Calculus Honors

Version Description

In Calculus Honors, instructional time will emphasize four areas:

(1) developing understanding of limits and continuity of functions;
(2) finding derivatives and applying them to motions, slopes, related rates and optimizations;
(3) applying limits and derivatives to graph and analyze functions and
(4) evaluating integrals and applying them to areas, volumes, average values and differential equations.

Curricular content for all subjects must integrate critical-thinking, problem-solving, and workforce-literacy skills; communication, reading, and writing skills; mathematics skills; collaboration skills; contextual and applied-learning skills; technology-literacy skills; information and media-literacy skills; and civic-engagement skills.

All clarifications stated, whether general or specific to Calculus Honors, are expectations for instruction of that benchmark.

General Notes

Honors and Accelerated Level Course Note: Accelerated courses require a greater demand on students through increased academic rigor. Academic rigor is obtained through the application, analysis, evaluation, and creation of complex ideas that are often abstract and multi-faceted. Students are challenged to think and collaborate critically on the content they are learning. Honors level rigor will be achieved by increasing text complexity through text selection, focus on high-level qualitative measures, and complexity of task. Instruction will be structured to give students a deeper understanding of conceptual themes and organization within and across disciplines. Academic rigor is more than simply assigning to students a greater quantity of work.

Florida's Benchmarks for Excellent Student Thinking (B.E.S.T.) Standards: This course includes Florida's B.E.S.T. ELA Expectations (EE) and Mathematical Thinking and Reasoning Standards (MTRs) for students. Florida educators should intentionally embed these standards within the content and their instruction as applicable. For guidance on the implementation of the EEs and MTRs, please visit https://www.cpalms.org/Standards/BEST_Standards.aspx and select the appropriate B.E.S.T. Standards package.

English Language Development ELD Standards Special Notes Section: Teachers are required to provide listening, speaking, reading and writing instruction that allows English language learners (ELL) to communicate information, ideas and concepts for academic success in the content area of Mathematics. For the given level of English language proficiency and with visual, graphic, or interactive support, students will interact with grade level words, expressions, sentences and discourse to process or produce language necessary for academic success. The ELD standard should specify a relevant content area concept or topic of study chosen by curriculum developers and teachers which maximizes an ELL's need for communication and social skills. To access an

ELL supporting document which delineates performance definitions and descriptors, please click on the following link: https://cpalmsmediaprod.blob.core.windows.net/uploads/docs/standards/eld/ma.pdf.

General Information

Course Number: 1202300	Course Type: Core Academic Course
Course Length: Year (Y)	Course Level: 3
Course Attributes: Honors	Grade Level(s): 9, 10, 11, 12
Graduation Requirement: Mathematics	Number of Credits: One (1) credit
Course Path: Section | Grades PreK to 12 Education Courses > Grade Group | Grades 9 to 12 and Adult Education Courses > Subject | Mathematics > SubSubject | Calculus > Abbreviated Title | CALC H	
Educator Certification: Mathematics (Grades 6-12)	

Course Standards and Benchmarks

Mathematical Thinking and Reasoning

MA.K12.MTR.1.1 Actively participate in effortful learning both individually and collectively.

Mathematicians who participate in effortful learning both individually and with others:

- Analyze the problem in a way that makes sense given the task.
- Ask questions that will help with solving the task.
- Build perseverance by modifying methods as needed while solving a challenging task.
- Stay engaged and maintain a positive mindset when working to solve tasks.
- Help and support each other when attempting a new method or approach.

Clarifications:
Teachers who encourage students to participate actively in effortful learning both individually and with others:

- Cultivate a community of growth mindset learners.
- Foster perseverance in students by choosing tasks that are challenging.
- Develop students' ability to analyze and problem solve.
- Recognize students' effort when solving challenging problems.

MA.K12.MTR.2.1 Demonstrate understanding by representing problems in multiple ways.

Mathematicians who demonstrate understanding by representing problems in multiple ways:
- Build understanding through modeling and using manipulatives.
- Represent solutions to problems in multiple ways using objects, drawings, tables, graphs and equations.
- Progress from modeling problems with objects and drawings to using algorithms and equations.
- Express connections between concepts and representations.
- Choose a representation based on the given context or purpose.

Clarifications:
Teachers who encourage students to demonstrate understanding by representing problems in multiple ways:
- Help students make connections between concepts and representations.
- Provide opportunities for students to use manipulatives when investigating concepts.
- Guide students from concrete to pictorial to abstract representations as understanding progresses.
- Show students that various representations can have different purposes and can be useful in different situations.

