NBT.1.1—Understand that the three digits of a three-digit number represent amounts of hundreds, tens, and ones; e.g., 706 equals 7 hundreds, 0 tens, and 6 ones. Understand the following as special cases:</td>
</tr>
<tr>
<td></td>
<td>MACC.2.NBT.1.1.a—100 can be thought of as a bundle of ten tens—called a “hundred.”</td>
</tr>
<tr>
<td></td>
<td>MACC.2.NBT.1.1.b—The numbers 100, 200, 300, 400, 500, 600, 700, 800, 900 refer to one, two, three, four, five, six, seven, eight, or nine hundreds (and 0 tens and 0 ones).</td>
</tr>
<tr>
<td></td>
<td>MACC.2.NBT.1.2—Count within 1,000; skip-count by 5s, 10s, and 100s.</td>
</tr>
<tr>
<td></td>
<td>MACC.2.NBT.1.3—Read and write numbers to 1,000 using base-ten numerals, number names, and expanded form.</td>
</tr>
<tr>
<td></td>
<td>MACC.2.NBT.1.4—Compare two three-digit numbers based on meanings of the hundreds, tens, and ones digits, using &gt;, =, and &lt; symbols to record the results of comparisons.</td>
</tr>
<tr>
<td><strong>Standards Clarifications/Content Limits</strong></td>
<td>As an explanation for some of the standards listed above, in Grade 2, students will</td>
</tr>
<tr>
<td></td>
<td>• extend their understanding of the base ten system</td>
</tr>
<tr>
<td></td>
<td>• understand the place value concepts of three-digit numbers as hundreds, tens, and ones. Base ten models will be important in helping to build and compare three-digit numbers.</td>
</tr>
<tr>
<td></td>
<td>• read, write, and count with numbers up to 1,000 in standard and expanded form</td>
</tr>
<tr>
<td></td>
<td>• skip count by fives, tens, and hundreds up to 1,000</td>
</tr>
<tr>
<td></td>
<td>• compare the magnitude of numbers by understanding the value of the hundreds, tens, and ones digits</td>
</tr>
<tr>
<td></td>
<td>• understand the meaning of the symbols &gt;, =, and &lt; and use them to compare two 3-digit numbers</td>
</tr>
</tbody>
</table>
### Domain

**NUMBER AND OPERATIONS IN BASE TEN**

<table>
<thead>
<tr>
<th>Cluster</th>
<th>Use place value understanding and properties of operations to add and subtract.</th>
</tr>
</thead>
</table>

| Standards | MACC.2.NBT.2.5—Fluently add and subtract within 100 using strategies based on place value, properties of operations, and/or the relationship between addition and subtraction.  
MACC.2.NBT.2.6—Add up to four two-digit numbers using strategies based on place value and properties of operations.  
MACC.2.NBT.2.7—Add and subtract within 1,000, using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; relate the strategy to a written method. Understand that in adding or subtracting three-digit numbers, one adds or subtracts hundreds and hundreds, tens and tens, ones and ones; and sometimes it is necessary to compose or decompose tens or hundreds.  
MACC.2.NBT.2.8—Mentally add 10 or 100 to a given number 100–900, and mentally subtract 10 or 100 from a given number 100–900.  
MACC.2.NBT.2.9—Explain why addition and subtraction strategies work, using place value and the properties of operations. |

