

Florida Interim Assessment Item Bank and Test Platform

Item Specifications

**Algebra 2
Grades 9–12**



FLORIDA DEPARTMENT OF EDUCATION
www.fldoe.org

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I. Introduction

In July 2010 the Florida Department of Education (FDOE) approved the adoption of the Common Core State Standards (CCSS) for Mathematics to support its pursuit of improved outcomes for all Florida mathematics students and participation in national educational initiatives, such as Race to the Top. The U.S. Department of Education awarded a Race to the Top grant to Florida in August 2010. An important component of this grant focused on the development of high-quality assessment items and balanced assessments for use by districts, schools, and teachers. The assessment items will be stored in the Florida Interim Assessment Item Bank and Test Platform (IBTP), a statewide secure system that allows Florida educators to search the item bank, export test items, and generate customized high-quality assessments for computer-based delivery or paper-and-pencil delivery. The IBTP allows Florida educators to determine what students know and are able to do relative to instruction based on Florida’s Next Generation Sunshine State Standards and the Common Core State Standards.

A. Purpose of the Item Specifications

The *Item Specifications* define the expectations for content, standards alignment, and format of assessment items for the Item Bank and Test Platform. The *Item Specifications* are intended for use by item writers and reviewers in the development of high-quality assessment items.

B. Scope

The *Item Specifications* provide general and grade-specific guidelines for the development of all Mathematics assessment items available in the Florida Interim Assessment Item Bank.

C. Standards Alignment

Items developed for the Florida Interim Assessment Item Bank and Test Platform will align to the Common Core State Standards for Mathematics. The Common Core State Standards for Mathematics are structured into three levels of specificity: Domains, Clusters, and Standards. These define what mathematics students should know and be able to do at every grade level/course, kindergarten through high school.

II. Criteria for Item Development

Mathematics item writers for the Florida Interim Assessment Item Bank must have a comprehensive knowledge of mathematics curriculum based on the Common Core State Standards and an understanding of the range of cognitive abilities of the target student population. Item writers should understand and consistently apply the guidelines established in this document. Item writers are expected to use their best judgment in writing items that measure the Mathematics standards of the CCSS without introducing extraneous elements that reflect bias for or against a group of students.

A. Overall Considerations for Item Development

These guidelines are provided to ensure the development of high-quality assessment items for the Florida Interim Assessment Item Bank.

1. Each item should be written to measure primarily one Common Core State Standard; however, other standards may also be addressed for some item types. In addition to the content standard alignment, each item should align to at least one Mathematical Practice Standard. Some items should be written reflecting the ELA Literacy standards cited in the course descriptions.
2. Items should be appropriate for students in terms of grade-level/course instruction, experience and difficulty, cognitive development, and reading level. The reading level of the test items should be on grade level.
3. Items should be written at or above the cognitive level (DOK) of the standard unless otherwise noted in the Individual Standard Specifications sections.
4. Each item should be written clearly and unambiguously to elicit the desired response.
5. Items should not disadvantage or exhibit disrespect to anyone in regard to age, gender, race, ethnicity, language, religion, socioeconomic status, disability, occupation, or geographic region.
6. At grades kindergarten through 5, items should be able to be answered without using a calculator. For grades 6 through 7, a four-function calculator may be used. For grade 8, a scientific calculator may be used. For Algebra 1, Geometry, and Algebra 2, both a scientific calculator and a graphing calculator (with functionalities similar to that of a TI-84) may be used. For all grades, calculators should not be used for items where computational skills or fluency are being assessed.

B. Item Contexts

The context in which an item is presented is called the item context or scenario. These guidelines are provided to assist item writers with development of items within an appropriate context.

1. The item context should be designed to interest students at the targeted level. Scenarios should be appropriate for students in terms of grade-level experience and difficulty, cognitive development, and reading level.
2. The context should be directly related to the question asked. The context should lead the student cognitively to the question. Every effort should be made to keep items as concise as possible without losing cognitive flow or missing the overall idea or concept.
3. Item contexts should include subject areas other than mathematics. Specifically, topics from grade-level/course Next Generation Sunshine State Standards for Science and Social Studies, and Common Core State Standards for English Language Arts may be used where appropriate.
4. Items including specific information or data must be accurate and verified against reliable sources. Source documentation must accompany these types of items.
5. Mathematics item stimuli should include written text and/or visual material, such as graphs, tables, diagrams, maps, models, and/or other illustrations.

6. All item scenarios, graphics, diagrams, and illustrations must be age-, grade-, and experience-appropriate.
7. All graphs used in item stems or answer options must be complete with title, scale, and labeled axes, except when these components are to be completed by the student.
8. Any graphics in items should be uncluttered and should clearly depict the necessary information. Graphics should contain relevant details that contribute to the students' understanding of the item or that support the context of the item. Graphics should not introduce bias to the item.
9. Item content should be timely but not likely to become dated too quickly.