MA.K12.MTR.3.1 Complete tasks with mathematical fluency.

Mathematicians who complete tasks with mathematical fluency:
- Select efficient and appropriate methods for solving problems within the given context.
- Maintain flexibility and accuracy while performing procedures and mental calculations.
- Complete tasks accurately and with confidence.
- Adapt procedures to apply them to a new context.
- Use feedback to improve efficiency when performing calculations.

Clarifications:
Teachers who encourage students to complete tasks with mathematical fluency:
- Provide students with the flexibility to solve problems by selecting a procedure that allows them to solve efficiently and accurately.
- Offer multiple opportunities for students to practice efficient and generalizable methods.
- Provide opportunities for students to reflect on the method they used and determine if a more efficient method could have been used.

MA.K12.MTR.4.1 Engage in discussions that reflect on the mathematical thinking of self and others.

Mathematicians who engage in discussions that reflect on the mathematical thinking of self and others:

- Communicate mathematical ideas, vocabulary and methods effectively.
- Analyze the mathematical thinking of others.
- Compare the efficiency of a method to those expressed by others.
- Recognize errors and suggest how to correctly solve the task.
- Justify results by explaining methods and processes.
- Construct possible arguments based on evidence.

Clarifications:
Teachers who encourage students to engage in discussions that reflect on the mathematical thinking of self and others:

- Establish a culture in which students ask questions of the teacher and their peers, and error is an opportunity for learning.
- Create opportunities for students to discuss their thinking with peers.
- Select, sequence and present student work to advance and deepen understanding of correct and increasingly efficient methods.
- Develop students' ability to justify methods and compare their responses to the responses of their peers.

MA.K12.MTR.5.1 Use patterns and structure to help understand and connect mathematical concepts.

Mathematicians who use patterns and structure to help understand and connect mathematical concepts:

- Focus on relevant details within a problem.
- Create plans and procedures to logically order events, steps or ideas to solve problems.
- Decompose a complex problem into manageable parts.
- Relate previously learned concepts to new concepts.
- Look for similarities among problems.
- Connect solutions of problems to more complicated large-scale situations.

Clarifications:
Teachers who encourage students to use patterns and structure to help understand and connect mathematical concepts:

- Help students recognize the patterns in the world around them and connect these patterns to mathematical concepts.
- Support students to develop generalizations based on the similarities found among problems.
- Provide opportunities for students to create plans and procedures to solve problems.
- Develop students' ability to construct relationships between their current understanding and more sophisticated ways of thinking.

MA.K12.MTR.6.1 Assess the reasonableness of solutions.

Mathematicians who assess the reasonableness of solutions:
- Estimate to discover possible solutions.
- Use benchmark quantities to determine if a solution makes sense.
- Check calculations when solving problems.
- Verify possible solutions by explaining the methods used.
- Evaluate results based on the given context.

Clarifications:
Teachers who encourage students to assess the reasonableness of solutions:
- Have students estimate or predict solutions prior to solving.
- Prompt students to continually ask, "Does this solution make sense? How do you know?"
- Reinforce that students check their work as they progress within and after a task.
- Strengthen students' ability to verify solutions through justifications.

MA.K12.MTR.7.1 Apply mathematics to real-world contexts.

Mathematicians who apply mathematics to real-world contexts:
- Connect mathematical concepts to everyday experiences.
- Use models and methods to understand, represent and solve problems.
- Perform investigations to gather data or determine if a method is appropriate.
- Redesign models and methods to improve accuracy or efficiency.

Clarifications:
Teachers who encourage students to apply mathematics to real-world contexts:
- Provide opportunities for students to create models, both concrete and abstract, and perform investigations.
- Challenge students to question the accuracy of their models and methods.
- Support students as they validate conclusions by comparing them to the given situation.
- Indicate how various concepts can be applied to other disciplines.

ELA Expectations

ELA.K12.EE.1.1 Cite evidence to explain and justify reasoning.

ELA.K12.EE.2.1 Read and comprehend grade-level complex texts proficiently.

ELA.K12.EE.3.1 Make inferences to support comprehension.

ELA.K12.EE.4.1 Use appropriate collaborative techniques and active listening skills when engaging in discussions in a variety of situations.

ELA.K12.EE.5.1 Use the accepted rules governing a specific format to create quality work.

ELA.K12.EE.6.1 Use appropriate voice and tone when speaking or writing.

English Language Development

ELD.K12.ELL.MA Language of Mathematics

ELD.K12.ELL.MA.1	English language learners communicate information, ideas and concepts necessary for academic success in the content area of Mathematics.

Calculus

MA.912.C.1 Develop an understanding for limits and continuity. Determine limits and continuity.

