Florida Interim Assessment Item Bank and Test Platform

Item Specifications

Geometry
Grades 9–12

FLORIDA DEPARTMENT OF EDUCATION

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I. Introduction

In July 2010 the Florida Department of Education (FDOE) approved the adoption of the Common Core State Standards (CCSS) for Mathematics to support its pursuit of improved outcomes for all Florida mathematics students and participation in national educational initiatives, such as Race to the Top. The U.S. Department of Education awarded a Race to the Top grant to Florida in August 2010. An important component of this grant focused on the development of high-quality assessment items and balanced assessments for use by districts, schools, and teachers. The assessment items will be stored in the Florida Interim Assessment Item Bank and Test Platform (IBTP), a statewide secure system that allows Florida educators to search the item bank, export test items, and generate customized high-quality assessments for computer-based delivery or paper-and-pencil delivery. The IBTP allows Florida educators to determine what students know and are able to do relative to instruction based on Florida's Next Generation Sunshine State Standards and the Common Core State Standards.

A. Purpose of the Item Specifications

The *Item Specifications* define the expectations for content, standards alignment, and format of assessment items for the Item Bank and Test Platform. The *Item Specifications* are intended for use by item writers and reviewers in the development of high-quality assessment items.

B. Scope

The *Item Specifications* provide general and grade-specific guidelines for the development of all Mathematics assessment items available in the Florida Interim Assessment Item Bank.

C. Standards Alignment

Items developed for the Florida Interim Assessment Item Bank and Test Platform will align to the Common Core State Standards for Mathematics. The Common Core State Standards for Mathematics are structured into three levels of specificity: Domains, Clusters, and Standards. These define what mathematics students should know and be able to do at every grade level/course, kindergarten through high school.

II. Criteria for Item Development

Mathematics item writers for the Florida Interim Assessment Item Bank must have a comprehensive knowledge of mathematics curriculum based on the Common Core State Standards and an understanding of the range of cognitive abilities of the target student population. Item writers should understand and consistently apply the guidelines established in this document. Item writers are expected to use their best judgment in writing items that measure the Mathematics standards of the CCSS without introducing extraneous elements that reflect bias for or against a group of students.

A. Overall Considerations for Item Development

These guidelines are provided to ensure the development of high-quality assessment items for the Florida Interim Assessment Item Bank.

1. Each item should be written to measure primarily one Common Core State Standard; however, other standards may also be addressed for some item types. In addition to the content standard alignment, each item should align to at least one Mathematical Practice Standard. Some items should be written reflecting the ELA Literacy standards cited in the course descriptions.

2. Items should be appropriate for students in terms of grade-level/course instruction, experience and difficulty, cognitive development, and reading level. The reading level of the test items should be on grade level.

3. Items should be written at or above the cognitive level (DOK) of the standard unless otherwise noted in the Individual Standard Specifications sections.

4. Each item should be written clearly and unambiguously to elicit the desired response.

5. Items should not disadvantage or exhibit disrespect to anyone in regard to age, gender, race, ethnicity, language, religion, socioeconomic status, disability, occupation, or geographic region.

6. At grades kindergarten through 5, items should be able to be answered without using a calculator. For grades 6 through 7, a four-function calculator may be used. For grade 8, a scientific calculator may be used. For Algebra 1, Geometry, and Algebra 2, both a scientific calculator and a graphing calculator (with functionalities similar to that of a TI-84) may be used. For all grades, calculators should not be used for items where computational skills or fluency are being assessed.

B. Item Contexts

The context in which an item is presented is called the item context or scenario. These guidelines are provided to assist item writers with development of items within an appropriate context.

1. The item context should be designed to interest students at the targeted level. Scenarios should be appropriate for students in terms of grade-level experience and difficulty, cognitive development, and reading level.

2. The context should be directly related to the question asked. The context should lead the student cognitively to the question. Every effort should be made to keep items as concise as possible without losing cognitive flow or missing the overall idea or concept.

3. Item contexts should include subject areas other than mathematics. Specifically, topics from grade-level/course Next Generation Sunshine State Standards for Science and Social Studies, and Common Core State Standards for English Language Arts may be used where appropriate.

4. Items including specific information or data must be accurate and verified against reliable sources. Source documentation must accompany these types of items.

5. Mathematics item stimuli should include written text and/or visual material, such as graphs, tables, diagrams, maps, models, and/or other illustrations.

6. All item scenarios, graphics, diagrams, and illustrations must be age-, grade-, and experience-appropriate.

7. All graphs used in item stems or answer options must be complete with title, scale, and labeled axes, except when these components are to be completed by the student.

8. Any graphics in items should be uncluttered and should clearly depict the necessary information. Graphics should contain relevant details that contribute to the students' understanding of the item or that support the context of the item. Graphics should not introduce bias to the item.

9. Item content should be timely but not likely to become dated too quickly.

C. Use of Media

Media can be used to provide either necessary or supplemental information—that is, some media contain information that is necessary for answering the question, while other media support the context of the question. Items may include diagrams, illustrations, charts, tables, audio files, or video files unless otherwise noted in the Individual Standard Specifications. Some standards require a heavier use of graphics than others. Geometry, for example, relies heavily on graphics to convey information.

1. Items should not begin with media. Media in items are always preceded by text.

2. All visual media (tables, charts, graphs, photographs, etc.) should be titled. Titles should be in all caps, boldfaced, and centered, and may be placed above or below the visual media.

D. Item Style and Format

This section presents stylistic guidelines and formatting directions that should be followed while developing items.

1. Items should be clear and concise, and they should use vocabulary and sentence structure appropriate for the assessed grade level.

2. The words *most likely* or *best* should be used only when appropriate to the question.

3. Items using the word *not* should emphasize the word *not* using all uppercase letters (e.g., Which of the following is NOT an example of . . .). The word *not* should be used sparingly.

4. For items that refer to an estimate (noun), lowercase letters should be used.

5. As appropriate, boldface type should be used to emphasize key words in the item (e.g., least, most, greatest, percent, mode, median, mean, range).

6. Masculine pronouns should NOT be used to refer to both sexes. Plural forms should be used whenever possible to avoid gender-specific pronouns (e.g., instead of "The student will make changes so that he . . . ," use "The students will make changes so that they . . .").

7. An equal balance of male and female names should be used, including names representing different ethnic groups appropriate for Florida.

8. For clarity, operation symbols, equality signs, and ordinates should be preceded and followed by one space.

9. Decimal numbers between –1 and 1 (including currency) should have a leading zero.

10. Metric numbers should be expressed in a single unit when possible (e.g., 1.4 kilograms instead of 1 kilogram 400 grams).

11. Decimal notation should be used for numbers with metric units (e.g., 1.2 grams instead of 151 grams).

12. Commas should be used within numbers greater than or equal to 1,000. Commas may be omitted within an equation or expression.

13. Units of measure should be spelled out, except in graphics, where an abbreviation may be used (e.g., *ft* or *yd*). Abbreviations that also spell a word must be followed by a period to avoid confusion. For example, to avoid confusion with the preposition *in*, the abbreviation *in.* should be used for the unit of measure *inches* and should include a period. If an abbreviation is used in a graphic, an explanation of the meaning of the abbreviation should be included in the stem.

14. In titles for tables and charts and in labels for axes, the units of measure should be included, preferably in lowercase letters and in parentheses, e.g., *height* (*in inches*).

15. Fractions should be typed with a horizontal fraction bar. The numerator and denominator should be centered with respect to each other. The bar should cover all portions (superscripts, parentheses, etc.) of the numerator and denominator. In a mixed number, a half space should appear between the whole number and the fraction. If a variable appears before or after a fraction bar, the variable should be centered with respect to the fraction bar. If a stimulus, stem, or set of responses contains a fraction in fractional notation, that portion of the item should be 1.5-spaced.

16. In general, numbers zero through nine should be presented as words and numbers 10 and above should be presented as numerals. In the item stem, any numbers needed to compute answers should be presented as numerals.

E. Item Types

This section presents guidelines for development of the following types of items:

1. Selected Response (SR) Items (1 point)

2. Gridded Response (GR) and Short Response (SHR) Items (1 point)

3. Constructed Response and Extended Response Items

   a. Constructed Response (CR) Items (2 points)

   b. Extended Response (ER) Items (4 points)

4. Essay Response (ESR) Items (6 points)

5. Performance Task (PT) Items (1–10 points)

1. **Selected Response (SR) Items (1 point)**

   Selected response items require students to choose an answer from the choices given. Each item consists of a stem and either three or four answer options, depending on the grade level/course (see c below). One of the answer options is the correct answer, and the remaining options are called distractors. Selected response items may include a stimulus and/or passage.

   a. SR items should take an average of 1 minute per item to solve.

   b. SR items are worth 1 point each.

   c. SR items in grades K, 1, and 2 should have three answer choices (A, B, and C). SR items for all other grades and courses should have four answer choices (A, B, C, and D).

   d. Answer options that are single words should be arranged in alphabetical or reverse alphabetical order.

   e. Answer options that are phrases or sentences should be arranged from shortest to longest or longest to shortest.

   f. Numerical answer options should be arranged in ascending or descending order.

   g. Numerical answer options that represent relative magnitude or size should be arranged as they are shown in the stem or some other logical order.

   h. When the item requires the identification of a choice from the item stem, table, chart, or illustration, the options should be arranged as they are presented in the item stem, table, chart, or illustration.

   i. If the answer options for an item are neither strictly numerical nor denominate numbers, the options should be arranged by the logic presented in the item, by alphabetical order, or by length.

   j. Distractor rationales should represent computational or conceptual errors or misconceptions commonly made by students who have not mastered the assessed concepts.

   k. Outliers (i.e., answer choices that are longer phrases or sentences than the other choices, or choices with significantly more/fewer digits than the other choices) should NOT be used.

   l. Options such as none of the above, all of the above, not here, not enough information, or cannot be determined should not be used as answer options.

