Probability and Statistics Honors

Version Description

In Probability and Statistics Honors, instructional time will emphasize four areas:

(1) creating and interpreting data displays for univariate and bivariate categorical and numerical data;
(2) comparing and making observations about populations using statistical data, including confidence intervals and hypothesis testing;
(3) extending understanding of probability and probability distributions and
(4) developing an understanding of methods for collecting statistical data, including randomized trials.

Curricular content for all subjects must integrate critical-thinking, problem-solving, and workforce-literacy skills; communication, reading, and writing skills; mathematics skills; collaboration skills; contextual and applied-learning skills; technology-literacy skills; information and media-literacy skills; and civic-engagement skills.

All clarifications stated, whether general or specific to Probability and Statistics Honors, are expectations for instruction of that benchmark.

General Notes

Honors and Accelerated Level Course Note: Accelerated courses require a greater demand on students through increased academic rigor. Academic rigor is obtained through the application, analysis, evaluation, and creation of complex ideas that are often abstract and multi-faceted. Students are challenged to think and collaborate critically on the content they are learning. Honors level rigor will be achieved by increasing text complexity through text selection, focus on high-level qualitative measures, and complexity of task. Instruction will be structured to give students a deeper understanding of conceptual themes and organization within and across disciplines. Academic rigor is more than simply assigning to students a greater quantity of work.

Florida's Benchmarks for Excellent Student Thinking (B.E.S.T.) Standards: This course includes Florida's B.E.S.T. ELA Expectations (EE) and Mathematical Thinking and Reasoning Standards (MTRs) for students. Florida educators should intentionally embed these standards within the content and their instruction as applicable. For guidance on the implementation of the EEs and MTRs, please visit https://www.cpalms.org/Standards/BEST_Standards.aspx and select the appropriate B.E.S.T. Standards package.

English Language Development ELD Standards Special Notes Section: Teachers are required to provide listening, speaking, reading and writing instruction that allows English language learners (ELL) to communicate information, ideas and concepts for academic success in the content area of Mathematics. For the given level of English language proficiency and with visual, graphic, or interactive support, students will interact with grade level words, expressions, sentences and discourse to process or produce language necessary for academic success. The ELD standard should specify a relevant content area concept or topic of study chosen by curriculum developers

and teachers which maximizes an ELL's need for communication and social skills. To access an ELL supporting document which delineates performance definitions and descriptors, please click on the following link: https://cpalmsmediaprod.blob.core.windows.net/uploads/docs/standards/eld/ma.pdf.

General Information

Course Number: 1210300	Course Type: Core Academic Course
Course Length: Year (Y)	Course Level: 3
Course Attributes: Honors	Grade Level(s): 9, 10, 11, 12
Graduation Requirement: Mathematics	Number of Credits: One (1) credit
Course Path: Section | Grades PreK to 12 Education Courses > Grade Group | Grades 9 to 12 and Adult Education Courses > Subject | Mathematics > SubSubject | Probability and Statistics > Abbreviated Title | PROB & STATS HONORS	
Educator Certification: Mathematics (Grades 6-12)	

Course Standards and Benchmarks

Mathematical Thinking and Reasoning

MA.K12.MTR.1.1 Actively participate in effortful learning both individually and collectively.

Mathematicians who participate in effortful learning both individually and with others:

- Analyze the problem in a way that makes sense given the task.
- Ask questions that will help with solving the task.
- Build perseverance by modifying methods as needed while solving a challenging task.
- Stay engaged and maintain a positive mindset when working to solve tasks.
- Help and support each other when attempting a new method or approach.

Clarifications:

Teachers who encourage students to participate actively in effortful learning both individually and with others:

- Cultivate a community of growth mindset learners.
- Foster perseverance in students by choosing tasks that are challenging.
- Develop students' ability to analyze and problem solve.
- Recognize students' effort when solving challenging problems.

MA.K12.MTR.2.1 Demonstrate understanding by representing problems in multiple ways.

Mathematicians who demonstrate understanding by representing problems in multiple ways:

- Build understanding through modeling and using manipulatives.
- Represent solutions to problems in multiple ways using objects, drawings, tables, graphs and equations.
- Progress from modeling problems with objects and drawings to using algorithms and equations.
- Express connections between concepts and representations.
- Choose a representation based on the given context or purpose.

Clarifications:

Teachers who encourage students to demonstrate understanding by representing problems in multiple ways:

- Help students make connections between concepts and representations.
- Provide opportunities for students to use manipulatives when investigating concepts.
- Guide students from concrete to pictorial to abstract representations as understanding progresses.
- Show students that various representations can have different purposes and can be useful in different situations.