MA.912.C.1.1	Demonstrate understanding of the concept of a limit and estimate limits from graphs and tables of values. *Example:* For $f(x) = \frac{x^2+2x-8}{x-2}$, estimate $\lim_{x\to 2}\left(\frac{x^2+2x-8}{x-2}\right)$ by calculating the function's values for $x = 2.1, 2.01, 2.001$ and for $x = 1.9, 1.99, 1.999$. Explain your answer.
MA.912.C.1.2	Determine algebraically the value of a limit if it exists using limits of sums, differences, products, quotients and compositions of continuous functions. *Example:* Find $\lim_{x\to\pi}(\sin x \cos x + \tan x)$.
MA.912.C.1.3	Find limits of rational functions that are undefined at a point. *Example:* The magnitude of the force between two positive charges, q_1 and q_2, can be described by the following function: $F(r) = k\frac{q_1 q_2}{r^2}$, where k is Coulomb's constant and r is the distance between the two charges. Find the limit as r approaches 0 of the function $F(r)$. Interpret the answer in terms of the context.

MA.912.C.1.4	Find one-sided limits. *Example:* Find $\lim_{x\to 4^-} -\sqrt{4-x}$.
MA.912.C.1.5	Find limits at infinity. *Example:* Find $\lim_{x\to\infty}(2x^3 - 500x^2)$.
MA.912.C.1.6	Decide when a limit is infinite and use limits involving infinity to describe asymptotic behavior. *Example:* Where does the function, $f(x) = \frac{1}{x^2-7x+10}$, have asymptote(s)?
MA.912.C.1.7	Find limits by using the Squeeze Theorem or algebraic manipulation. *Example:* Find $\lim_{x\to 0}\frac{\sin^2 x}{x}$.
MA.912.C.1.8	Find limits of indeterminate forms using L'Hôpital's Rule.
MA.912.C.1.9	Define continuity in terms of limits. *Example:* Given that the limit of $g(x)$ as x approaches to 5 exists, is the statement "$g(x)$ is continuous at $x = 5$" necessarily true? Provide example functions to support your conclusion.
MA.912.C.1.10	Given the graph of a function, identify whether a function is continuous at a point. If not, identify the type of discontinuity for the given function.
MA.912.C.1.11	Apply the Intermediate Value Theorem and the Extreme Value Theorem. *Example:* Use the Intermediate Value Theorem to show that $g(x) = x^3 + 3x^2 - 9x - 2$ has a zero between $x = 0$ and $x = 3$. *Example:* Use the Extreme Value Theorem to decide whether $f(x) = tan(x)$ has a minimum and maximum on the interval $\left[-\frac{\pi}{4}, \frac{\pi}{4}\right]$. What about on the interval $[-\pi, \pi]$?

MA.912.C.2 Develop an understanding for and determine derivatives.

MA.912.C.2.1	State, understand and apply the definition of derivative. Apply and interpret derivatives geometrically and numerically. *Example:* Find $\lim_{h\to 0}\frac{(5+h)^2-5^2}{h}$. What does the result tell you? Use the limit to determine the derivative function for $f(x) = x^2$.

MA.912.C.2.2	Interpret the derivative as an instantaneous rate of change or as the slope of the tangent line.

MA.912.C.2.3	Prove the rules for finding derivatives of constants, sums, products, quotients and the Chain Rule.

Benchmark Clarifications:
Clarification 1: Special cases of rules include a constant multiple of a function and the power of a function.

MA.912.C.2.4	Apply the rules for finding derivatives of constants, sums, products, quotients and the Chain Rule to solve problems with functions limited to algebraic, trigonometric, inverse trigonometric, logarithmic and exponential.

Example: Find $\frac{dy}{dx}$ for the function $y = \ln x$.
Example: Show that the derivative of $f(x) = \tan x$ is $f'(x) = \sec^2 x$ using the quotient rule for derivatives.
Example: Find $f'(x)$ for $f(x) = (x^2 + 2)^{\frac{1}{2}}$.

Benchmark Clarifications:
Clarification 1: Special cases of rules include a constant multiple of a function and the power of a function.

MA.912.C.2.5	Find the derivatives of implicitly defined functions.

Example: For the equation $xy - x^2y^2 = 5$, find $\frac{dy}{dx}$ at the point $(2, 3)$.

MA.912.C.2.6	Find derivatives of inverse functions.

Example: Let $f(x) = 2x^3$ and $g(x) = f^{-1}(x)$, find $g'(2)$.

MA.912.C.2.7	Find second derivatives and derivatives of higher order.