| Standards Clarifications/Content Limits | As an explanation for some of the standards listed above, in Grade 2, students will  
• use their knowledge of fluency facts for addition and subtraction to add and subtract one- and two-digit whole numbers within 100  
• identify and use pictorial representations and models in conjunction with problem-solving strategies to add and subtract up to three-digit whole numbers within 1,000  
• apply place value understanding to add or subtract the same place values and compose or decompose hundreds, tens, and ones appropriately  
• use the properties of operations, such as the Commutative property, and/or the relationship between addition and subtraction to solve problems  
• demonstrate their understanding of performing addition and subtraction, and explain the strategy they used and why it works |
<table>
<thead>
<tr>
<th>Domain</th>
<th>MEASUREMENT AND DATA</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cluster</td>
<td>Measure and estimate lengths in standard units.</td>
</tr>
</tbody>
</table>
| Standards | **MACC.2.MD.1.1**—Measure the length of an object by selecting and using appropriate tools, such as rulers, yardsticks, meter sticks, and measuring tapes.  
**MACC.2.MD.1.2**—Measure the length of an object twice, using length units of different lengths for the two measurements; describe how the two measurements relate to the size of the unit chosen.  
**MACC.2.MD.1.3**—Estimate lengths using units of inches, feet, centimeters, and meters.  
**MACC.2.MD.1.4**—Measure to determine how much longer one object is than another, expressing the length difference in terms of a standard length unit. |
| Standards Clarifications/Content Limits | As an explanation for some of the standards listed above, in Grade 2, students will  
• measure and compare objects by length  
• recognize the need for standard units of measure and how to select and use rulers and other measurement tools  
• demonstrate an understanding of length as the distance between both ends of an object and an understanding that linear measurement involves an iteration of units  
• recognize that larger units (e.g., yard) can be subdivided into equivalent units (e.g., inches)  
• estimate lengths and use them as a benchmark for accurately determining real measurements |
<table>
<thead>
<tr>
<th>Domain</th>
<th>MEASUREMENT AND DATA</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cluster</td>
<td>Relate addition and subtraction to length.</td>
</tr>
<tr>
<td>Standards</td>
<td>MACC.2.MD.2.5—Use addition and subtraction within 100 to solve word problems involving lengths that are given in the same units, e.g., by using drawings (such as drawings of rulers) and equations with a symbol for the unknown number to represent the problem. MACC.2.MD.2.6—Represent whole numbers as lengths from 0 on a number line diagram with equally spaced points corresponding to the numbers 0, 1, 2..., and represent whole-number sums and differences within 100 on a number line diagram.</td>
</tr>
</tbody>
</table>
| Standards Clarifications/Content Limits | As an explanation for some of the standards listed above, in Grade 2, students will
  • solve straightforward one-step contextual word problems involving addition and subtraction of whole number measurement quantities within 100
  • solve word problems involving lengths given the same unit by using drawings and/or equations
  • identify whole number lengths using a number line and be able to recognize the similarities to a ruler
  • model and apply strategies for addition and subtraction of lengths within 100 using a number line |
<table>
<thead>
<tr>
<th>Domain</th>
<th>MEASUREMENT AND DATA</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cluster</td>
<td>Work with time and money.</td>
</tr>
</tbody>
</table>
| Standards        | MACC.2.MD.3.7—Tell and write time from analog and digital clocks to the nearest five minutes, using a.m. and p.m.  
MACC.2.MD.3.8—Solve word problems involving dollar bills, quarters, dimes, nickels, and pennies, using $ and ¢ symbols appropriately. Example: If you have 2 dimes and 3 pennies, how many cents do you have? |
| Standards Clarifications/Content Limits | As an explanation for some of the standards listed above, in Grade 2, students will  
• use their knowledge of the hour and minute hands to identify the time on a clock  
• apply their understanding of fractional halves and wholes when telling time on an analog clock to be able to tell time to the nearest five minutes using their experience with skip counting by fives  
• match the times on analog clocks to digital clocks and vice versa  
• distinguish between morning times (a.m.) and afternoon or evening times (p.m.)  
• identify money amounts using whole dollars or cents  
• use their knowledge of the value of dollar bills and different coins to solve simple word problems |
<table>
<thead>
<tr>
<th>Domain</th>
<th>MEASUREMENT AND DATA</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cluster</td>
<td>Represent and interpret data.</td>
</tr>
</tbody>
</table>
| Standards | MACC.2.MD.4.9—Generate measurement data by measuring lengths of several objects to the nearest whole unit, or by making repeated measurements of the same object. Show the measurements by making a line plot, where the horizontal scale is marked off in whole-number units.  
MACC.2.MD.4.10—Draw a picture graph and a bar graph (with single-unit scale) to represent a data set with up to four categories. Solve simple put-together, take-apart, and compare problems using information presented in a bar graph. |
| Standards Clarifications/Content Limits | As an explanation for some of the standards listed above, in Grade 2, students will  
• apply their knowledge of measuring lengths in standard units to collect, analyze, and organize data in a line plot  
• sort and represent/display data in appropriate line plots, picture graphs, and single bar graphs  
• interpret bar graphs to solve simple one-step word problems involving the combination or comparison of the representation of the data (see Table 1 in the CCSS-M Glossary) |
<table>
<thead>
<tr>
<th><strong>Domain</strong></th>
<th><strong>GEOMETRY</strong></th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Cluster</strong></td>
<td>Reason with shapes and their attributes.</td>
</tr>
<tr>
<td><strong>Standards</strong></td>
<td>MACC.2.G.1.1—Recognize and draw shapes having specified attributes, such as a given number of angles or a given number of equal faces. Identify triangles, quadrilaterals, pentagons, hexagons, and cubes. MACC.2.G.1.2—Partition a rectangle into rows and columns of same-size squares and count to find the total number of them. MACC.2.G.1.3—Partition circles and rectangles into two, three, or four equal shares, describe the shares using the words <em>halves, thirds, half of, a third of</em>, etc., and describe the whole as two halves, three thirds, four fourths. Recognize that equal shares of identical wholes need not have the same shape.</td>
</tr>
</tbody>
</table>
| **Standards Clarifications/Content Limits** | As an explanation for some of the standards listed above, in Grade 2, students will  
• sort, compare, identify, and draw shapes based on their attributes  
• begin to identify and model the foundations for calculating the area of rectangles through tiling based on their experiences with arrays in multiplication  
• demonstrate an understanding of the concept of fractions by applying the idea of equal-sized parts or fair shares through context and models |
Appendices