2. **Gridded Response (GR) and Short Response (SHR) Items (1 point)**

   a. Gridded response and short response items are worth 1 point.

   b. The GR format is designed for items that require a positive numeric solution (whole numbers, decimals, percents, or fractions).

   c. The bubble grids used with GR items should contain eight columns. Each column will contain the digits 0 through 9, decimal point (.), and fraction bar (/) enclosed in bubbles.

   d. Gridded response items should include instructions that specify the unit in which the answer is to be provided (e.g., inches). If several units of measure are in the item (e.g., in an item involving a conversion), the final unit needed for the answer should be written in boldface.

   e. The short response format is designed for items that result in a value or answer that cannot be answered in the gridded response format (negative numbers, expressions, etc.).

3. **Constructed Response and Extended Response Items**

   Mathematics constructed response and extended response items require students to produce a response in words, pictures, diagrams, and/or numbers. As such, these items are especially suited to assessing many of the more complex tasks and high-level thinking skills demanded by the Common Core State Standards for Mathematics. The Florida Interim Assessment Item Bank will include 2-point constructed response items (CR) and 4-point extended response items (ER).

   Overall characteristics for mathematics CRs and ERs are as follows:

   a. The item should measure understanding and insight of mathematical concepts rather than rote memory or factual recall.

   b. Real-world, factual stimulus material (charts, graphs, tables, etc.) must cite the source used.

   c. Items requiring students to produce responses as pictures, diagrams, graphs, tables, etc., should provide workspace and/or templates where appropriate.

a. **Constructed Response (CR) Items (2 points)**

Constructed response items usually include a scenario and instructions on how to respond. The recommended time allotment for a student to respond is 5 minutes. A complete answer is worth 2 points, and a partial answer is worth 1 point. The constructed response holistic rubric and exemplar specific to each item are used for scoring as follows:

SCORING RUBRIC	
2	Work demonstrates a **clear and complete** understanding of the mathematical concepts and/or procedures required by the task. Appropriate strategy is shown with clear and complete explanations and interpretations.
1	Response demonstrates a **partial** understanding of the mathematical concepts and/or procedures. Appropriate strategy is shown, but explanation or interpretation has minor flaws. OR Response is incorrect because of calculation errors. Work and strategy indicate a **clear** understanding of the mathematical concepts and/or procedures required by the task.
0	Response is irrelevant, inappropriate, or not provided.

Exemplars: A specific exemplar should be developed for each constructed response item. Exemplars will be used as scoring guides and should be specific to the item, but not so specific as to discount multiple correct answers. Exemplars should include a clear and defensible description of the top score point, and contain straightforward language that is accurate, complete, and easy to interpret.

b. Extended Response (ER) Items (4 points)

Extended response items include a scenario and instructions on how to respond and are worth 4 points. However, ER items are usually more complex than SHR and 2-point CR items. The recommended time allotment for a student to respond is 10–15 minutes. The extended response holistic rubric and exemplar specific to each item are used for scoring as follows:

SCORING RUBRIC	
4	Work demonstrates a **clear and complete** understanding of the mathematical concepts and/or procedures required by the task. Appropriate strategy is shown with clear and complete explanations and interpretations.
3	Work demonstrates a **clear** understanding of the mathematical concepts and/or procedures but is not complete. Appropriate strategy is shown, but explanation or interpretation has minor flaws. OR Response is incorrect because of calculation errors. Work and strategy indicate a **clear** demonstration of the problem.
2	Response demonstrates a **partial** understanding of the mathematical concepts and/or procedures. Appropriate strategy is shown, but explanation or interpretation has minor flaws.
1	Response shows **minimal** understanding of the mathematical concepts and/or procedures or provides no explanation or interpretation for the solution or shows major flaws.
0	Response is irrelevant, inappropriate, or not provided.

Exemplars: A specific exemplar should be developed for each extended response item. Exemplars will be used as scoring guides and should be specific to the item, but not so specific as to discount multiple correct answers. Exemplars should include a clear and defensible description of the top score point, and contain straightforward language that is accurate, complete, and easy to interpret.

4. **Essay Response (ESR) Items (6 points)**

The essay response item consists of asking a general question or providing a stimulus (such as an article or research paper on a relevant topic), and asking students to express their thoughts or provide facts about the topic using logic and reason. Essay response items encompass a higher level of thinking and a broader range of skills that includes CCSS literacy standards, which is critical to future success in higher education and the workforce.

In most cases, essay responses will go beyond a single paragraph in length, with a distinct introduction, body, and conclusion. An essay response will be worth a total of 6 points, with a rubric structure similar to that of the 4-point extended response. Students should be given about 20 to 30 minutes to complete each item.

Exemplars: A specific exemplar should be developed for each essay response item. Exemplars will be used as scoring guides and should be specific to the item, but not so specific as to discount multiple correct answers. Exemplars should include a clear and defensible description of the top score point, and contain straightforward language that is accurate, complete, and easy to interpret.

5. **Performance Tasks (PT) (1–10 points)**

Performance tasks are used to measure students' ability to *demonstrate* knowledge and skills from one or more CCSS. Specifically, performance tasks may require students to create a product, demonstrate a process, or perform an activity that demonstrates proficiency in Mathematics. They are evaluated using customized scoring exemplars, and each task may be worth 1–10 points.

Performance tasks may have the following characteristics:

a. Performance tasks may cover a short time period or may cover an extended period.

b. Performance tasks must contain clear and explicit directions for understanding and completing the required component tasks and producing the objective output.

c. All tasks, skills, and/or behaviors required by the performance tasks must be objective, observable, and measurable.

d. All necessary equipment, materials, and resources should be referenced within the text of the performance task.

e. Performance tasks should elicit a range of score points.

f. Performance tasks generally require students to organize, apply, analyze, synthesize, and/or evaluate concepts.

g. Performance tasks may measure performance in authentic situations and outside the classroom, where appropriate and practical.

h. Typical response formats include demonstrations, laboratory performance, oral presentations, exhibits, or other products.

i. Every performance task requires a companion exemplar to be used for scoring purposes. Exemplars should meet the following criteria.

   i   The exemplars and performance tasks should be developed in tandem to ensure compatibility.

   ii Exemplars must be specific to the individual requirements of each performance task; generic rubrics are not acceptable.

   iii The exemplar must allow for efficient and consistent scoring.

   iv Each part of the performance task must have a clearly stated score point in the exemplar and when a part of the task is divided into sections or requirements, each of those must have a maximum score indicated.

   v   The exemplar descriptors consist of an ideal response exemplar and should allow for all foreseeable methods of correctly and thoroughly completing all requirements of the performance task.

F. Readability

Items must be written with readability in mind. In addition, vocabulary must be appropriate for the grade level being tested. The following sources provide information about the reading level of individual words:

Taylor, Stanford E. *EDL Core Vocabularies: Reading, Mathematics, Science, and Social Studies*. Austin, TX: Steck-Vaughn-EDL, 1989.

Mogilner, Alijandra. *Children's Writer's Word Book*. Cincinnati, OH: Writer's Digest Books, 1992.

G. Cognitive Complexity

1. Overview

Florida's adoption of the Common Core State Standards (CCSS) for Mathematics and English Language Arts & Literacy in History/Social Studies, Science, and Technical Subjects presents Florida with an opportunity to revise its current Depth of Knowledge (DOK) Model of Cognitive Complexity. More information about Florida's Depth of Knowledge levels is available online at *http://www.cpalms.org/cpalms/dok.aspx*.

2. Levels of Depth of Knowledge for Mathematics

Level 1 (Recall) includes the recall of information such as a fact, definition, term, or a simple procedure, as well as performing a simple algorithm or applying a formula. That is, in mathematics a one-step, well-defined, or straight algorithmic procedure should be included at this lowest level.

Some examples that represent but do not constitute all of Level 1 performance are:

- Count to 100 by ones and by tens.
- Fluently multiply and divide within 100, using strategies such as the relationship between multiplication and division (e.g., knowing that $8 \times 5 = 40$, one knows $40 \div 5 = 8$).

- Write simple expressions that record calculations with numbers, and interpret numerical expressions without evaluating them. For example, express the calculation "add 8 and 7 and then multiply by 2" as $2 \times (8 + 7)$. Recognize that $3 \times (18932 + 921)$ is three times as large as $18932 + 921$ without having to calculate the indicated sum or product.