MA.K12.MTR.3.1 Complete tasks with mathematical fluency.

Mathematicians who complete tasks with mathematical fluency:

- Select efficient and appropriate methods for solving problems within the given context.
- Maintain flexibility and accuracy while performing procedures and mental calculations.
- Complete tasks accurately and with confidence.
- Adapt procedures to apply them to a new context.
- Use feedback to improve efficiency when performing calculations.

Clarifications:

Teachers who encourage students to complete tasks with mathematical fluency:

- Provide students with the flexibility to solve problems by selecting a procedure that allows them to solve efficiently and accurately.
- Offer multiple opportunities for students to practice efficient and generalizable methods.
- Provide opportunities for students to reflect on the method they used and determine if a more efficient method could have been used.

MA.K12.MTR.4.1 Engage in discussions that reflect on the mathematical thinking of self and others.

Mathematicians who engage in discussions that reflect on the mathematical thinking of self and others:

- Communicate mathematical ideas, vocabulary and methods effectively.
- Analyze the mathematical thinking of others.
- Compare the efficiency of a method to those expressed by others.
- Recognize errors and suggest how to correctly solve the task.
- Justify results by explaining methods and processes.
- Construct possible arguments based on evidence.

Clarifications:
Teachers who encourage students to engage in discussions that reflect on the mathematical thinking of self and others:

- Establish a culture in which students ask questions of the teacher and their peers, and error is an opportunity for learning.
- Create opportunities for students to discuss their thinking with peers.
- Select, sequence and present student work to advance and deepen understanding of correct and increasingly efficient methods.
- Develop students' ability to justify methods and compare their responses to the responses of their peers.

MA.K12.MTR.5.1 Use patterns and structure to help understand and connect mathematical concepts.

Mathematicians who use patterns and structure to help understand and connect mathematical concepts:

- Focus on relevant details within a problem.
- Create plans and procedures to logically order events, steps or ideas to solve problems.
- Decompose a complex problem into manageable parts.
- Relate previously learned concepts to new concepts.
- Look for similarities among problems.
- Connect solutions of problems to more complicated large-scale situations.

Clarifications:
Teachers who encourage students to use patterns and structure to help understand and connect mathematical concepts:

- Help students recognize the patterns in the world around them and connect these patterns to mathematical concepts.
- Support students to develop generalizations based on the similarities found among problems.
- Provide opportunities for students to create plans and procedures to solve problems.
- Develop students' ability to construct relationships between their current understanding and more sophisticated ways of thinking.

MA.K12.MTR.6.1 Assess the reasonableness of solutions.

Mathematicians who assess the reasonableness of solutions:

- Estimate to discover possible solutions.
- Use benchmark quantities to determine if a solution makes sense.
- Check calculations when solving problems.
- Verify possible solutions by explaining the methods used.
- Evaluate results based on the given context.

Clarifications:

Teachers who encourage students to assess the reasonableness of solutions:

- Have students estimate or predict solutions prior to solving.
- Prompt students to continually ask, "Does this solution make sense? How do you know?"
- Reinforce that students check their work as they progress within and after a task.
- Strengthen students' ability to verify solutions through justifications.

MA.K12.MTR.7.1 Apply mathematics to real-world contexts.

Mathematicians who apply mathematics to real-world contexts:

- Connect mathematical concepts to everyday experiences.
- Use models and methods to understand, represent and solve problems.
- Perform investigations to gather data or determine if a method is appropriate.
- Redesign models and methods to improve accuracy or efficiency.

Clarifications:

Teachers who encourage students to apply mathematics to real-world contexts:

- Provide opportunities for students to create models, both concrete and abstract, and perform investigations.
- Challenge students to question the accuracy of their models and methods.
- Support students as they validate conclusions by comparing them to the given situation.
- Indicate how various concepts can be applied to other disciplines.

ELA Expectations

ELA.K12.EE.1.1 Cite evidence to explain and justify reasoning.

ELA.K12.EE.2.1 Read and comprehend grade-level complex texts proficiently.

ELA.K12.EE.3.1 Make inferences to support comprehension.

ELA.K12.EE.4.1 Use appropriate collaborative techniques and active listening skills when engaging in discussions in a variety of situations.

ELA.K12.EE.5.1 Use the accepted rules governing a specific format to create quality work.

ELA.K12.EE.6.1 Use appropriate voice and tone when speaking or writing.