Appendix A

Sample Items

Grade: Kindergarten
Item Type: Selected Response
Correct Answer: A
Possible Points: 1
DOK: 1
Calculator Usage: No
CCSS Standard:
MACC.K.G.1.1—Describe objects in the environment using names of shapes, and describe the relative positions of these objects using terms such as above, below, beside, in front of, behind, and next to.

Standards for Mathematical Practice:
2. Reason abstractly and quantitatively.
Which picture shows a cube in front of a cylinder?

A. 🔘

B. 🔘

C. 🔘

Distractor Rationales

A. Correct answer

B. There is confusion between the terms \textit{in front of} and \textit{behind}.

C. There is confusion between the terms \textit{in front of} and \textit{next to}.
Grade: 2
Item Type: Short Response
Correct Answer: 42
Possible Points: 1
DOK: 1
Calculator Usage: No
CCSS Standard:
MACC.2.NBT.2.5—Fluently add and subtract within 100 using strategies based on place value, properties of operations, and/or the relationship between addition and subtraction.
Standards for Mathematical Practice:
2. Reason abstractly and quantitatively.

Which number makes this equation true? ____________

65 − 23 = ?
Grade: 1
Item Type: Short Response
Correct Answer: 6 leaves long
Possible Points: 1
DOK: 1
Calculator Usage: No
CCSS Standard:
MACC.1.MD.1.2—Express the length of an object as a whole number of length units by laying multiple copies of a shorter object (the length unit) end to end; understand that the length measurement of an object is the number of same-size length units that span it with no gaps or overlaps. Limit to contexts where the object being measured is spanned by a whole number of length units with no gaps or overlaps.
Standards for Mathematical Practice:
1. Make sense of problems and persevere in solving them.

Pete used leaves to measure the length of one window.

How many leaves long are two windows?
_____ leaves long
Grade: Kindergarten
Item Type: Constructed Response
Correct Answer: See Scoring Exemplar
Possible Points: 2
DOK: 1
Calculator Usage: No
CCSS Standard:
MACC.K.CC.2.5—Count to answer “how many?” questions about as many as 20 things arranged in a line, a rectangular array, or a circle, or as many as 10 things in a scattered configuration; given a number from 1–20, count out that many objects.
Standards for Mathematical Practice:
4. Model with mathematics.

Part A. Draw a circle around 9 of the balloons.
Part B. Put an X on 3 of the balloons.
### SCORING RUBRIC

<table>
<thead>
<tr>
<th>Score</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>Work demonstrates a <strong>clear and complete</strong> understanding of the mathematical concepts and/or procedures required by the task. Appropriate strategy is shown with clear and complete explanations and interpretations.</td>
</tr>
<tr>
<td>1</td>
<td>Response demonstrates a <strong>partial</strong> understanding of the mathematical concepts and/or procedures. Appropriate strategy is shown, but explanation or interpretation has minor flaws. <strong>OR</strong> Response is incorrect because of calculation errors. Work and strategy indicate a <strong>clear</strong> understanding of the mathematical concepts and/or procedures required by the task.</td>
</tr>
<tr>
<td>0</td>
<td>Response is irrelevant, inappropriate, or not provided.</td>
</tr>
</tbody>
</table>

### SCORING EXEMPLAR

**Maximum Points—2**

**Part A—1 point**

- Each of 9 balloons is circled, or a large circle around a set of 9 balloons is drawn.

or equivalent work

**Part B—1 point**

- An X is drawn on each of 3 balloons, or a large X is drawn to cover 3 balloons. The X can be drawn on any balloon even if it is circled.

or equivalent work
Lisha had 14 goldfish in her fishbowl. She put some of the goldfish in a jar. Now Lisha has just 11 goldfish in her fishbowl.