Example: Let $f(x) = e^{5x}$, find $f''(x)$ and $f'''(x)$.

MA.912.C.2.8	Find derivatives using logarithmic differentiation.

Example: Find the derivative of $f(x) = (3x^2 + 5)^x$.

MA.912.C.2.9	Demonstrate and use the relationship between differentiability and continuity.

Example: Is $f(x) = |x|$ continuous at $x = 0$? Is $f(x)$ differentiable at $x = 0$? Explain your answers.

MA.912.C.2.10	Apply the Mean Value Theorem. *Example:* At a car race, two cars join the race at the same point at the same time. They finish the race in a tie. Prove that sometime during the race, the two cars had exactly the same speed. (Hint: Define $f(t)$, $g(t)$, and $h(t)$, where $f(t)$ is the distance that Car A has traveled at time t; $g(t)$ is the distance that Car B has travelled at time t; and $h(t) = f(t) - g(t)$.)

MA.912.C.3 Apply derivatives to solve problems.

MA.912.C.3.1	Find the slope of a curve at a point, including points at which there are vertical tangent lines. *Example:* Find the slope of the line tangent to the graph of $f(x) = \sqrt[3]{1-x}$ at $x = 1$.
MA.912.C.3.2	Find an equation for the tangent line to a curve at a point and a local linear approximation. *Example:* Use a local linear approximation to estimate the value of $f(x) = x^x$ at $x = 2.1$.
MA.912.C.3.3	Determine where a function is decreasing and increasing using its derivative. *Example:* For what values of x is the function $f(x) = \frac{x}{x^2+1}$ decreasing?
MA.912.C.3.4	Find local and absolute maximum and minimum points of a function. *Example:* For the graph of the function $f(x) = x^3 - 3x$, find the local maximum and local minimum points of $f(x)$ on $[-2,3]$.
MA.912.C.3.5	Determine the concavity and points of inflection of a function using its second derivative. *Example:* For the graph of the function $f(x) = x^3 - 3x$, find the points of inflection of $f(x)$ and determine where $f(x)$ is concave upward and concave downward.
MA.912.C.3.6	Sketch graphs by using first and second derivatives. Compare the corresponding characteristics of the graphs of f, f' and f''. *Example:* Sketch the graph of $f(x) = x^4 + 3x^2 - 2x + 1$ using information from the first and second derivatives.
MA.912.C.3.7	Solve optimization problems using derivatives. *Example:* Find the shortest length of fencing you can use to enclose a rectangular field with and area of $5000\ m^2$. *Example:* Find the dimensions of an equilateral triangle and a square that will produce the least area is the sum of their perimeters is 20 centimeters.

MA.912.C.3.8	Find average and instantaneous rates of change. Explain the instantaneous rate of change as the limit of the average rate of change. Interpret a derivative as a rate of change in applications, including velocity, speed and acceleration. *Example:* The vertical distance traveled by an object within the earth's gravitational field, neglecting air resistance, is given by the equation $x = 0.5gt^2 + v_ot + x_o$, where g is the force on the object due to earth's gravity, v_o is the initial velocity, x_0 is the initial height above the ground, t is the time in seconds and down is the negative vertical direction. Determine the instantaneous speed and the average speed for an object, initially at rest, 3 seconds after it is dropped from a 100 meter tall cliff. Describe the object 5 seconds after it is dropped from the same height. Use $g = -10\frac{m}{s^2}$.
MA.912.C.3.9	Find the velocity and acceleration of a particle moving in a straight line. *Example:* A bead on a wire moves so that, after t seconds, its distance s cm. from the midpoint of the wire is given by $s = 5\sin\left(t - \frac{\pi}{4}\right)$. Find its maximum velocity and where along the wire this occurs.
MA.912.C.3.10	Model and solve problems involving rates of change, including related rates. *Example:* One boat is heading due south at 10 mph. Another boat is heading due west at 15 mph. Both boats are heading toward the same point. If the boats maintain their speeds and directions, they will meet in two hours. Find the rate, in miles per hour, that the distance between them is decreasing exactly one hour before they meet.

MA.912.C.4 Develop an understanding for and determine integrals.

MA.912.C.4.1	Interpret a definite integral as a limit of Riemann sums. Calculate the values of Riemann sums over equal subdivisions using left, right and midpoint evaluation points. *Example:* Find the values of the Riemann sums over the interval [0,1] using 12 and 24 subintervals of equal width for $f(x) = e^x$ evaluated at the midpoint of each subinterval. Write an expression for the Riemann sums using n intervals of equal width. Find the limit of this Riemann Sums as n goes to infinity. *Example:* Estimate $\int_0^\pi \sin x\, dx$ using a Riemann midpoint sum with 4 subintervals. *Example:* Find an approximate value for $\int_0^3 x^2 dx$ using 6 rectangles of equal width under the graph of $f(x) = x^2$ between $x = 0$ and $x = 3$. How did you form your rectangles? Compute this approximation three times using at least three different methods to form the rectangles.