C. Use of Media

Media can be used to provide either necessary or supplemental information—that is, some media contain information that is necessary for answering the question, while other media support the context of the question. Items may include diagrams, illustrations, charts, tables, audio files, or video files unless otherwise noted in the Individual Standard Specifications. Some standards require a heavier use of graphics than others. Geometry, for example, relies heavily on graphics to convey information.

1. Items should not begin with media. Media in items are always preceded by text.
2. All visual media (tables, charts, graphs, photographs, etc.) should be titled. Titles should be in all caps, boldfaced, and centered, and may be placed above or below the visual media.

D. Item Style and Format

This section presents stylistic guidelines and formatting directions that should be followed while developing items.

1. Items should be clear and concise, and they should use vocabulary and sentence structure appropriate for the assessed grade level.
2. The words *most likely* or *best* should be used only when appropriate to the question.
3. Items using the word *not* should emphasize the word *not* using all uppercase letters (e.g., Which of the following is NOT an example of . . .). The word *not* should be used sparingly.
4. For items that refer to an estimate (noun), lowercase letters should be used.
5. As appropriate, boldface type should be used to emphasize key words in the item (e.g., least, most, greatest, percent, mode, median, mean, range).
6. Masculine pronouns should NOT be used to refer to both sexes. Plural forms should be used whenever possible to avoid gender-specific pronouns (e.g., instead of “The student will make changes so that he . . .,” use “The students will make changes so that they . . .”).
7. An equal balance of male and female names should be used, including names representing different ethnic groups appropriate for Florida.

8. For clarity, operation symbols, equality signs, and ordinals should be preceded and followed by one space.
9. Decimal numbers between -1 and 1 (including currency) should have a leading zero.
10. Metric numbers should be expressed in a single unit when possible (e.g., 1.4 kilograms instead of 1 kilogram 400 grams).
11. Decimal notation should be used for numbers with metric units (e.g., 1.2 grams instead of 151 grams).
12. Commas should be used within numbers greater than or equal to 1,000. Commas may be omitted within an equation or expression.
13. Units of measure should be spelled out, except in graphics, where an abbreviation may be used (e.g., *ft* or *yd*). Abbreviations that also spell a word must be followed by a period to avoid confusion. For example, to avoid confusion with the preposition *in*, the abbreviation *in.* should be used for the unit of measure *inches* and should include a period. If an abbreviation is used in a graphic, an explanation of the meaning of the abbreviation should be included in the stem.
14. In titles for tables and charts and in labels for axes, the units of measure should be included, preferably in lowercase letters and in parentheses, e.g., *height (in inches)*.
15. Fractions should be typed with a horizontal fraction bar. The numerator and denominator should be centered with respect to each other. The bar should cover all portions (superscripts, parentheses, etc.) of the numerator and denominator. In a mixed number, a half space should appear between the whole number and the fraction. If a variable appears before or after a fraction bar, the variable should be centered with respect to the fraction bar. If a stimulus, stem, or set of responses contains a fraction in fractional notation, that portion of the item should be 1.5-spaced.
16. In general, numbers zero through nine should be presented as words and numbers 10 and above should be presented as numerals. In the item stem, any numbers needed to compute answers should be presented as numerals.

E. Item Types

This section presents guidelines for development of the following types of items:

1. Selected Response (SR) Items (1 point)
2. Gridded Response (GR) and Short Response (SHR) Items (1 point)
3. Constructed Response and Extended Response Items
 - a. Constructed Response (CR) Items (2 points)
 - b. Extended Response (ER) Items (4 points)
4. Essay Response (ESR) Items (6 points)
5. Performance Task (PT) Items (1–10 points)

1. Selected Response (SR) Items (1 point)

Selected response items require students to choose an answer from the choices given. Each item consists of a stem and either three or four answer options, depending on the grade level/course (see c below). One of the answer options is the correct answer, and the remaining options are called distractors. Selected response items may include a stimulus and/or passage.

- a. SR items should take an average of 1 minute per item to solve.
- b. SR items are worth 1 point each.
- c. SR items in grades K, 1, and 2 should have three answer choices (A, B, and C). SR items for all other grades and courses should have four answer choices (A, B, C, and D).
- d. Answer options that are single words should be arranged in alphabetical or reverse alphabetical order.
- e. Answer options that are phrases or sentences should be arranged from shortest to longest or longest to shortest.
- f. Numerical answer options should be arranged in ascending or descending order.
- g. Numerical answer options that represent relative magnitude or size should be arranged as they are shown in the stem or some other logical order.
- h. When the item requires the identification of a choice from the item stem, table, chart, or illustration, the options should be arranged as they are presented in the item stem, table, chart, or illustration.
- i. If the answer options for an item are neither strictly numerical nor denominate numbers, the options should be arranged by the logic presented in the item, by alphabetical order, or by length.
- j. Distractor rationales should represent computational or conceptual errors or misconceptions commonly made by students who have not mastered the assessed concepts.
- k. Outliers (i.e., answer choices that are longer phrases or sentences than the other choices, or choices with significantly more/fewer digits than the other choices) should NOT be used.
- l. Options such as none of the above, all of the above, not here, not enough information, or cannot be determined should not be used as answer options.