- Enter measurement data into a data table.

- Identify the variables indicated in a two-dimensional graph.

Level 2 (Basic Application of Concepts & Skills) includes the engagement of some mental processing beyond a habitual response. A Level 2 standard or assessment item requires students to make some decisions as to how to approach the problem or activity, whereas Level 1 requires students to demonstrate a rote response, perform a well-known algorithm, follow a set procedure (like a recipe), or perform a clearly defined series of steps. For example, to compare data requires first identifying characteristics of the objects or phenomenon and then grouping or ordering the objects. Interpreting information from a simple graph, requiring reading information from the graph, also is a Level 2. Interpreting information from a complex graph that requires some decisions on what features of the graph need to be considered and how information from the graph can be aggregated is a Level 3. Caution is warranted in interpreting Level 2 as only skills because some reviewers will interpret skills very narrowly as primarily numerical skills, and such interpretation excludes from this level other skills such as visualization skills and probability skills, which may be more complex simply because they are less common and require more mental processing.

Some examples that represent but do not constitute all of Level 2 performance are:

- Understand that the last number name said tells the number of objects counted. The number of objects is the same regardless of their arrangement or the order in which they were counted.

- Express the length of an object as a whole number of length units, by laying multiple copies of a shorter object (the length unit) end to end.

- Measure and estimate liquid volumes and masses of objects using standard units of grams (g), kilograms (kg), and liters (l).

- Apply properties of operations as strategies to add and subtract rational numbers.

- Measure and record data and produce graphs of relevant variables.

- Graph proportional relationships, interpreting the unit rate as the slope of the graph.

Level 3 (Strategic Thinking & Complex Reasoning) requires reasoning, planning, using evidence, and a higher level of thinking than the previous two levels. In most instances, requiring students to explain their thinking is a Level 3. Activities that require students to make conjectures are also at this level. The cognitive demands at Level 3 are complex and abstract. The complexity does not result from the fact that there are multiple answers, a possibility for both levels 1 and 2, but because the task requires more demanding reasoning. However, an activity that has more than one possible answer and requires students to justify the response they give would most likely be a Level 3.

Some examples that represent but do not constitute all of Level 3 performance are:

- Explain why addition and subtraction strategies work, using place value and the properties of operations.

- Interpret the rate of change and initial value of a linear function in terms of the situation it models, and in terms of its graph or a table of values.

- Given a real-world situation, formulate a problem.

- Organize, represent, and interpret data obtained through experiments or observations.

- Formulate a mathematical model to describe a complex phenomenon.

- Justify a solution to a problem.

- Analyze a deductive argument.

Level 4 (Extended Thinking & Complex Reasoning) in mathematics involves the application of Level 3 processes and skills over an extended period. This is likely to incorporate demands from other content areas (e.g., English language arts, science) in the development and support of mathematical arguments that describe some real-world phenomenon or situation.

Some examples that represent but do not constitute all of Level 4 performance are:

- Derive a mathematical model to explain a complex phenomenon or make a prediction.

- Complete a project requiring the formulation of questions, devising a plan, collecting data, analyzing the data, and preparing a written report describing the justification of the conclusions reached.

H. Item Difficulty

Item writers will not be expected to make a prediction of difficulty for each item created. However, item writers should develop items that reflect a range of difficulty.

I. Universal Design

The application of universal design principles helps develop assessments that are usable to the greatest number of students, including students with disabilities and nonnative speakers of English. To support the goal of providing access to all students, the items in the Florida Interim Assessment Item Bank maximize readability, legibility, and compatibility with accommodations, and item development includes a review for potential bias and sensitivity issues.

Items must allow for the widest possible range of student participation. Item writers must attend to the best practices suggested by universal design, including, but not limited to,

1. reduction in wordiness
2. avoidance of ambiguity
3. selection of reader-friendly construction and terminology
4. consistently applied concept names and graphic conventions

Universal design principles also inform decisions about item layout and design, including, but not limited to, type size, line length, spacing, and graphics.

J. Sample Items

Appendix A of this document contains a selection of sample items. The sample items represent a range of cognitive complexities and item types.

III. Review Procedures for Florida Interim Assessment Item Bank Items

Prior to being included in the Florida Interim Assessment Item Bank, all mathematics items must pass several levels of review as part of the item development process.

A. Review for Item Quality

Assessment items developed for the Florida Interim Assessment Item Bank will be reviewed by Florida educators, the FDOE, and the contractors to ensure the quality of the items, including grade-level/course appropriateness, alignment to the standard, accuracy, and other criteria for overall item quality.

B. Review for Bias and Sensitivity

Items are reviewed by groups of Florida educators generally representative of Florida's geographic regions and culturally diverse population. Items are reviewed for the following kinds of bias: gender, racial, ethnic, linguistic, religious, geographic, and socioeconomic. Item reviews also include consideration of issues related to individuals with disabilities.

This review is to ensure that the primary purpose of assessing student achievement is not undermined by inadvertently including in the item bank any material that students, parents, or other stakeholders may deem inappropriate. Reviewers are asked to consider the variety of cultural, regional, philosophical, political, and religious backgrounds throughout Florida and to determine whether the subject matter will be acceptable to Florida students, their parents, and other members of Florida communities.

IV. Guide to the Individual Standard Specifications

A. CCSS Mathematics Standards Classification System

The graphic below demonstrates the coding schema for the Common Core State Standards for Mathematics.

MACC.K.CC.1.1

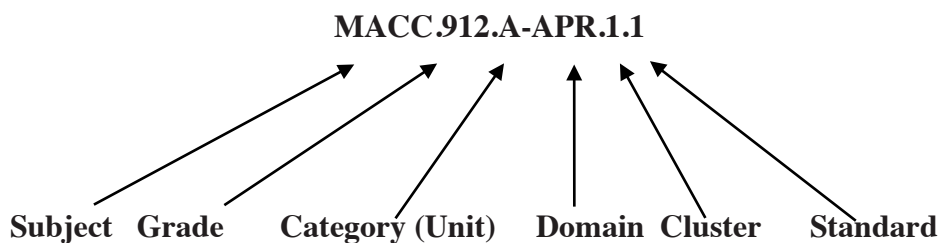| Subject | Grade | Domain | Cluster | Standard |
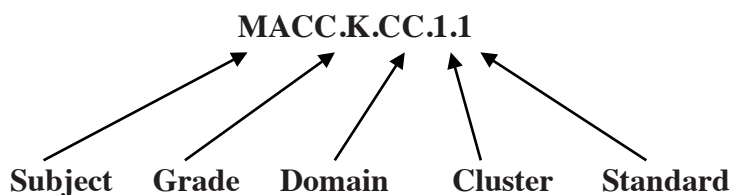
Using this schema:

Subject Code MACC: Mathematics Common Core

Grade: Kindergarten

Domain CC: Counting and Cardinality

Cluster 1: Know number names and the count sequence.

Standard 1: Count to 100 by ones and by tens.

| Subject | Grade | Category (Unit) | Domain | Cluster | Standard |

Using the schema, the bottom row refers to:

Subject Code MACC: Mathematics Common Core

Grade: High school Grades 9–12

Category A: Algebra

Domain APR: Arithmetic with Polynomials and Rational Expressions

Cluster 1: Perform arithmetic operations on polynomials.

Standard 1: Understand that polynomials form a system analogous to the integers, namely, they are closed under the operations of addition, subtraction, and multiplication; add, subtract, and multiply polynomials.

B. Definitions of Cluster and Standard Specifications

The *Item Specifications* identify how the standards in the CCSS are assessed by items in the Florida Interim Assessment Item Bank. For each assessed standard, the following information is provided in the Individual Standards Specifications section.

Domain	refers to larger groups of related standards. Standards from different domains may sometimes be closely related.
Cluster	refers to groups of related standards. Note that standards from different clusters may sometimes be closely related because mathematics is a connected subject.
Standards	define what students should understand and be able to do.
Standards Clarifications/ Content Limits for Course	Standards clarifications, when needed as an explanation for some of the standards listed above, explain the type of behavior that the student should exhibit for mastery of the standard. The clarification statements explain what students are expected to do when responding to the question.
	Course limits define the range of content knowledge and degree of difficulty that should be assessed in the items for the standard. Course limits may be used to identify content beyond the scope of the targeted standard if the content is more appropriately assessed by another standard. These statements help to provide validity by ensuring the test items are clearly aligned to the targeted standard.

V. Individual Standards Specifications for Florida Interim Assessment Item Bank Mathematics Items

This section of the *Item Specifications* provides standard-specific guidance for assessment item development for the Florida Interim Assessment Item Bank based on the Common Core State Standards.

Each item developed for the Florida Interim Assessment Item Bank and Test Platform should assess one or more of the Mathematical Practice Standards listed in Appendix B.