MA.912.C.4.2	Apply Riemann sums, the Trapezoidal Rule and technology to approximate definite integrals of functions represented algebraically, geometrically and by tables of values. *Example:* Approximate the value of $\int_0^3 x^2 dx$ using the Trapezoidal Rule with 6 subintervals over [0,3] for $f(x) = x^2$. *Example:* Find an approximation to $\int_{-3}^0 \sqrt{9 - x^2} dx$.
MA.912.C.4.3	Interpret a definite integral of the rate of change of a quantity over an interval as the change of the quantity over the interval. *Example:* Explain why $\int_4^5 2x dx = 5^2 - 4^2$.

Benchmark Clarifications:

Clarification 1: Instruction focuses on the relationship $\int_a^b f'(x)dx = f(b) - f(a)$ which is the Fundamental Theorem of Calculus.

MA.912.C.4.4	Evaluate definite integrals by using the Fundamental Theorem of Calculus. *Example:* Evaluate $\int_1^5 e^x dx$.
MA.912.C.4.5	Analyze function graphs by using derivative graphs and the Fundamental Theorem of Calculus.
MA.912.C.4.6	Evaluate or solve problems using the properties of definite integrals. Properties are limited to the following: • $\int_a^b [f(x) + g(x)]dx = \int_a^b f(x)dx + \int_a^b g(x)dx$ • $\int_a^b k \cdot f(x)dx = k \int_a^b f(x)dx$ • $\int_a^a f(x)dx = 0$ • $\int_a^b f(x)dx = -\int_b^a f(x)dx$ • $\int_a^b f(x)dx + \int_b^c f(x)dx = \int_a^c f(x)dx$ • If $f(x) \leq g(x)$ on $[a, b]$, then $\int_a^b f(x)dx \leq \int_a^b g(x)dx$.
MA.912.C.4.7	Evaluate definite and indefinite integrals by using integration by substitution. *Example:* Find $\int x^2(x^3 + 1)^4 dx$.

MA.912.C.5 Apply integrals to solve problems.

MA.912.C.5.1	Find specific antiderivatives using initial conditions, including finding velocity functions from acceleration functions, finding position functions from velocity functions and solving applications related to motion along a line. *Example:* A bead on a wire moves so that its velocity, in cm/s, after t seconds, is given by $v(t) = 3\cos 3t$. Given that it starts 2 cm to the left of the midpoint of the wire, find its position after 5 seconds.
MA.912.C.5.2	Solve separable differential equations. *Example:* A certain amount of money, P, is earning interest continually at a rate of r. Write a separable differential equation to model the rate of change of the amount of money with respect to time.
MA.912.C.5.3	Solve differential equations of the form $\frac{dy}{dt} = ky$ as applied to growth and decay problems. *Example:* The amount of a certain radioactive material was 10 kg a year ago. The amount is now 9 kg. When will it be reduced to 1 kg? Explain your answer.
MA.912.C.5.4	Display a graphic representation of the solution to a differential equation by using slope fields, and locate particular solutions to the equation. *Example:* Draw a slope field for $\frac{dy}{dx} = x^2$ and graph the particular solution that passes through the point $(2,4)$.
MA.912.C.5.5	Find the area between a curve and the x-axis or between two curves by using definite integrals. *Example:* Find the area bounded by $y = \sqrt{x}$, $y = 0$ and $x = 2$.
MA.912.C.5.6	Find the average value of a function over a closed interval by using definite integrals. *Example:* The daytime temperature, in degrees Fahrenheit, in a certain city t hours after 8 AM can be modeled by the function $T = 54 + 15\sin\left(\frac{\pi t}{12}\right)$. What is the average temperature in this city during the time period from 8 AM to 8 PM?
MA.912.C.5.7	Find the volume of a figure with known cross-sectional area, including figures of revolution, by using definite integrals. *Example:* A cone with its vertex at the origin lies symmetrically along the x-axis. The base of the cone is at $x = 5$ and the base radius is 7. Use integration to find the volume of the cone. *Example:* What is the volume of the solid created when the area between the curves $f(x) = x$ and $g(x) = x^2$ for $0 \leq x \leq 1$ is revolved around the y-axis?