2. Gridded Response (GR) and Short Response (SHR) Items (1 point)

- a. Gridded response and short response items are worth 1 point.
- b. The GR format is designed for items that require a positive numeric solution (whole numbers, decimals, percents, or fractions).
- c. The bubble grids used with GR items should contain eight columns. Each column will contain the digits 0 through 9, decimal point (.), and fraction bar (/) enclosed in bubbles.
- d. Gridded response items should include instructions that specify the unit in which the answer is to be provided (e.g., inches). If several units of measure are in the item (e.g., in an item involving a conversion), the final unit needed for the answer should be written in boldface.
- e. The short response format is designed for items that result in a value or answer that cannot be answered in the gridded response format (negative numbers, expressions, etc.).

3. Constructed Response and Extended Response Items

Mathematics constructed response and extended response items require students to produce a response in words, pictures, diagrams, and/or numbers. As such, these items are especially suited to assessing many of the more complex tasks and high-level thinking skills demanded by the Common Core State Standards for Mathematics. The Florida Interim Assessment Item Bank will include 2-point constructed response items (CR) and 4-point extended response items (ER).

Overall characteristics for mathematics CRs and ERs are as follows:

- a. The item should measure understanding and insight of mathematical concepts rather than rote memory or factual recall.
- b. Real-world, factual stimulus material (charts, graphs, tables, etc.) must cite the source used.
- c. Items requiring students to produce responses as pictures, diagrams, graphs, tables, etc., should provide workspace and/or templates where appropriate.

a. Constructed Response (CR) Items (2 points)

Constructed response items usually include a scenario and instructions on how to respond. The recommended time allotment for a student to respond is 5 minutes. A complete answer is worth 2 points, and a partial answer is worth 1 point. The constructed response holistic rubric and exemplar specific to each item are used for scoring as follows:

SCORING RUBRIC	
2	Work demonstrates a clear and complete understanding of the mathematical concepts and/or procedures required by the task. Appropriate strategy is shown with clear and complete explanations and interpretations.
1	Response demonstrates a partial understanding of the mathematical concepts and/or procedures. Appropriate strategy is shown, but explanation or interpretation has minor flaws. OR Response is incorrect because of calculation errors. Work and strategy indicate a clear understanding of the mathematical concepts and/or procedures required by the task.
0	Response is irrelevant, inappropriate, or not provided.

Exemplars: A specific exemplar should be developed for each constructed response item. Exemplars will be used as scoring guides and should be specific to the item, but not so specific as to discount multiple correct answers. Exemplars should include a clear and defensible description of the top score point, and contain straightforward language that is accurate, complete, and easy to interpret.

b. Extended Response (ER) Items (4 points)

Extended response items include a scenario and instructions on how to respond and are worth 4 points. However, ER items are usually more complex than SHR and 2-point CR items. The recommended time allotment for a student to respond is 10–15 minutes. The extended response holistic rubric and exemplar specific to each item are used for scoring as follows:

SCORING RUBRIC	
4	Work demonstrates a clear and complete understanding of the mathematical concepts and/or procedures required by the task. Appropriate strategy is shown with clear and complete explanations and interpretations.
3	Work demonstrates a clear understanding of the mathematical concepts and/or procedures but is not complete. Appropriate strategy is shown, but explanation or interpretation has minor flaws. OR Response is incorrect because of calculation errors. Work and strategy indicate a clear demonstration of the problem.
2	Response demonstrates a partial understanding of the mathematical concepts and/or procedures. Appropriate strategy is shown, but explanation or interpretation has minor flaws.
1	Response shows minimal understanding of the mathematical concepts and/or procedures or provides no explanation or interpretation for the solution or shows major flaws.
0	Response is irrelevant, inappropriate, or not provided.

Exemplars: A specific exemplar should be developed for each extended response item. Exemplars will be used as scoring guides and should be specific to the item, but not so specific as to discount multiple correct answers. Exemplars should include a clear and defensible description of the top score point, and contain straightforward language that is accurate, complete, and easy to interpret.

4. Essay Response (ESR) Items (6 points)

The essay response item consists of asking a general question or providing a stimulus (such as an article or research paper on a relevant topic), and asking students to express their thoughts or provide facts about the topic using logic and reason. Essay response items encompass a higher level of thinking and a broader range of skills that includes CCSS literacy standards, which is critical to future success in higher education and the workforce.