English Language Development

ELD.K12.ELL.MA Language of Mathematics

ELD.K12.ELL.MA.1	English language learners communicate information, ideas and concepts necessary for academic success in the content area of Mathematics.

Data Analysis and Probability

MA.912.DP.1 Summarize, represent and interpret categorical and numerical data with one and two variables.

MA.912.DP.1.1	Given a set of data, select an appropriate method to represent the data, depending on whether it is numerical or categorical data and on whether it is univariate or bivariate.

Benchmark Clarifications:
Clarification 1: Instruction includes discussions regarding the strengths and weaknesses of each data display.
Clarification 2: Numerical univariate includes histograms, stem-and-leaf plots, box plots and line plots; numerical bivariate includes scatter plots and line graphs; categorical univariate includes bar charts, circle graphs, line plots, frequency tables and relative frequency tables; and categorical bivariate includes segmented bar charts, joint frequency tables and joint relative frequency tables.
Clarification 3: Instruction includes the use of appropriate units and labels and, where appropriate, using technology to create data displays.

MA.912.DP.1.2	Interpret data distributions represented in various ways. State whether the data is numerical or categorical, whether it is univariate or bivariate and interpret the different components and quantities in the display.

Benchmark Clarifications:
Clarification 1: Within the Probability and Statistics course, instruction includes the use of spreadsheets and technology.

MA.912.DP.1.3	Explain the difference between correlation and causation in the contexts of both numerical and categorical data.

Algebra 1 Example: There is a strong positive correlation between the number of Nobel prizes won by country and the per capita chocolate consumption by country. Does this mean that increased chocolate consumption in America will increase the United States of America's chances of a Nobel prize winner?

MA.912.DP.1.4	Estimate a population total, mean or percentage using data from a sample survey; develop a margin of error through the use of simulation.

Algebra 1 Example: Based on a survey of 100 households in Twin Lakes, the newspaper reports that the average number of televisions per household is 3.5 with a margin of error of ± 0.6. The actual population mean can be estimated to be between 2.9 and 4.1 television per household. Since there are 5,500 households in Twin Lakes the estimated number of televisions is between 15,950 and 22,550.

Benchmark Clarifications:
Clarification 1: Within the Algebra 1 course, the margin of error will be given.

MA.912.DP.1.5	Interpret the margin of error of a mean or percentage from a data set. Interpret the confidence level corresponding to the margin of error.

MA.912.DP.2 Solve problems involving univariate and bivariate numerical data.

MA.912.DP.2.1	For two or more sets of numerical univariate data, calculate and compare the appropriate measures of center and measures of variability, accounting for possible effects of outliers. Interpret any notable features of the shape of the data distribution.

Benchmark Clarifications:
Clarification 1: The measure of center is limited to mean and median. The measure of variation is limited to range, interquartile range, and standard deviation.
Clarification 2: Shape features include symmetry or skewness and clustering.
Clarification 3: Within the Probability and Statistics course, instruction includes the use of spreadsheets and technology.

MA.912.DP.2.2	Use the mean and standard deviation of a data set to fit it to a normal distribution and to estimate population percentages. Recognize that there are data sets for which such a procedure is not appropriate.

Benchmark Clarifications:
Clarification 1: Instruction includes the connection to the binomial distribution and surveys.

MA.912.DP.2.3	Estimate population percentages from data that has been fit to the normal distribution.

Benchmark Clarifications:
Clarification 1: Instruction includes using technology, empirical rules or tables to estimate areas under the normal curve.

MA.912.DP.2.4	Fit a linear function to bivariate numerical data that suggests a linear association and interpret the slope and y-intercept of the model. Use the model to solve real-world problems in terms of the context of the data.

Benchmark Clarifications:
Clarification 1: Instruction includes fitting a linear function both informally and formally with the use of technology.
Clarification 2: Problems include making a prediction or extrapolation, inside and outside the range of the data, based on the equation of the line of fit.

MA.912.DP.2.5	Given a scatter plot that represents bivariate numerical data, assess the fit of a given linear function by plotting and analyzing residuals.

Benchmark Clarifications:
Clarification 1: Within the Algebra 1 course, instruction includes determining the number of positive and negative residuals; the largest and smallest residuals; and the connection between outliers in the data set and the corresponding residuals.

MA.912.DP.2.6	Given a scatter plot with a line of fit and residuals, determine the strength and direction of the correlation. Interpret strength and direction within a real-world context.