A. Geometry Item Specifications

Conceptual Category	GEOMETRY
Domain	Congruence
Cluster	Experiment with transformations in the plane.
Standards	MACC.912.G-CO.1.1—Know precise definitions of angle, circle, perpendicular line, parallel line, and line segment, based on the undefined notions of point, line, distance along a line, and distance around a circular arc.
	MACC.912.G-CO.1.2—Represent transformations in the plane using, e.g., transparencies and geometry software; describe transformations as functions that take points in the plane as inputs and give other points as outputs. Compare transformations that preserve distance and angle to those that do not (e.g., translation versus horizontal stretch).
	MACC.912.G-CO.1.3—Given a rectangle, parallelogram, trapezoid, or regular polygon, describe the rotations and reflections that carry it onto itself.
	MACC.912.G-CO.1.4—Develop definitions of rotations, reflections, and translations in terms of angles, circles, perpendicular lines, parallel lines, and line segments.
	MACC.912.G-CO.1.5—Given a geometric figure and a rotation, reflection, or translation, draw the transformed figure using, e.g., graph paper, tracing paper, or geometry software. Specify a sequence of transformations that will carry a given figure onto another.

Standards Clarifications/ Content Limits for Course	As an explanation for some of the standards listed above, in Geometry, students will
	• understand and use definitions of geometric terms (including, but not limited to, angle, circle, perpendicular line, parallel line, parallel plane, rhombus, and trapezoid) based on the undefined notions of point, line, distance along a line, and distance around a circular arc
	• complete and compare transformations in the coordinate plane, including describing the transformations as functions. Students will use a variety of tools to complete transformations. They will determine and explain the difference between the transformations that preserve distance and angle and those that do not.
	• identify and describe specific transformations that carry figures onto themselves
	• develop definitions of rotations, reflections, and translations in terms of angles, circles, perpendicular lines, parallel lines, and line segments. For example, students will be able to explain that any rotation of a circle about its center preserves the location of the circle and that rotating a line by 90 degrees produces a line that is perpendicular to the first line.
	• perform transformations in the coordinate plane. Students will also determine which series of transformations could take a figure to a specified location or which other transformations could produce a transformation that has occurred. For example, a 180-degree rotation about the origin produces the same result as a reflection across the x-axis and then the y-axis.

Conceptual Category	GEOMETRY
Domain	Congruence
Cluster	Understand congruence in terms of rigid motions.
Standards	MACC.912.G-CO.2.6—Use geometric descriptions of rigid motions to transform figures and to predict the effect of a given rigid motion on a given figure; given two figures, use the definition of congruence in terms of rigid motions to decide if they are congruent.
	MACC.912.G-CO.2.7—Use the definition of congruence in terms of rigid motions to show that two triangles are congruent if and only if corresponding pairs of sides and corresponding pairs of angles are congruent.
	MACC.912.G-CO.2.8—Explain how the criteria for triangle congruence (ASA, SAS, and SSS) follow from the definition of congruence in terms of rigid motions.
Standards Clarifications/ Content Limits for Course	As an explanation for some of the standards listed above, in Geometry, students will
	• use definitions of rigid motions to determine whether two figures are congruent, transform figures using rigid motions, and predict the effect of a given rigid motion transformation
	• use the definition of congruence and rigid motions to show that two triangles are congruent if corresponding pairs of sides and angles are congruent
	• use the definitions of congruence and rigid motion to explain how ASA, AAS, SAS, SSS, and HL prove triangle congruence
	• build on rigid motions as a familiar starting point for development of concept of geometric proof

Conceptual Category	GEOMETRY
Domain	Congruence
Cluster	Prove geometric theorems.
Standards	MACC.912.G-CO.3.9—Prove theorems about lines and angles. *Theorems include: vertical angles are congruent; when a transversal crosses parallel lines, alternate interior angles are congruent and corresponding angles are congruent; points on a perpendicular bisector of a line segment are exactly those equidistant from the segment's endpoints.* MACC.912.G-CO.3.10—Prove theorems about triangles. *Theorems include: measures of interior angles of a triangle sum to 180°; base angles of isosceles triangles are congruent; the segment joining midpoints of two sides of a triangle is parallel to the third side and half the length; the medians of a triangle meet at a point.* MACC.912.G-CO.3.11—Prove theorems about parallelograms. *Theorems include: opposite sides are congruent, opposite angles are congruent, the diagonals of a parallelogram bisect each other, and conversely, rectangles are parallelograms with congruent diagonals.*

Standards Clarifications/ Content Limits for Course	As an explanation for some of the standards above, in Geometry, students will
	• prove geometric theorems about lines and angles and use these theorems to solve mathematical problems. Theorems include vertical angles are congruent; when a transversal crosses parallel lines, alternate interior angles are congruent and corresponding angles are congruent; and points on a perpendicular bisector of a line segment are exactly those equidistant from the segment's endpoints.
	• prove geometric theorems about triangles and use these theorems to solve mathematical problems. Theorems include measures of interior angles of a triangle sum to 180°, base angles of isosceles triangles are congruent, the segment joining midpoints of two sides of a triangle is parallel to the third side and half the length, and the medians of a triangle meet at a point. This standard may be extended to include concurrence of perpendicular bisectors and angle bisectors.
	• prove geometric theorems about parallelograms and use these theorems to solve mathematical problems. Theorems include opposite sides are congruent, opposite angles are congruent, the diagonals of a parallelogram bisect each other, and, conversely, rectangles are parallelograms with congruent diagonals.
	• focus on validity of underlying reasoning while using a variety of ways of writing proofs
	Theorems involving similar or congruent triangles are not included; those are addressed in standards MACC.912.G-SRT.2.4 and MACC.912.G-SRT.2.5.

Conceptual Category	GEOMETRY
Domain	Congruence
Cluster	Make geometric constructions.
Standards	MACC.912.G-CO.4.12—Make formal geometric constructions with a variety of tools and methods (compass and straightedge, string, reflective devices, paper folding, dynamic geometric software, etc.). *Copying a segment; copying an angle; bisecting a segment; bisecting an angle; constructing perpendicular lines, including the perpendicular bisector of a line segment; and constructing a line parallel to a given line through a point not on the line.* MACC.912.G-CO.4.13—Construct an equilateral triangle, a square, and a regular hexagon inscribed in a circle.
Standards Clarifications/ Content Limits for Course	As an explanation for some of the standards above, in Geometry, students will • construct geometric figures using a variety of tools and methods and will be able to explain the steps in the constructions • formalize and explain processes Course limit: As indicated in italics in standard MACC.912.G-CO.4.12

Conceptual Category	GEOMETRY
Domain	Similarity, Right Triangles, and Trigonometry
Cluster	Understand similarity in terms of similarity transformations.
Standards	MACC.912.G-SRT.1.1—Verify experimentally the properties of dilations given by a center and a scale factor: MACC.912.G-SRT.1.1.a—A dilation takes a line not passing through the center of the dilation to a parallel line, and leaves a line passing through the center unchanged. MACC.912.G-SRT.1.1.b—The dilation of a line segment is longer or shorter in the ratio given by the scale factor. MACC.912.G-SRT.1.2—Given two figures, use the definition of similarity in terms of similarity transformations to decide if they are similar; explain using similarity transformations the meaning of similarity for triangles as the equality of all corresponding pairs of angles and the proportionality of all corresponding pairs of sides. MACC.912.G-SRT.1.3—Use the properties of similarity transformations to establish the AA criterion for two triangles to be similar.
Standards Clarifications/ Content Limits for Course	As an explanation for some of the standards above, in Geometry, students will • demonstrate understanding of properties of dilations, including the facts that a dilation results in a figure that is longer or shorter in the ratio given by the scale factor, the location of a line that passes through the center of a dilation is unchanged other than its length, and the location of a line that does not pass through the center of dilation is parallel to the location of the line before it was dilated • relate similarity to dilations and use the definitions of similarity transformations to explain whether two figures are similar. Students will define triangle similarity based on dilations and similarity transformations and will understand that corresponding pairs of angles are congruent and corresponding pairs of sides are proportional. • use the properties of similarity transformations to show that AA proves triangle similarity Items may include diagrams and/or descriptions of figures and the dilations performed. Items may also include results of dilations and may ask which similarity transformations were performed.

Conceptual Category	GEOMETRY
Domain	Similarity, Right Triangles, and Trigonometry
Cluster	Prove theorems involving similarity.
Standards	MACC.912.G-SRT.2.4—Prove theorems about triangles. *Theorems include: a line parallel to one side of a triangle divides the other two proportionally, and conversely; the Pythagorean Theorem proved using triangle similarity.* MACC.912.G-SRT.2.5—Use congruence and similarity criteria for triangles to solve problems and to prove relationships in geometric figures.
Standards Clarifications/ Content Limits for Course	As an explanation for some of the standards above, in Geometry, students will • prove theorems related to similar triangles, including using similar triangles to prove the Pythagorean theorem • use similar and congruent triangles to find missing side lengths and/or angle measures. Students will use postulates and theorems about similar and congruent triangles to prove that triangles are similar or congruent or to prove parts congruent. They may also use these theorems by creating triangles in other shapes (e.g., drawing a diagonal in a parallelogram to prove that opposite sides are congruent by ASA and CPCTC).