In most cases, essay responses will go beyond a single paragraph in length, with a distinct introduction, body, and conclusion. An essay response will be worth a total of 6 points, with a rubric structure similar to that of the 4-point extended response. Students should be given about 20 to 30 minutes to complete each item.

Exemplars: A specific exemplar should be developed for each essay response item. Exemplars will be used as scoring guides and should be specific to the item, but not so specific as to discount multiple correct answers. Exemplars should include a clear and defensible description of the top score point, and contain straightforward language that is accurate, complete, and easy to interpret.

5. Performance Tasks (PT) (1–10 points)

Performance tasks are used to measure students' ability to *demonstrate* knowledge and skills from one or more CCSS. Specifically, performance tasks may require students to create a product, demonstrate a process, or perform an activity that demonstrates proficiency in Mathematics. They are evaluated using customized scoring exemplars, and each task may be worth 1–10 points.

Performance tasks may have the following characteristics:

- a. Performance tasks may cover a short time period or may cover an extended period.
- b. Performance tasks must contain clear and explicit directions for understanding and completing the required component tasks and producing the objective output.
- c. All tasks, skills, and/or behaviors required by the performance tasks must be objective, observable, and measurable.
- d. All necessary equipment, materials, and resources should be referenced within the text of the performance task.
- e. Performance tasks should elicit a range of score points.
- f. Performance tasks generally require students to organize, apply, analyze, synthesize, and/or evaluate concepts.
- g. Performance tasks may measure performance in authentic situations and outside the classroom, where appropriate and practical.
- h. Typical response formats include demonstrations, laboratory performance, oral presentations, exhibits, or other products.

- i. Every performance task requires a companion exemplar to be used for scoring purposes. Exemplars should meet the following criteria.
 - i The exemplars and performance tasks should be developed in tandem to ensure compatibility.
 - ii Exemplars must be specific to the individual requirements of each performance task; generic rubrics are not acceptable.
 - iii The exemplar must allow for efficient and consistent scoring.
 - iv Each part of the performance task must have a clearly stated score point in the exemplar and when a part of the task is divided into sections or requirements, each of those must have a maximum score indicated.
 - v The exemplar descriptors consist of an ideal response exemplar and should allow for all foreseeable methods of correctly and thoroughly completing all requirements of the performance task.

F. Readability

Items must be written with readability in mind. In addition, vocabulary must be appropriate for the grade level being tested. The following sources provide information about the reading level of individual words:

Taylor, Stanford E. *EDL Core Vocabularies: Reading, Mathematics, Science, and Social Studies*. Austin, TX: Steck-Vaughn-EDL, 1989.

Mogilner, Alijandra. *Children's Writer's Word Book*. Cincinnati, OH: Writer's Digest Books, 1992.

G. Cognitive Complexity

1. Overview

Florida's adoption of the Common Core State Standards (CCSS) for Mathematics and English Language Arts & Literacy in History/Social Studies, Science, and Technical Subjects presents Florida with an opportunity to revise its current Depth of Knowledge (DOK) Model of Cognitive Complexity. More information about Florida's Depth of Knowledge levels is available online at <http://www.cpalms.org/cpalms/dok.aspx>.

2. Levels of Depth of Knowledge for Mathematics

Level 1 (Recall) includes the recall of information such as a fact, definition, term, or a simple procedure, as well as performing a simple algorithm or applying a formula. That is, in mathematics a one-step, well-defined, or straight algorithmic procedure should be included at this lowest level.

Some examples that represent but do not constitute all of Level 1 performance are:

- Count to 100 by ones and by tens.
- Fluently multiply and divide within 100, using strategies such as the relationship between multiplication and division (e.g., knowing that $8 \times 5 = 40$, one knows $40 \div 5 = 8$).

- Write simple expressions that record calculations with numbers, and interpret numerical expressions without evaluating them. For example, express the calculation “add 8 and 7 and then multiply by 2” as $2 \times (8 + 7)$. Recognize that $3 \times (18932 + 921)$ is three times as large as $18932 + 921$ without having to calculate the indicated sum or product.
- Enter measurement data into a data table.
- Identify the variables indicated in a two-dimensional graph.

Level 2 (Basic Application of Concepts & Skills) includes the engagement of some mental processing beyond a habitual response. A Level 2 standard or assessment item requires students to make some decisions as to how to approach the problem or activity, whereas Level 1 requires students to demonstrate a rote response, perform a well-known algorithm, follow a set procedure (like a recipe), or perform a clearly defined series of steps. For example, to compare data requires first identifying characteristics of the objects or phenomenon and then grouping or ordering the objects. Interpreting information from a simple graph, requiring reading information from the graph, also is a Level 2. Interpreting information from a complex graph that requires some decisions on what features of the graph need to be considered and how information from the graph can be aggregated is a Level 3. Caution is warranted in interpreting Level 2 as only skills because some reviewers will interpret skills very narrowly as primarily numerical skills, and such interpretation excludes from this level other skills such as visualization skills and probability skills, which may be more complex simply because they are less common and require more mental processing.