Benchmark Clarifications:
Clarification 1: Instruction focuses on determining the direction by analyzing the slope and informally determining the strength by analyzing the residuals.

MA.912.DP.2.7	Compute the correlation coefficient of a linear model using technology. Interpret the strength and direction of the correlation coefficient.

MA.912.DP.2.9	Fit an exponential function to bivariate numerical data that suggests an exponential association. Use the model to solve real-world problems in terms of the context of the data.

Benchmark Clarifications:
Clarification 1: Instruction focuses on determining whether an exponential model is appropriate by taking the logarithm of the dependent variable using spreadsheets and other technology.
Clarification 2: Instruction includes determining whether the transformed scatterplot has an appropriate line of best fit, and interpreting the y-intercept and slope of the line of best fit.
Clarification 3: Problems include making a prediction or extrapolation, inside and outside the range of the data, based on the equation of the line of fit.

MA.912.DP.3 Solve problems involving categorical data.

MA.912.DP.3.1 Construct a two-way frequency table summarizing bivariate categorical data. Interpret joint and marginal frequencies and determine possible associations in terms of a real-world context.

Algebra 1 Example: Complete the frequency table below.

	Has an A in math	Doesn't have an A in math	Total
Plays an instrument	20		90
Doesn't play an instrument	20		
Total			350

Using the information in the table, it is possible to determine that the second column contains the numbers 70 and 240. This means that there are 70 students who play an instrument but do not have an A in math and the total number of students who play an instrument is 90. The ratio of the joint frequencies in the first column is 1 to 1 and the ratio in the second column is 7 to 24, indicating a strong positive association between playing an instrument and getting an A in math.

MA.912.DP.3.2 Given marginal and conditional relative frequencies, construct a two-way relative frequency table summarizing categorical bivariate data.

Algebra 1 Example: A study shows that 9% of the population have diabetes and 91% do not. The study also shows that 95% of the people who do not have diabetes, test negative on a diabetes test while 80% who do have diabetes, test positive. Based on the given information, the following relative frequency table can be constructed.

	Positive	Negative	Total
Has diabetes	7.2%	1.8%	9%
Doesn't have diabetes	4.55%	86.45%	91%

Benchmark Clarifications:
Clarification 1: Construction includes cases where not all frequencies are given but enough are provided to be able to construct a two-way relative frequency table.
Clarification 2: Instruction includes the use of a tree diagram when calculating relative frequencies to construct tables.

MA.912.DP.3.3 Given a two-way relative frequency table or segmented bar graph summarizing categorical bivariate data, interpret joint, marginal and conditional relative frequencies in terms of a real-world context.

Algebra 1 Example: Given the relative frequency table below, the ratio of true positives to false positives can be determined as 7.2 to 4.55, which is about 3 to 2, meaning that a randomly selected person who tests positive for diabetes is about 50% more likely to have diabetes than not have it.

	Positive	Negative	Total
Has diabetes	7.2%	1.8%	9%
Doesn't have diabetes	4.55%	86.45%	91%

Benchmark Clarifications:
Clarification 1: Instruction includes problems involving false positive and false negatives.

MA.912.DP.3.4 Given a relative frequency table, construct and interpret a segmented bar graph.

MA.912.DP.3.5 Solve real-world problems involving univariate and bivariate categorical data.

Benchmark Clarifications:
Clarification 1: Instruction focuses on the connection to probability.
Clarification 2: Instruction includes calculating joint relative frequencies or conditional relative frequencies using tree diagrams.
Clarification 3: Graphical representations include frequency tables, relative frequency tables, circle graphs and segmented bar graphs.

MA.912.DP.4 Use and interpret independence and probability.

MA.912.DP.4.1 Describe events as subsets of a sample space using characteristics, or categories, of the outcomes, or as unions, intersections or complements of other events.

MA.912.DP.4.2 Determine if events A and B are independent by calculating the product of their probabilities.

MA.912.DP.4.3 Calculate the conditional probability of two events and interpret the result in terms of its context.

MA.912.DP.4.4 Interpret the independence of two events using conditional probability.

MA.912.DP.4.5	Given a two-way table containing data from a population, interpret the joint and marginal relative frequencies as empirical probabilities and the conditional relative frequencies as empirical conditional probabilities. Use those probabilities to determine whether characteristics in the population are approximately independent. *Example:* A company has a commercial for their new grill. A population of people are surveyed to determine whether or not they have seen the commercial and whether or not they have purchased the product. Using this data, calculate the empirical conditional probabilities that a person who has seen the commercial did or did not purchase the grill.