Conceptual Category	GEOMETRY
Domain	Similarity, Right Triangles, and Trigonometry
Cluster	Define trigonometric ratios and solve problems involving right triangles.
Standards	MACC.912.G-SRT.3.6—Understand that by similarity, side ratios in right triangles are properties of the angles in the triangle, leading to definitions of trigonometric ratios for acute angles. MACC.912.G-SRT.3.7—Explain and use the relationship between the sine and cosine of complementary angles. MACC.912.G-SRT.3.8—Use trigonometric ratios and the Pythagorean Theorem to solve right triangles in applied problems.
Standards Clarifications/ Content Limits for Course	As an explanation for some of the standards above, in Geometry, students will • use right triangles to explain and use the fact that the sine of an angle is the same as the cosine of its complement • use trigonometric ratios and the Pythagorean theorem to find unknown side lengths or angle measures in real-world and mathematical problems

Conceptual Category	GEOMETRY
Domain	Right Triangles and Trigonometry
Cluster	Apply trigonometry to general triangles
Standards	+MACC.912.G-SRT.4.9—Derive the formula $A = \frac{1}{2} ab \sin(C)$ for the area of a triangle by drawing an auxiliary line from a vertex perpendicular to the opposite side. +MACC.912.G-SRT.4.10—Prove the Laws of Sines and Cosines and use them to solve problems. +MACC.912.G-SRT.4.11—Understand and apply the Law of Sines and the Law of Cosines to find unknown measurements in right and non-right triangles (e.g., surveying problems, resultant forces).
Standards Clarifications/ Content Limits for Course	As an explanation for some of the standards listed above, in Geometry, students will • prove and explain the Laws of Sines and Cosines and identify the cases in which it would be appropriate to use the Law of Sines or the Law of Cosines to solve a problem • apply the Laws of Sines and Cosines to find unknown measurements in contextual situations relating to right and non-right triangles. With respect to the general case of the Laws of Sines and Cosines, the definitions of sine and cosine must be extended to obtuse angles.

Conceptual Category	GEOMETRY
Domain	Circles
Cluster	Understand and apply theorems about circles.
Standards	MACC.912.G-C.1.1—Prove that all circles are similar. MACC.912.G-C.1.2—Identify and describe relationships among inscribed angles, radii, and chords. *Include the relationship between central, inscribed, and circumscribed angles; inscribed angles on a diameter are right angles; the radius of a circle is perpendicular to the tangent where the radius intersects the circle.* MACC.912.G-C.1.3—Construct the inscribed and circumscribed circles of a triangle, and prove properties of angles for a quadrilateral inscribed in a circle. +MACC.912.G-C.1.4—Construct a tangent line from a point outside a given circle to the circle.

Standards Clarifications/ Content Limits for Course	As an explanation for some of the standards above, in Geometry, students will
	• identify and apply properties among inscribed angles, radii, and chords to solve problems about tangents, secants, radii, chords, and arcs and inscribed, circumscribed, and central angles
	• justify steps in constructions described above

Conceptual Category	GEOMETRY
Domain	Circles
Cluster	Find arc lengths and areas of sectors of circles.
Standards	MACC.912.G-C.2.5—Derive using similarity the fact that the length of the arc intercepted by an angle is proportional to the radius, and define the radian measure of the angle as the constant of proportionality; derive the formula for the area of a sector.
Standards Clarifications/ Content Limits for Course	In Geometry, students will
	• use similarity to determine that the length of an arc intercepted by an angle is proportional to the radius. Students will derive the formulas for the area of a sector of a circle and for arc length; they will find the areas of sectors and the lengths of arcs.
	Course limit: Radian is introduced only as a unit of measure.

Conceptual Category	GEOMETRY
Domain	Geometric Properties with Equations
Cluster	Translate between the geometric description and the equation for a conic section.
Standards	MACC.912.G-GPE.1.1—Derive the equation of a circle of given center and radius using the Pythagorean Theorem; complete the square to find the center and radius of a circle given by an equation.
Standards Clarifications/ Content Limits for Course	In Geometry, students will
	• for standard MACC.912.G-GPE.1.1, derive equations for circles when given the center and radius length, when given endpoints of the diameter or radius, or when circles are shown on the coordinate plane. Students will complete the square in a given equation to find the center and radius of a circle and will compare the equation with the graph of the circle on the coordinate plane.

Conceptual Category	GEOMETRY
Domain	Expressing Geometric Properties with Equations
Cluster	Use coordinates to prove simple geometric theorems algebraically.
Standards	MACC.912.G-GPE.2.4—Use coordinates to prove simple geometric theorems algebraically. *For example, prove or disprove that a figure defined by four given points in the coordinate plane is a rectangle; prove or disprove that the point (1, √3) lies on the circle centered at the origin and containing the point (0, 2).* MACC.912.G-GPE.2.5—Prove the slope criteria for parallel and perpendicular lines and use them to solve geometric problems (e.g., find the equation of a line parallel or perpendicular to a given line that passes through a given point). MACC.912.G-GPE.2.6—Find the point on a directed line segment between two given points that partitions the segment in a given ratio. MACC.912.G-GPE.2.7—Use coordinates to compute perimeters of polygons and areas of triangles and rectangles, e.g., using the distance formula.
Standards Clarifications/ Content Limits for Course	As an explanation for some of the standards above, in Geometry, students will • use coordinates to prove geometric theorems or to determine properties of figures on the coordinate plane. Items may include proofs like the ones listed above; the derivation of the distance formula from the Pythagorean theorem should also be included. Simple proofs involving circles should be included. • find equations for lines that pass through given points and that are parallel or perpendicular to other lines. Students will prove that parallel lines have the same slope and that perpendicular lines have slopes that are negative reciprocals of each other. • use the distance formula and criteria for perpendicular lines to compute the perimeters and areas of polygons • include the distance formula and relate it to the Pythagorean theorem

Conceptual Category	GEOMETRY
Domain	Geometric Measurement and Dimension
Cluster	Explain volume formulas and use them to solve problems.
Standards	MACC.912.G-GMD.1.1—Give an informal argument for the formulas for the circumference of a circle, area of a circle, volume of a cylinder, pyramid, and cone. *Use dissection arguments, Cavalieri's principle, and informal limit arguments.* MACC.912.G-GMD.1.3—Use volume formulas for cylinders, pyramids, cones, and spheres to solve problems.
Standards Clarifications/ Content Limits for Course	As an explanation for some of the standards above, in Geometry, students will • explain the significance of pi and give reasonable estimations of pi; explain the formulas for the area and circumference of a circle and for the volume of a cylinder, pyramid, and cone. Students will relate the volume of a cylinder to the volume of a cone and also relate the volume of a pyramid to the volume of a prism. They will explain Cavalieri's principle and use dissection arguments, limits, and similarity scale arguments to explain the volume of figures. • fluently use the volume formulas for cylinders, pyramids, cones, and spheres to solve real-world and mathematical problems

Conceptual Category	GEOMETRY
Domain	Geometric Measurement and Dimension
Cluster	Visualize relationships between two-dimensional and three-dimensional objects.
Standards	MACC.912.G-GMD.2.4—Identify the shapes of two-dimensional cross-sections of three-dimensional objects, and identify three-dimensional objects generated by rotations of two-dimensional objects.
Standards Clarifications/ Content Limits for Course	In Geometry, students will • identify a cross section or cross sections of three-dimensional figures. Students will identify results of rotating two-dimensional shapes.

Conceptual Category	GEOMETRY
Domain	Modeling with Geometry
Cluster	Apply geometric concepts in modeling situations.
Standards	MACC.912.G-MG.1.1—Use geometric shapes, their measures, and their properties to describe objects (e.g., modeling a tree trunk or a human torso as a cylinder). MACC.912.G-MG.1.2—Apply concepts of density based on area and volume in modeling situations (e.g., persons per square mile, BTUs per cubic foot). MACC.912.G-MG.1.3—Apply geometric methods to solve design problems (e.g., designing an object or structure to satisfy physical constraints or minimize cost; working with typographic grid systems based on ratios).
Standards Clarifications/ Content Limits for Course	As an explanation for some of the standards above, in Geometry, students will • understand two- and three-dimensional shapes and their properties and identify, describe, and provide real-life examples of these shapes • calculate density by calculating the volume or area of a figure and find another measure when given density (e.g., when given population density, length, and width of a city block, students can calculate the population). Students will also apply concepts of density in real-world contexts. • apply a variety of geometric methods to solve modeling and design problems, such as optimization problems or using a scale grid

Appendices

Appendix A

Sample Items

Item Type: Selected Response

Correct Answer: B

Possible Points: 1

DOK: 2

Calculator Usage: Allowed but Not Required

CCSS Standard:

MACC.912.G-GPE.2.6—Find the point on a directed line segment between two given points that partitions the segment in a given ratio.

Standards for Mathematical Practice:

1. Make sense of problems and persevere in solving them.

The endpoints of $\overline{MP}$ are $M(-7, 4)$ and $P(3, -1)$. What are the coordinates of the point, N, that divides $\overline{MP}$ such that $MN{:}NP$ is equal to 2:3?