Part A. Draw a picture to show what Lisha did.

Part B. Write a number sentence you could use to find the number of fish Lisha put in the jar.

Part C. How many goldfish did Lisha put in the jar? _______
### SCORING RUBRIC

<table>
<thead>
<tr>
<th>Score</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>4</td>
<td>Work demonstrates a <strong>clear and complete</strong> understanding of the mathematical concepts and/or procedures required by the task. Appropriate strategy is shown with clear and complete explanations and interpretations.</td>
</tr>
<tr>
<td>3</td>
<td>Work demonstrates a <strong>clear</strong> understanding of the mathematical concepts and/or procedures but is not complete. Appropriate strategy is shown, but explanation or interpretation has minor flaws. OR Response is incorrect because of calculation errors. Work and strategy indicate a <strong>clear</strong> demonstration of the problem.</td>
</tr>
<tr>
<td>2</td>
<td>Response demonstrates a <strong>partial</strong> understanding of the mathematical concepts and/or procedures. Appropriate strategy is shown, but explanation or interpretation has minor flaws.</td>
</tr>
<tr>
<td>1</td>
<td>Response shows <strong>minimal</strong> understanding of the mathematical concepts and/or procedures or provides no explanation or interpretation for the solution or shows major flaws.</td>
</tr>
<tr>
<td>0</td>
<td>Response is irrelevant, inappropriate, or not provided.</td>
</tr>
</tbody>
</table>

### SCORING EXEMPLAR

**Maximum Points—4**

**Part A—2 points**
Eleven fish are drawn in the bowl and 3 in the jar, or 14 fish are drawn in the bowl with some indication that a total of 3 are being moved to the jar.

or equivalent work

**Part B—1 point**

\[ 14 - 11 = 3 \]

OR

\[ 14 - □ = 11 \]

OR

\[ 11 + ___ = 14 \]

Other correct number sentences are possible.

**Part C—1 point**

The correct number of goldfish that Lisha put in the jar is identified as 3. All work is appropriate.

or equivalent work
Grade: 1
Item Type: Performance Task
Correct Answer: See Scoring Exemplar
Possible Points: 6
DOK: 4
Calculator Usage: No
CCSS Standard:
MACC.1.OA.3.6—Add and subtract within 20, demonstrating fluency for addition and subtraction within 10. Use strategies such as counting on; making ten (e.g., 8 + 6 = 8 + 2 + 4 = 10 + 4 = 14); decomposing a number leading to a ten (e.g., 13 − 4 = 13 − 3 − 1 = 10 − 1 = 9); using the relationship between addition and subtraction (e.g., knowing that 8 + 4 = 12, one knows 12 − 8 = 4); and creating equivalent but easier or known sums (e.g., adding 6 + 7 by creating the known equivalent 6 + 6 + 1 = 12 + 1 = 13).
Standards for Mathematical Practice:
1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
4. Model with mathematics.

Books in Bags

Teacher Directions:
Read the tasks aloud to the students.
Be sure that all vocabulary is understood.
Be sure that students understand that all 10 books must be used each time.
Instruct students to use words, numbers, and/or pictures to show their work.
 Allow 30 to 40 minutes for this task.
This task can be modified by using different numbers of books.
Make all necessary materials available.