Benchmark Clarifications:
Clarification 1: Instruction includes the connection between mathematical probability and applied statistics.

MA.912.DP.4.6	Recognize and explain the concepts of conditional probability and independence in everyday language and everyday situations.
MA.912.DP.4.7	Apply the addition rule for probability, taking into consideration whether the events are mutually exclusive, and interpret the result in terms of the model and its context.
MA.912.DP.4.8	Apply the general multiplication rule for probability, taking into consideration whether the events are independent, and interpret the result in terms of the context.
MA.912.DP.4.9	Apply the addition and multiplication rules for counting to solve mathematical and real-world problems, including problems involving probability.
MA.912.DP.4.10	Given a mathematical or real-world situation, calculate the appropriate permutation or combination.

MA.912.DP.5 Determine methods of data collection and make inferences from collected data.

MA.912.DP.5.1	Distinguish between a population parameter and a sample statistic.
MA.912.DP.5.2	Explain how random sampling produces data that is representative of a population.

MA.912.DP.5.3	Compare and contrast sampling methods.

Benchmark Clarifications:
Clarification 1: Instruction includes understanding the connection between probability and sampling methods.
Clarification 2: Sampling methods include simple random, stratified, cluster, systematic, judgement, quota and convenience.

MA.912.DP.5.4	Generate multiple samples or simulated samples of the same size to measure the variation in estimates or predictions.
MA.912.DP.5.5	Determine if a specific model is consistent within a given process by analyzing the data distribution from a data-generating process.
MA.912.DP.5.6	Determine the appropriate design, survey, experiment or observational study, based on the purpose. Articulate the types of questions appropriate for each type of design.
MA.912.DP.5.7	Compare and contrast surveys, experiments and observational studies.

Benchmark Clarifications:
Clarification 1: Instruction includes understanding how randomization relates to sample surveys, experiments and observational studies.

MA.912.DP.5.8	Draw inferences about two populations using data and statistical analysis from two random samples.
MA.912.DP.5.9	Compare two treatments using data from an experiment in which the treatments are assigned randomly.

Benchmark Clarifications:
Clarification 1: Instruction includes the understanding that if one wants to validate a causal relationship, then randomized assignment of treatment groups must occur.

MA.912.DP.5.10	Determine whether differences between parameters are significant using simulations.

MA.912.DP.5.11	Evaluate reports based on data from diverse media, print and digital resources by interpreting graphs and tables; evaluating data-based arguments; determining whether a valid sampling method was used; or interpreting provided statistics. *Example:* A local news station changes the y-axis on a data display from 0 to 10,000 to include data only within the range 7,000 to 10,000. Depending on the purpose, this could emphasize differences in data values in a misleading way.

Benchmark Clarifications:
Clarification 1: Instruction includes determining whether or not data displays could be misleading.

MA.912.DP.6 Use probability distributions to solve problems.

MA.912.DP.6.1	Define a random variable for a quantity of interest by assigning a numerical value to each individual outcome in a sample space; graph the corresponding probability distribution using the same graphical displays as for data distributions.
MA.912.DP.6.2	Develop a probability distribution for a discrete random variable using theoretical probabilities. Find the expected value and interpret it as the mean of the discrete distribution.
MA.912.DP.6.3	Develop a probability distribution for a discrete random variable using empirical probabilities. Find the expected value and interpret it as the mean of the discrete distribution.
MA.912.DP.6.4	Given a binomial distribution, calculate and interpret the expected value. Solve real-world problems involving binomial distributions.

Benchmark Clarifications:
Clarification 1: Instruction focuses on the connection between binomial distributions and coin tossing and the connection to one-question surveys in which the question has two possible responses.

MA.912.DP.6.5 Solve real-world problems involving geometric distributions.

Benchmark Clarifications:
Clarification 1: Instruction focuses on the connection between geometric distributions and tossing a coin until the first heads appears and the connection to making repeated attempts at a task until it is successfully completed.

MA.912.DP.6.7	Weigh the possible outcomes of a decision by assigning probabilities to payoff values and finding expected values and standard deviations. Evaluate and compare strategies on the basis of the calculated expected values and standard deviations.

Benchmark Clarifications:
Clarification 1: Instruction includes the relationship between expected values and standard deviations on one hand and the rewards and risks on the other hand.
Clarification 2: Instruction includes reducing risk through diversification.

MA.912.DP.6.8	Apply probabilities to make fair decisions, such as drawing from lots or using a random number generator.