   A. $(-5, 1)$

* **B.** $(-3, 2)$

   C. $(-2, 1.5)$

   D. $(-1, 1)$

Distractor Rationales

A. This is the result of adding 2 to the x-coordinate of M and subtracting 3 from the y-coordinate.

B. Correct answer

C. This is the midpoint of $\overline{MP}$. It divides the segment into a ratio of 1:1.

D. This is the point such that $MN{:}NP = 3{:}2$ instead of 2:3.

Item Type: Gridded Response

Correct Answer: 16 inches

Possible Points: 1

DOK: 2

Calculator Usage: Allowed but Not Required

CCSS Standard:

MACC.912.G-SRT 3.6—Understand that by similarity, side ratios in right triangles are properties of the angles in the triangle, leading to definitions of trigonometric ratios for acute angles.

Standards for Mathematical Practice:

1. Make sense of problems and persevere in solving them.

2. Reason abstractly and quantitatively.

Right triangles A and B are similar. In triangle A, $\cos(\theta) = \frac{3}{4}$. If the length of the side adjacent to angle θ in triangle B is 12 inches, what is the length of the hypotenuse of triangle B?

Item Type: Short Response

Correct Answer: Any TWO of the Following: Square, Rectangle, Trapezoid, Parallelogram, Triangle, Pentagon, Hexagon

Possible Points: 1

DOK: 1

Calculator Usage: Allowed but Not Required

CCSS Standard:

MACC.912.G-GMD.2.4—Identify the shapes of two-dimensional cross-sections of three-dimensional objects, and identify three-dimensional objects generated by rotations of two-dimensional objects.

Standards for Mathematical Practice:

2. Reason abstractly and quantitatively.

6. Attend to precision.

What are two different shapes of cross sections of cubes? Use the most specific geometric terms possible.

_____   _____

Item Type: Constructed Response

Correct Answer: See Scoring Exemplar

Possible Points: 2

DOK: 3

Calculator Usage: Allowed but Not Required

CCSS Standard:

MACC.912.G-CO.1.2—Represent transformations in the plane using, e.g., transparencies and geometry software; describe transformations as functions that take points in the plane as inputs and give other points as outputs. Compare transformations that preserve distance and angle to those that do not (e.g., translation versus horizontal stretch).

Standards for Mathematical Practice:

2. Reason abstractly and quantitatively.

Given: $\triangle ABC$, shown on the coordinate plane below
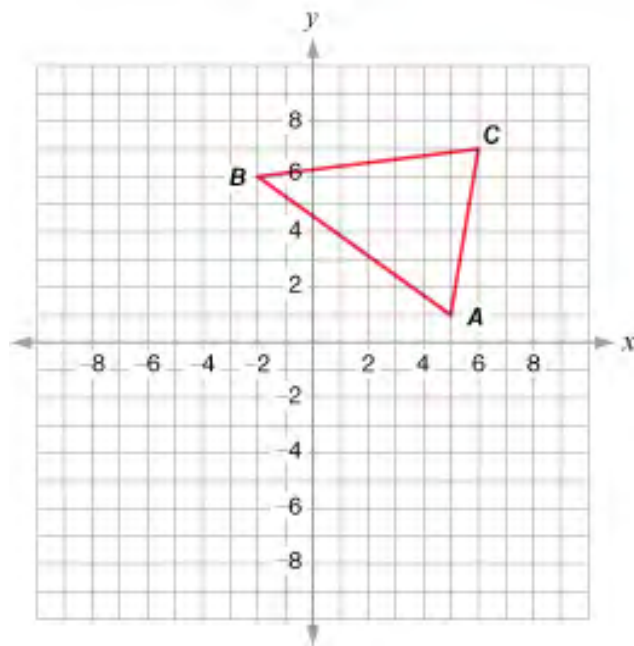
Part A. If $\triangle ABC$ is reflected over the x-axis to yield $\triangle A'B'C'$, what are the coordinates of the vertices of $\triangle A'B'C'$?

Part B. Using this reflection, write a general rule that will map $\triangle ABC$ onto $\triangle A'B'C'$.

Part C. If $\triangle ABC$ is translated 4 units to the left and 3 units down to yield $\triangle A''B''C''$, draw $\triangle A''B''C''$ on the coordinate plane below.
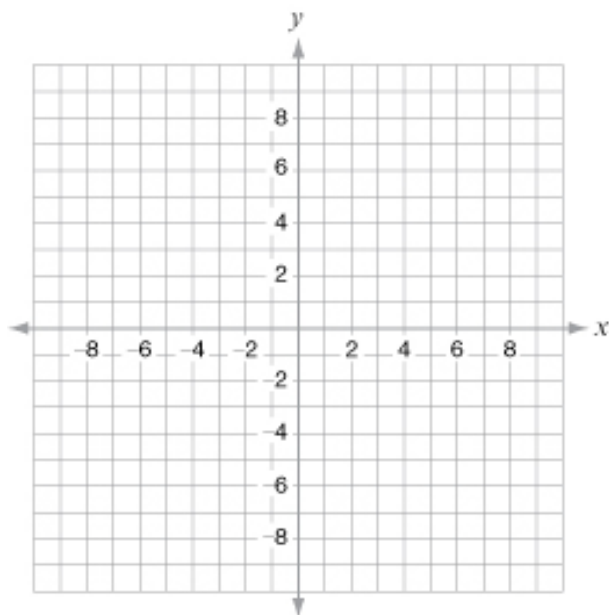
Part D. Using your translation, write a general rule that will map $\triangle ABC$ onto $\triangle A''B''C''$.

Use words, numbers, and/or pictures to show your work.

| SCORING RUBRIC	
2	Work demonstrates a **clear and complete** understanding of the mathematical concepts and/or procedures required by the task. Appropriate strategy is shown with clear and complete explanations and interpretations.
1	Response demonstrates a **partial** understanding of the mathematical concepts and/or procedures. Appropriate strategy is shown, but explanation or interpretation has minor flaws. OR Response is incorrect because of calculation errors. Work and strategy indicate a **clear** understanding of the mathematical concepts and/or procedures required by the task.
0	Response is irrelevant, inappropriate, or not provided.

Maximum Points—2

Part A—$\frac{1}{2}$ point

The coordinates of the vertices of $\triangle A'B'C'$ are $A'(5, -1)$, $B'(-2, -6)$, and $C'(6, -7)$.

Part B—$\frac{1}{2}$ point

The rule could be described as $(x, -y)$.

Part C—$\frac{1}{2}$ point
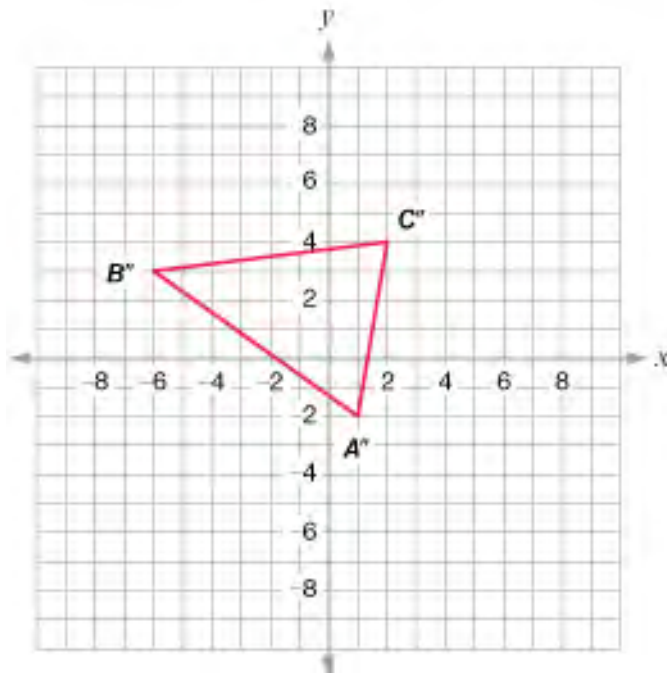
Part D—$\frac{1}{2}$ point

The rule could be described as $(x - 4, y - 3)$.

Other appropriate strategies are acceptable.

Item Type: Extended Response

Correct Answer: See Scoring Exemplar

Possible Points: 4

DOK: 3

Calculator Usage: Required (for Trigonometric Functions)

CCSS Standard:

MACC.912.G-SRT.3.8—Use trigonometric ratios and the Pythagorean Theorem to solve right triangles in applied problems.

Standards for Mathematical Practice:

1. Make sense of problems and persevere in solving them.

6. Attend to precision.

Omar drew two isosceles triangles. Both triangles have a base of 120 cm. The height of the first triangle is 2.5 times the height of the second triangle. The combined area of both triangles is 1,428 cm^2.

Part A. What is the height of the larger triangle?

Part B. What is the measure of one of the base angles of the smaller triangle, to the nearest tenth of a degree?

Part C. What is the length of one of the legs of the larger triangle, to the nearest tenth of a centimeter?

Part D. What is the measure of the vertex angle in the larger triangle, to the nearest tenth of a degree?