Suggested Materials: Two color linking cubes, two color counters, extra paper, and pencils

TASK:
Mike has 10 books. He will put all of the books into 2 bags.
Part A. Show all the ways Mike can put some of his 10 books in Ann’s bag and the rest of the books in Ben’s bag.
Part B. How many different ways did you find to put the books into the 2 bags?
Part C. Mike will put 10 more books in Ben’s bag. What is the greatest number of books that could be in Ben’s bag if Ann still has a book in her bag? Explain your answer.
Maximum Points—6

Part A—3 points

Any recording method that shows all possible combinations that total 10 is acceptable excluding 0 + 10 and 10 + 0, such as:

<table>
<thead>
<tr>
<th>Ben</th>
<th>Ann</th>
</tr>
</thead>
<tbody>
<tr>
<td>9</td>
<td>1</td>
</tr>
<tr>
<td>8</td>
<td>2</td>
</tr>
<tr>
<td>7</td>
<td>3</td>
</tr>
<tr>
<td>6</td>
<td>4</td>
</tr>
<tr>
<td>5</td>
<td>5</td>
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<tr>
<td>4</td>
<td>6</td>
</tr>
<tr>
<td>3</td>
<td>7</td>
</tr>
<tr>
<td>2</td>
<td>8</td>
</tr>
<tr>
<td>1</td>
<td>9</td>
</tr>
</tbody>
</table>

Pictures, expressions, number sentences, or an organized list are acceptable. Other recording methods may be possible.

Part B—1 point

- The correct number of different ways to put the 10 books into the 2 bags is identified as 9 ways.

Part C—2 points

- The greatest number of books that could be in Ben’s bag if Ann still has a book in her bag is 19. There are a total of 20 books and if Ann has 1, there are 19 left for Ben.

OR

- $9 + 10 = 19$ so Ben can have 19 books

OR

- 19 because Ann has to get at least 1; $20 - 1 = 19$
Appendix B

Standards for Mathematical Practice

The Standards for Mathematical Practice describe varieties of expertise that mathematics educators at all levels should seek to develop in their students. These practices rest on important “processes and proficiencies” with longstanding importance in mathematics education. The first of these are the NCTM process standards of problem solving, reasoning and proof, communication, representation, and connections. The second are the strands of mathematical proficiency specified in the National Research Council’s report *Adding It Up*: adaptive reasoning, strategic competence, conceptual understanding (comprehension of mathematical concepts, operations and relations), procedural fluency (skill in carrying out procedures flexibly, accurately, efficiently and appropriately), and productive disposition (habitual inclination to see mathematics as sensible, useful, and worthwhile, coupled with a belief in diligence and one’s own efficacy).1

MACC.K12.MP.1.1 Make sense of problems and persevere in solving them.

Mathematically proficient students start by explaining to themselves the meaning of a problem and looking for entry points to its solution. They analyze givens, constraints, relationships, and goals. They make conjectures about the form and meaning of the solution and plan a solution pathway rather than simply jumping into a solution attempt. They consider analogous problems, and try special cases and simpler forms of the original problem in order to gain insight into its solution. They monitor and evaluate their progress and change course if necessary. Older students might, depending on the context of the problem, transform algebraic expressions or change the viewing window on their graphing calculator to get the information they need. Mathematically proficient students can explain correspondences between equations, verbal descriptions, tables, and graphs or draw diagrams of important features and relationships, graph data, and search for regularity or trends. Younger students might rely on using concrete objects or pictures to help conceptualize and solve a problem. Mathematically proficient students check their answers to problems using a different method, and they continually ask themselves, “Does this make sense?” They can understand the approaches of others to solving complex problems and identify correspondences between different approaches.

MACC.K12.MP.2.1 Reason abstractly and quantitatively.

Mathematically proficient students make sense of quantities and their relationships in problem situations. They bring two complementary abilities to bear on problems involving quantitative relationships: the ability to decontextualize—to abstract a given situation and represent it symbolically and manipulate the representing symbols as if they have a life of their own, without necessarily attending to their referents—and the ability to contextualize, to pause as needed during the manipulation process in order to probe into the referents for the symbols involved. Quantitative reasoning entails habits of creating a coherent representation of the problem at hand; considering the units involved; attending to the meaning of quantities, not just how to compute them; and knowing and flexibly using different properties of operations and objects.

MACC.K12.MP.3.1 Construct viable arguments and critique the reasoning of others.