Some examples that represent but do not constitute all of Level 2 performance are:

- Understand that the last number name said tells the number of objects counted. The number of objects is the same regardless of their arrangement or the order in which they were counted.
- Express the length of an object as a whole number of length units, by laying multiple copies of a shorter object (the length unit) end to end.
- Measure and estimate liquid volumes and masses of objects using standard units of grams (g), kilograms (kg), and liters (l).
- Apply properties of operations as strategies to add and subtract rational numbers.
- Measure and record data and produce graphs of relevant variables.
- Graph proportional relationships, interpreting the unit rate as the slope of the graph.

Level 3 (Strategic Thinking & Complex Reasoning) requires reasoning, planning, using evidence, and a higher level of thinking than the previous two levels. In most instances, requiring students to explain their thinking is a Level 3. Activities that require students to make conjectures are also at this level. The cognitive demands at Level 3 are complex and abstract. The complexity does not result from the fact that there are multiple answers, a possibility for both levels 1 and 2, but because the task requires more demanding reasoning. However, an activity that has more than one possible answer and requires students to justify the response they give would most likely be a Level 3.

Some examples that represent but do not constitute all of Level 3 performance are:

- Explain why addition and subtraction strategies work, using place value and the properties of operations.
- Interpret the rate of change and initial value of a linear function in terms of the situation it models, and in terms of its graph or a table of values.
- Given a real-world situation, formulate a problem.
- Organize, represent, and interpret data obtained through experiments or observations.
- Formulate a mathematical model to describe a complex phenomenon.
- Justify a solution to a problem.
- Analyze a deductive argument.

Level 4 (Extended Thinking & Complex Reasoning) in mathematics involves the application of Level 3 processes and skills over an extended period. This is likely to incorporate demands from other content areas (e.g., English language arts, science) in the development and support of mathematical arguments that describe some real-world phenomenon or situation.

Some examples that represent but do not constitute all of Level 4 performance are:

- Derive a mathematical model to explain a complex phenomenon or make a prediction.
- Complete a project requiring the formulation of questions, devising a plan, collecting data, analyzing the data, and preparing a written report describing the justification of the conclusions reached.

H. Item Difficulty

Item writers will not be expected to make a prediction of difficulty for each item created. However, item writers should develop items that reflect a range of difficulty.

I. Universal Design

The application of universal design principles helps develop assessments that are usable to the greatest number of students, including students with disabilities and nonnative speakers of English. To support the goal of providing access to all students, the items in the Florida Interim Assessment Item Bank maximize readability, legibility, and compatibility with accommodations, and item development includes a review for potential bias and sensitivity issues.

Items must allow for the widest possible range of student participation. Item writers must attend to the best practices suggested by universal design, including, but not limited to,

1. reduction in wordiness
2. avoidance of ambiguity
3. selection of reader-friendly construction and terminology
4. consistently applied concept names and graphic conventions

Universal design principles also inform decisions about item layout and design, including, but not limited to, type size, line length, spacing, and graphics.

J. Sample Items

Appendix A of this document contains a selection of sample items. The sample items represent a range of cognitive complexities and item types.

III. Review Procedures for Florida Interim Assessment Item Bank Items

Prior to being included in the Florida Interim Assessment Item Bank, all mathematics items must pass several levels of review as part of the item development process.

A. Review for Item Quality

Assessment items developed for the Florida Interim Assessment Item Bank will be reviewed by Florida educators, the FDOE, and the contractors to ensure the quality of the items, including grade-level/course appropriateness, alignment to the standard, accuracy, and other criteria for overall item quality.

B. Review for Bias and Sensitivity

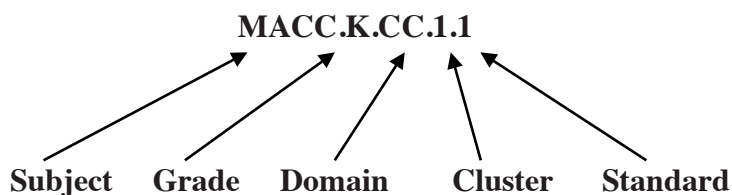
Items are reviewed by groups of Florida educators generally representative of Florida’s geographic regions and culturally diverse population. Items are reviewed for the following kinds of bias: gender, racial, ethnic, linguistic, religious, geographic, and socioeconomic. Item reviews also include consideration of issues related to individuals with disabilities.