Use words, numbers, and/or pictures to show your work and explain your steps.

SCORING RUBRIC	
4	Work demonstrates a **clear and complete** understanding of the mathematical concepts and/or procedures required by the task. Appropriate strategy is shown with clear and complete explanations and interpretations.
3	Work demonstrates a **clear** understanding of the mathematical concepts and/or procedures but is not complete. Appropriate strategy is shown, but explanation or interpretation has minor flaws. OR Response is incorrect because of calculation errors. Work and strategy indicate a **clear** demonstration of the problem.
2	Response demonstrates a **partial** understanding of the mathematical concepts and/or procedures. Appropriate strategy is shown, but explanation or interpretation has minor flaws.
1	Response shows **minimal** understanding of the mathematical concepts and/or procedures or provides no explanation or interpretation for the solution or shows major flaws.
0	Response is irrelevant, inappropriate, or not provided.

Maximum Points—4

Part A—1 point

- The height is 17 cm.

  The formula for the area of a triangle is $A = \frac{1}{2}bh$, where b is the base of the triangle and h is the height of the triangle. The triangles have the same bases, b, but have different heights. We can use h for the height of the second, smaller triangle and H for the height of the first, larger triangle. Because the combined area is 1,428 cm², we can set up the following equation:

  Combined area = area of larger triangle + area of smaller triangle, so
  $1{,}428 = \frac{1}{2}bh + \frac{1}{2}bH$

  The bases are both 120 cm and $H = 2.5h$, so
  $1{,}428 = \frac{1}{2}120 \cdot h + \frac{1}{2}120 \cdot 2.5h \rightarrow$
  $1{,}428 = 60h + 150h = 210h$; $h = 6.8$ cm and $H = 17$ cm.

Part B—1 point

- The measure of one of the base angles is 6.5°.

  To find the measure of one of the base angles, we use the right triangle formed by the height and half of the base. The height of the smaller triangle is 6.8 cm, and half of the base is 60 cm. If the base angle is x, we can set up the equation $\tan(x) = 6.8/60$; $x = 6.5°$.

Part C—1 point

- The length of one of the legs is 62.4 cm.

  To find the length of one of the congruent sides of the larger triangle, we use the Pythagorean theorem and the fact that the height divides the isosceles triangle into two congruent right triangles. The height and the base of the isosceles triangle are perpendicular, and half of the base forms one of the legs of the right triangle.

  $x^2 = (0.5b)^2 + (17)^2$
  $x^2 = (60)^2 + 289$
  $x^2 = 3{,}600 + 289$
  $x^2 = 3{,}889$
  $x = 62.4$ cm

Part D—1 point

- The measure of the vertex angle of the larger triangle is 148.4°.

  If the right triangle formed by the height and half of the base is used, half of the vertex angle can be found using the equation $\tan(y) = \left(\frac{60}{17}\right)$. The result is that
  $y = 74.18\ldots$; $2y = 148.3616\ldots$

or equivalent work

Item Type: Performance Task

Correct Answer: See Scoring Exemplar

Possible Points: 7

DOK: 4

Calculator Usage: Allowed but Not Required

CCSS Standard:

MACC.912.G-MG.1.3—Apply geometric methods to solve design problems (e.g., designing an object or structure to satisfy physical constraints or minimize cost; working with typographic grid systems based on ratios).

Standards for Mathematical Practice:

1. Make sense of problems and persevere in solving them.

4. Model with mathematics.

5. Use appropriate tools strategically.

6. Attend to precision.

8. Look for and express regularity in repeated reasoning.

The Packaging Company

Teacher Directions:

Before administration, discuss nets, volume, and surface area. Students should have experience calculating volume and surface area of solids; they should also have experience designing nets and creating solids from nets.

Read the problem aloud and respond to any questions.

Instruct students to use words, numbers, pictures, and/or models to show their work.

Allow 60 minutes for this task.

Make all necessary materials available.

Guide students and answer questions, but encourage independent thinking.

After the task, discuss answers as time permits.

Suggested Materials: Grid paper OR large sheets of paper OR cardboard and rulers; scissors; tape; calculators

TASK:

As part of her job at a packaging company, Calista puts items into boxes. She can use only cube-shaped boxes, as shown below. These boxes are available in different sizes with whole-centimeter edge lengths.
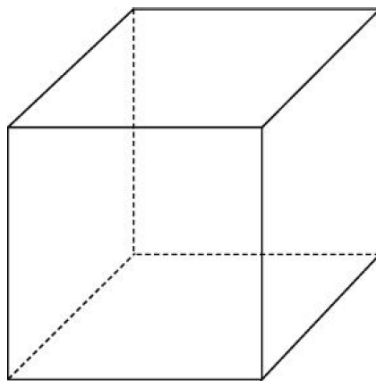
Part A. Calista needs to select a box to package her first order. The items for Calista's order will need approximately 8 cm³ of space. What would the length of each edge of this box need to be?

Part B. Calista now needs to select a box for her second order. Calista has three cube-shaped objects that need to be packaged into the same box. Two of the objects have a volume of 8 cm³. The third object has a volume of 27 cm³. What is the total volume of these objects?

Part C. What is the minimum edge length of a box that will hold the **total volume** of the objects for the second order?

Part D. Construct a net for each of the three objects in the second order, and then use the nets to construct these three solids. What is the minimum edge length of a box that will hold **these three objects**?

Part E. Compare your answer to part C with your answer to part D. Why are the answers the same or different? Explain.

Part F. After the three objects have been placed in the box for the second order, what is the minimum amount of empty space in the box? Explain how the amount of empty space changes based on the way the cube-shaped objects are arranged and what would happen if a larger or a smaller box were chosen.

Part G. If the boxes could be rectangular prisms instead of cubes, what would be the measurements of the box that would hold these three objects and have the smallest amount of empty space?

Maximum Points—7

Part A—1 point

- Each edge would need to be 2 cm long.

  This box needs to have 8 cm³ of space.

  The box is a cube, so $V = s^3$.

  $8 = s^3$

  $\sqrt[3]{8}$

  or equivalent work.

Part B—1 point

- The total volume of these objects is 43 cm³.
- Student reasoning might include:

  There are two boxes with a volume of 8 cm³ and one box with a volume of 27 cm³.

  $8 + 8 + 27 = 43$
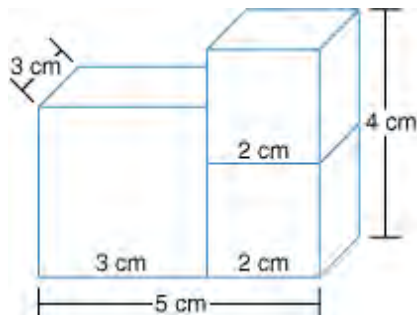
Part C—1 point

- The shortest possible length of each edge is 4 cm.
- Student reasoning might include:

  The box must be a cube with whole-number edges. The cube root of 43 is approximately 3.5, so the shortest possible edge length would be the next whole centimeter, or 4 cm.

Part D—1 point

- One possible net:

- The shortest possible length of each edge of the packing box that would hold all of the small objects is 5 cm.
- Because the length of an edge of a cube is the cube root of the volume, the length of an edge of each cube-shaped object with a volume of 8 cm³ is 2 cm, and the length of an edge of the cube-shaped object with a volume of 27 cm³ is 3 cm. When the 2 cm cubes are stacked on top of each other and the 3 cm cube is placed next to them, the edge of the packing box must be at least 5 cm.

Part E—1 point

- Student reasoning might include:

  These answers are different.

- Even though the total volume of the smaller cube-shaped objects is 43 cm^3 and the volume of a packing box with 4 cm edges is 64 cm^3, the smaller boxes could not be arranged inside a box with side lengths of 4 cm. There is no way to arrange a 2 cm cube and a 3 cm cube so that they fit together in a cube-shaped box with 4 cm edges. To best use space, the smaller boxes should be stacked so that two boxes are next to each other and one box is on top of the two others. One face of one of the 2 cm cubes should be flush with one face of the other 2 cm cube, and the adjacent face of the 2 cm cube should be flush with one face of the 3 cm cube. This would create a figure that is 5 cm at its widest point, as shown above.

Part F—1 point

- The packing box has a minimum edge length of 5 cm, so the volume is $5^3 = 125$ cm^3. The smaller cubes have a combined volume of 43 cm^3, and $125 - 43 = 82$.

- A larger box would have even more empty space, and the small objects would not be placed directly next to one another.

- If a smaller cube-shaped box were chosen, the objects would not fit even though the volume may be enough. The shape of the objects is the limiting factor.

Part G—1 point

- At the widest point, the prism would need to be 5 cm to account for a 2 cm cube being placed flush with the 3 cm cube. The other 2 cm cube would be flush with the first 2 cm cube's adjacent side, making another edge of the prism 4 cm. The last edge of the prism would need to be 3 cm long.

Other appropriate strategies are acceptable.