Mathematically proficient students understand and use stated assumptions, definitions, and previously established results in constructing arguments. They make conjectures and build a logical progression of statements to explore the truth of their conjectures. They are able to analyze situations by breaking them into cases, and can recognize and use counterexamples. They justify their conclusions, communicate them to others, and respond to the arguments of others. They reason inductively about data, making plausible arguments that take into account the context from which the data arose. Mathematically proficient students are also able to compare the effectiveness of two plausible arguments, distinguish correct logic or reasoning from that which is flawed, and—if there is a flaw in an argument—explain what it is. Elementary students can construct arguments using concrete referents such as objects, drawings, diagrams, and actions. Such arguments can make sense and be correct, even though they are not generalized or made formal until later grades. Later, students learn to determine domains to which an argument applies. Students at all grades can listen or read the arguments of others, decide whether they make sense, and ask useful questions to clarify or improve the arguments.

MACC.K12.MP.4.1 Model with mathematics.

Mathematically proficient students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. In early grades, this might be as simple as writing an addition equation to describe a situation. In middle grades, a student might apply proportional reasoning to plan a school event or analyze a problem in the community. By high school, a student might use geometry to solve a design problem or use a function to describe how one quantity of interest depends on another. Mathematically proficient students who can apply what they know are comfortable making assumptions and approximations to simplify a complicated situation, realizing that these may need revision later. They are able to identify important quantities in a practical situation and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts and formulas. They can analyze those relationships mathematically to draw conclusions. They routinely interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.

MACC.K12.MP.5.1 Use appropriate tools strategically.

Mathematically proficient students consider the available tools when solving a mathematical problem. These tools might include pencil and paper, concrete models, a ruler, a protractor, a calculator, a spreadsheet, a computer algebra system, a statistical package, or dynamic geometry software. Proficient students are sufficiently familiar with tools appropriate for their grade or course to make sound decisions about when each of these tools might be helpful, recognizing both the insight to be gained and their limitations. For example, mathematically proficient high school students analyze graphs of functions and solutions generated using a graphing calculator. They detect possible errors by strategically using estimation and other mathematical knowledge. When making mathematical models, they know that technology can enable them to visualize the results of varying assumptions, explore consequences, and compare predictions with data. Mathematically proficient students at various grade levels are able to identify relevant external mathematical resources, such as digital content located on a website, and use them to pose or solve problems. They are able to use technological tools to explore and deepen their understanding of concepts.
MACC.K12.MP.6.1 Attend to precision.
Mathematically proficient students try to communicate precisely to others. They try to use clear definitions in discussion with others and in their own reasoning. They state the meaning of the symbols they choose, including using the equal sign consistently and appropriately. They are careful about specifying units of measure, and labeling axes to clarify the correspondence with quantities in a problem. They calculate accurately and efficiently, express numerical answers with a degree of precision appropriate for the problem context. In the elementary grades, students give carefully formulated explanations to each other. By the time they reach high school they have learned to examine claims and make explicit use of definitions.

MACC.K12.MP.7.1 Look for and make use of structure.
Mathematically proficient students look closely to discern a pattern or structure. Young students, for example, might notice that three and seven more is the same amount as seven and three more, or they may sort a collection of shapes according to how many sides the shapes have. Later, students will see $7 \times 8$ equals the well remembered $7 \times 5 + 7 \times 3$, in preparation for learning about the distributive property. In the expression $x^2 + 9x + 14$, older students can see the 14 as $2 \times 7$ and the 9 as $2 + 7$. They recognize the significance of an existing line in a geometric figure and can use the strategy of drawing an auxiliary line for solving problems. They also can step back for an overview and shift perspective. They can see complicated things, such as some algebraic expressions, as single objects or as being composed of several objects. For example, they can see $5 - 3(x - y)^2$ as 5 minus a positive number times a square and use that to realize that its value cannot be more than 5 for any real numbers $x$ and $y$.

MACC.K12.MP.8.1 Look for and express regularity in repeated reasoning.
Mathematically proficient students notice if calculations are repeated, and look both for general methods and for shortcuts. Upper elementary students might notice when dividing 25 by 11 that they are repeating the same calculations over and over again, and conclude they have a repeating decimal. By paying attention to the calculation of slope as they repeatedly check whether points are on the line through $(1, 2)$ with slope 3, middle school students might abstract the equation $(y - 2)/(x - 1) = 3$. Noticing the regularity in the way terms cancel when expanding $(x - 1)(x + 1)$, $(x - 1)(x^2 + x + 1)$, and $(x - 1)(x^3 + x^2 + x + 1)$ might lead them to the general formula for the sum of a geometric series. As they work to solve a problem, mathematically proficient students maintain oversight of the process, while attending to the details. They continually evaluate the reasonableness of their intermediate results.