This review is to ensure that the primary purpose of assessing student achievement is not undermined by inadvertently including in the item bank any material that students, parents, or other stakeholders may deem inappropriate. Reviewers are asked to consider the variety of cultural, regional, philosophical, political, and religious backgrounds throughout Florida and to determine whether the subject matter will be acceptable to Florida students, their parents, and other members of Florida communities.

IV. Guide to the Individual Standard Specifications

A. CCSS Mathematics Standards Classification System

The graphic below demonstrates the coding schema for the Common Core State Standards for Mathematics.



Using this schema:

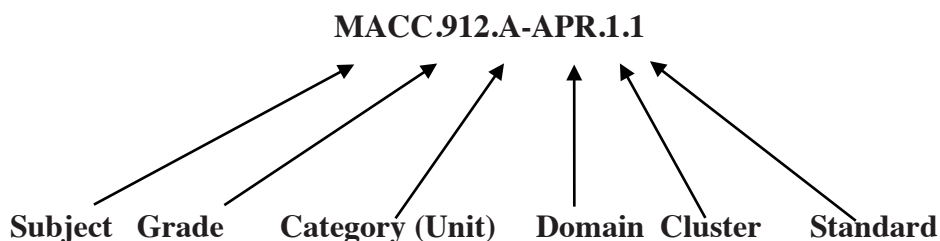
Subject Code MACC: Mathematics Common Core

Grade: Kindergarten

Domain CC: Counting and Cardinality

Cluster 1: Know number names and the count sequence.

Standard 1: Count to 100 by ones and by tens.



Using the schema, the bottom row refers to:

Subject Code MACC: Mathematics Common Core

Grade: High school Grades 9–12

Category A: Algebra

Domain APR: Arithmetic with Polynomials and Rational Expressions

Cluster 1: Perform arithmetic operations on polynomials.

Standard 1: Understand that polynomials form a system analogous to the integers, namely, they are closed under the operations of addition, subtraction, and multiplication; add, subtract, and multiply polynomials.

B. Definitions of Cluster and Standard Specifications

The *Item Specifications* identify how the standards in the CCSS are assessed by items in the Florida Interim Assessment Item Bank. For each assessed standard, the following information is provided in the Individual Standards Specifications section.

Domain	refers to larger groups of related standards. Standards from different domains may sometimes be closely related.
Cluster	refers to groups of related standards. Note that standards from different clusters may sometimes be closely related because mathematics is a connected subject.
Standards	define what students should understand and be able to do.
Standards Clarifications/ Content Limits for Course	Standards clarifications, when needed as an explanation for some of the standards listed above, explain the type of behavior that the student should exhibit for mastery of the standard. The clarification statements explain what students are expected to do when responding to the question. Course limits define the range of content knowledge and degree of difficulty that should be assessed in the items for the standard. Course limits may be used to identify content beyond the scope of the targeted standard if the content is more appropriately assessed by another standard. These statements help to provide validity by ensuring the test items are clearly aligned to the targeted standard.

V. Individual Standards Specifications for Florida Interim Assessment Item Bank Mathematics Items

This section of the *Item Specifications* provides standard-specific guidance for assessment item development for the Florida Interim Assessment Item Bank based on the Common Core State Standards.

Each item developed for the Florida Interim Assessment Item Bank and Test Platform should assess one or more of the Mathematical Practice Standards listed in Appendix B.

A. Algebra 2 Item Specifications

Conceptual Category	NUMBER AND QUANTITY
Domain	The Complex Number System
Cluster	Extend the properties of exponents to rational exponents.
Standards	<p>MACC.912.N-RN.1.1—Explain how the definition of the meaning of rational exponents follows from extending the properties of integer exponents to those values, allowing for a notation for radicals in terms of rational exponents. <i>For example, we define $5^{\frac{1}{3}}$ to be the cube root of 5 because we want $(5^{\frac{1}{3}})^3 = 5^{(\frac{1}{3})^3}$ to hold, so $5^{(\frac{1}{3})^3}$ must equal 5.</i></p> <p>MACC.912.N-RN.1.2—Rewrite expressions involving radicals and rational exponents using the properties of exponents.</p>
Standards Clarifications/ Content Limits for Course	<p>As an explanation for some of the standards listed above, in Algebra 2, students will</p> <ul style="list-style-type: none"> • extend the concept of simple roots and exponent rules to expressions with rational exponents and nth roots and understand how they are related • use the properties of exponents and properties of radicals to simplify expressions

Conceptual Category	NUMBER AND QUANTITY
Domain	Quantities
Cluster	Reason quantitatively and use units to solve problems.
Standards	MACC.912.N-Q.1.2—Define appropriate quantities for the purpose of descriptive modeling.
Standards Clarifications/ Content Limits for Course	In Algebra 2, students will <ul style="list-style-type: none"> analyze and solve mathematical models that require the solution to be written in appropriate units of measure