Appendix B

Standards for Mathematical Practice

The Standards for Mathematical Practice describe varieties of expertise that mathematics educators at all levels should seek to develop in their students. These practices rest on important "processes and proficiencies" with longstanding importance in mathematics education. The first of these are the NCTM process standards of problem solving, reasoning and proof, communication, representation, and connections. The second are the strands of mathematical proficiency specified in the National Research Council's report *Adding It Up*: adaptive reasoning, strategic competence, conceptual understanding (comprehension of mathematical concepts, operations and relations), procedural fluency (skill in carrying out procedures flexibly, accurately, efficiently and appropriately), and productive disposition (habitual inclination to see mathematics as sensible, useful, and worthwhile, coupled with a belief in diligence and one's own efficacy).[1]

MACC.K12.MP.1.1 Make sense of problems and persevere in solving them.

Mathematically proficient students start by explaining to themselves the meaning of a problem and looking for entry points to its solution. They analyze givens, constraints, relationships, and goals. They make conjectures about the form and meaning of the solution and plan a solution pathway rather than simply jumping into a solution attempt. They consider analogous problems, and try special cases and simpler forms of the original problem in order to gain insight into its solution. They monitor and evaluate their progress and change course if necessary. Older students might, depending on the context of the problem, transform algebraic expressions or change the viewing window on their graphing calculator to get the information they need. Mathematically proficient students can explain correspondences between equations, verbal descriptions, tables, and graphs or draw diagrams of important features and relationships, graph data, and search for regularity or trends. Younger students might rely on using concrete objects or pictures to help conceptualize and solve a problem. Mathematically proficient students check their answers to problems using a different method, and they continually ask themselves, "Does this make sense?" They can understand the approaches of others to solving complex problems and identify correspondences between different approaches.

MACC.K12.MP.2.1 Reason abstractly and quantitatively.

Mathematically proficient students make sense of quantities and their relationships in problem situations. They bring two complementary abilities to bear on problems involving quantitative relationships: the ability to *decontextualize*—to abstract a given situation and represent it symbolically and manipulate the representing symbols as if they have a life of their own, without necessarily attending to their referents—and the ability to *contextualize*, to pause as needed during the manipulation process in order to probe into the referents for the symbols involved. Quantitative reasoning entails habits of creating a coherent representation of the problem at hand; considering the units involved; attending to the meaning of quantities, not just how to compute them; and knowing and flexibly using different properties of operations and objects.

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[1] Common Core State Standards Initiative (CCSSI), 2010, Common Core State Standards for Mathematics. Washington, DC: National Governors Association Center for Best Practices and the Council of Chief State School Officers. *http://www.corestandards.org/assets/CCSSI_Math%20Standards.pdf*

MACC.K12.MP.3.1 Construct viable arguments and critique the reasoning of others.

Mathematically proficient students understand and use stated assumptions, definitions, and previously established results in constructing arguments. They make conjectures and build a logical progression of statements to explore the truth of their conjectures. They are able to analyze situations by breaking them into cases, and can recognize and use counterexamples. They justify their conclusions, communicate them to others, and respond to the arguments of others. They reason inductively about data, making plausible arguments that take into account the context from which the data arose. Mathematically proficient students are also able to compare the effectiveness of two plausible arguments, distinguish correct logic or reasoning from that which is flawed, and—if there is a flaw in an argument—explain what it is. Elementary students can construct arguments using concrete referents such as objects, drawings, diagrams, and actions. Such arguments can make sense and be correct, even though they are not generalized or made formal until later grades. Later, students learn to determine domains to which an argument applies. Students at all grades can listen or read the arguments of others, decide whether they make sense, and ask useful questions to clarify or improve the arguments.

MACC.K12.MP.4.1 Model with mathematics.

Mathematically proficient students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. In early grades, this might be as simple as writing an addition equation to describe a situation. In middle grades, a student might apply proportional reasoning to plan a school event or analyze a problem in the community. By high school, a student might use geometry to solve a design problem or use a function to describe how one quantity of interest depends on another. Mathematically proficient students who can apply what they know are comfortable making assumptions and approximations to simplify a complicated situation, realizing that these may need revision later. They are able to identify important quantities in a practical situation and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts and formulas. They can analyze those relationships mathematically to draw conclusions. They routinely interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.

MACC.K12.MP.5.1 Use appropriate tools strategically.

Mathematically proficient students consider the available tools when solving a mathematical problem. These tools might include pencil and paper, concrete models, a ruler, a protractor, a calculator, a spreadsheet, a computer algebra system, a statistical package, or dynamic geometry software. Proficient students are sufficiently familiar with tools appropriate for their grade or course to make sound decisions about when each of these tools might be helpful, recognizing both the insight to be gained and their limitations. For example, mathematically proficient high school students analyze graphs of functions and solutions generated using a graphing calculator. They detect possible errors by strategically using estimation and other mathematical knowledge. When making mathematical models, they know that technology can enable them to visualize the results of varying assumptions, explore consequences, and compare predictions with data. Mathematically proficient students at various grade levels are able to identify relevant external mathematical resources, such as digital content located on a website, and use them to pose or solve problems. They are able to use technological tools to explore and deepen their understanding of concepts.

MACC.K12.MP.6.1 Attend to precision.

Mathematically proficient students try to communicate precisely to others. They try to use clear definitions in discussion with others and in their own reasoning. They state the meaning of the symbols they choose, including using the equal sign consistently and appropriately. They are careful about specifying units of measure, and labeling axes to clarify the correspondence with quantities in a problem. They calculate accurately and efficiently, express numerical answers with a degree of precision appropriate for the problem context. In the elementary grades, students give carefully formulated explanations to each other. By the time they reach high school they have learned to examine claims and make explicit use of definitions.

MACC.K12.MP.7.1 Look for and make use of structure.

Mathematically proficient students look closely to discern a pattern or structure. Young students, for example, might notice that three and seven more is the same amount as seven and three more, or they may sort a collection of shapes according to how many sides the shapes have. Later, students will see 7×8 equals the well remembered $7 \times 5 + 7 \times 3$, in preparation for learning about the distributive property. In the expression $x^2 + 9x + 14$, older students can see the 14 as 2×7 and the 9 as $2 + 7$. They recognize the significance of an existing line in a geometric figure and can use the strategy of drawing an auxiliary line for solving problems. They also can step back for an overview and shift perspective. They can see complicated things, such as some algebraic expressions, as single objects or as being composed of several objects. For example, they can see $5 - 3(x - y)^2$ as 5 minus a positive number times a square and use that to realize that its value cannot be more than 5 for any real numbers x and y.

MACC.K12.MP.8.1 Look for and express regularity in repeated reasoning.

Mathematically proficient students notice if calculations are repeated, and look both for general methods and for shortcuts. Upper elementary students might notice when dividing 25 by 11 that they are repeating the same calculations over and over again, and conclude they have a repeating decimal. By paying attention to the calculation of slope as they repeatedly check whether points are on the line through (1, 2) with slope 3, middle school students might abstract the equation $(y - 2)/(x - 1) = 3$. Noticing the regularity in the way terms cancel when expanding $(x - 1)(x + 1)$, $(x - 1)(x^2 + x + 1)$, and $(x - 1)(x^3 + x^2 + x + 1)$ might lead them to the general formula for the sum of a geometric series. As they work to solve a problem, mathematically proficient students maintain oversight of the process, while attending to the details. They continually evaluate the reasonableness of their intermediate results.

Appendix C

Literacy Standards for Geometry

LACC.910.RST.1.3—Key Ideas and Details

Follow precisely a complex multi-step procedure when carrying out experiments, taking measurements, or performing technical tasks, attending to special cases or exceptions defined in the text.

LACC.910.RST.2.4—Craft and Structure

Determine the meaning of symbols, key terms, and other domain-specific words and phrases as they are used in a specific scientific or technical context relevant to grades 9−10 texts and topics.

LACC.910.RST.3.7—Integration of Knowledge and Ideas

Translate quantitative or technical information expressed in words in a text into visual form (e.g., a table or chart) and translate information expressed visually or mathematically (e.g., in an equation) into words.

LACC.910.WHST.1.1—Text Types and Purposes

Write arguments focused on discipline-specific content.

 a. Introduce precise claim(s), distinguish the claim(s) from alternate or opposing claims, and create an organization that establishes clear relationships among the claim(s), counterclaims, reasons, and evidence.

 b. Develop claim(s) and counterclaims fairly, supplying data and evidence for each while pointing out the strengths and limitations of both claim(s) and counterclaims in a discipline-appropriate form and in a manner that anticipates the audience's knowledge level and concerns.

 c. Use words, phrases, and clauses to link the major sections of the text, create cohesion, and clarify the relationships between claim(s) and reasons, between reasons and evidence, and between claim(s) and counterclaims.

 d. Establish and maintain a formal style and objective tone while attending to the norms and conventions of the discipline in which they are writing.

 e. Provide a concluding statement or section that follows from or supports the argument presented.

LACC.910.WHST.2.4—Production and Distribution of Writing

Produce clear and coherent writing in which the development, organization, and style are appropriate to task, purpose, and audience.

LACC.910.WHST.3.9—Research to Build and Present Knowledge

Draw evidence from informational texts to support analysis, reflection, and research