Conceptual Category	NUMBER AND QUANTITY
Domain	The Complex Number System
Cluster	Perform arithmetic operations with complex numbers.
Standards	MACC.912.N-CN.1.1—Know there is a complex number i such that $i^2 = -1$, and every complex number has the form $a + bi$ with a and b real. MACC.912.N-CN.1.2—Use the relation $i^2 = -1$ and the commutative, associative, and distributive properties to add, subtract, and multiply complex numbers.
Standards Clarifications/ Content Limits for Course	As an explanation for some of the standards listed above, in Algebra 2, students will <ul style="list-style-type: none"> determine the real part and imaginary part of a complex number justify the steps in simplifying expressions with complex numbers using the commutative, associative, and distributive properties; add, subtract, and multiply complex numbers

Conceptual Category	NUMBER AND QUANTITY
Domain	The Complex Number System
Cluster	Use complex numbers in polynomial identities and equations.
Standards	<p>MACC.912.N-CN.3.7—Solve quadratic equations with real coefficients that have complex solutions.</p> <p>+MACC.912.N-CN.3.8—Extend polynomial identities to the complex numbers. <i>For example, rewrite $x^2 + 4$ as $(x + 2i)(x - 2i)$.</i></p> <p>+MACC.912.N-CN.3.9—Know the Fundamental Theorem of Algebra; show that it is true for quadratic polynomials.</p>
Standards Clarifications/ Content Limits for Course	<p>As an explanation for some of the standards listed above, in Algebra 2, students will</p> <ul style="list-style-type: none"> • for standard MACC.912.N-CN.3.7, solve quadratic equations with real coefficients that have complex solutions; factor binomials and trinomials over the complex numbers; demonstrate understanding of the connection between the number of roots and the degree of the polynomial, considering multiple roots, complex roots, and distinct real roots; analyze the discriminant to determine the nature of the solutions, including complex solutions • use polynomial identities to write equivalent expressions in the form of complex numbers. Polynomial identities include $(a + b)^2 = a^2 + 2ab + b^2$ (perfect square trinomial); $(a + b)(c + d) = ac + ad + bc + bd$ (distributive property); $(a - b)(a + b) = a^2 - b^2$ (difference of squares); sum or difference of two cubes; and quadratic formula. • find the zeros of a quadratic equation and explain the answer in terms of the Fundamental Theorem of Algebra <p>Course limit: Polynomials with real coefficients</p>

Conceptual Category	ALGEBRA
Domain	Seeing Structure in Expressions
Cluster	Interpret the structure of expressions.
Standards	<p>MACC.912.A-SSE.1.1—Interpret expressions that represent a quantity in terms of its context.</p> <p>MACC.912.A-SSE.1.1.a—Interpret parts of an expression, such as terms, factors, and coefficients.</p> <p>MACC.912.A-SSE.1.1.b—Interpret complicated expressions by viewing one or more of their parts as a single entity. <i>For example, interpret $P(1 + r)^n$ as the product of P and a factor not depending on P.</i></p> <p>MACC.912.A-SSE.1.2—Use the structure of an expression to identify ways to rewrite it. <i>For example, see $x^4 - y^4$ as $(x^2)^2 - (y^2)^2$, thus recognizing it as a difference of squares that can be factored as $(x^2 - y^2)(x^2 + y^2)$.</i></p>
Standards Clarifications/ Content Limits for Course	<p>As an explanation for some of the standards listed above, in Algebra 2, students will</p> <ul style="list-style-type: none"> • recognize equivalent forms of an expression as determined by its structure; interpret expressions or parts of expressions in the context of a problem; make connections between symbolic representations and proper mathematics vocabulary • use properties of mathematics to alter the structure of an expression and select and use an appropriate factoring technique; for example, in the equation $x^2 + 2x + 1 + y^2 = 9$, students will see an opportunity to rewrite the first three terms as $(x + 1)^2$, thus recognizing the equation of a circle with radius 3 and center $(-1, 0)$ <p>Course limit: Polynomial and rational</p>

Conceptual Category	ALGEBRA
Domain	Seeing Structure in Expressions
Cluster	Write expressions in equivalent forms to solve problems.
Standards	<p>MACC.912.A-SSE.2.3—Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression.</p> <p>MACC.912.A-SSE.2.3.c—Use the properties of exponents to transform expressions for exponential functions. <i>For example, the expression 1.15^t can be rewritten as $(1.15^{\frac{1}{12}})^{12t} \approx 1.012^{12t}$ to reveal the approximate equivalent monthly interest rate if the annual rate is 15%.</i></p> <p>MACC.912.A-SSE.2.4—Derive the formula for the sum of a finite geometric series (when the common ratio is not 1), and use the formula to solve problems. <i>For example, calculate mortgage payments.</i></p>
Standards Clarifications/ Content Limits for Course	<p>As an explanation for some of the standards listed above, in Algebra 2, students will</p> <ul style="list-style-type: none"> • show the difference between an infinite and a finite series; apply the formula for the sum of a finite geometric series to solve problems

Conceptual Category	ALGEBRA
Domain	Arithmetic with Polynomials and Rational Expressions
Cluster	Perform arithmetic operations on polynomials.
Standards	<p>MACC.912.A-APR.1.1—Understand that polynomials form a system analogous to the integers; namely, they are closed under the operations of addition, subtraction, and multiplication; add, subtract, and multiply polynomials.</p>
Standards Clarifications/ Content Limits for Course	<p>In Algebra 2, students will</p> <ul style="list-style-type: none"> • simplify polynomials using addition, subtraction, and multiplication <p>Course limit: Beyond quadratic</p>

Conceptual Category	ALGEBRA
Domain	Arithmetic with Polynomials and Rational Expressions
Cluster	Understand the relationship between zeros and factors of polynomials.
Standards	<p>MACC.912.A-APR.2.2—Know and apply the Remainder Theorem: For a polynomial $p(x)$ and a number a, the remainder on division by $x - a$ is $p(a)$, so $p(a) = 0$ if and only if $(x - a)$ is a factor of $p(x)$.</p> <p>MACC.912.A-APR.2.3—Identify zeros of polynomials when suitable factorizations are available, and use the zeros to construct a rough graph of the function defined by the polynomial.</p>
Standards Clarifications/ Content Limits for Course	N/A

Conceptual Category	ALGEBRA
Domain	Arithmetic with Polynomials and Rational Expressions
Cluster	Use polynomial identities to solve problems.
Standards	<p>MACC.912.A-APR.3.4—Prove polynomial identities and use them to describe numerical relationships. <i>For example, the polynomial identity $(x^2 + y^2)^2 = (x^2 - y^2)^2 + (2xy)^2$ can be used to generate Pythagorean triples.</i></p> <p>+MACC.912.A-APR.3.5—Know and apply the Binomial Theorem for the expansion of $(x + y)^n$ in powers of x and y for a positive integer n, where x and y are any numbers, with coefficients determined, for example, by Pascal's triangle.</p>
Standards Clarifications/ Content Limits for Course	<p>As an explanation for some of the standards listed above, in Algebra 2, students will</p> <ul style="list-style-type: none"> • use their knowledge of the process for proving identities to describe numerical relationships and use and manipulate the structure in an expression as needed to prove an identity. Polynomial identities include $(a + b)^2 = a^2 + 2ab + b^2$ (perfect square trinomial); $(a + b)(c + d) = ac + ad + bc + bd$ (distributive property); $(a - b)(a + b) = a^2 - b^2$ (difference of squares); sum or difference of two cubes; and quadratic formula.

Conceptual Category	ALGEBRA
Domain	Arithmetic with Polynomials and Rational Expressions
Cluster	Rewrite rational expressions.
Standards	<p>MACC.912.A-APR.4.6—Rewrite simple rational expressions in different forms; write $\frac{a(x)}{b(x)}$ in the form $q(x) + \frac{r(x)}{b(x)}$, where $a(x)$, $b(x)$, $q(x)$, and $r(x)$ are polynomials with the degree of $r(x)$ less than the degree of $b(x)$, using inspection, long division, or, for the more complicated examples, a computer algebra system.</p> <p>+MACC.912.A-APR.4.7—Understand that rational expressions form a system analogous to the rational numbers, closed under addition, subtraction, multiplication, and division by a nonzero rational expression; add, subtract, multiply, and divide rational expressions.</p>
Standards Clarifications/ Content Limits for Course	<p>As an explanation for some of the standards listed above, in Algebra 2, students will</p> <ul style="list-style-type: none"> • use long division and synthetic division to rewrite rational expressions; make connections to the Remainder Theorem and simple rational expressions <p>Course limit: Linear and quadratic denominators</p>

Conceptual Category	ALGEBRA
Domain	Creating Equations
Cluster	Create equations that describe numbers or relationships.
Standards	<p>MACC.912.A-CED.1.1—Create equations and inequalities in one variable and use them to solve problems. Include equations arising from linear and quadratic functions, and simple rational and exponential functions.</p> <p>MACC.912.A-CED.1.2—Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.</p> <p>MACC.912.A-CED.1.3—Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or nonviable options in a modeling context. <i>For example, represent inequalities describing nutritional and cost constraints on combinations of different foods.</i></p> <p>MACC.912.A-CED.1.4—Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations. <i>For example, rearrange Ohm’s law $V = IR$ to highlight resistance R.</i></p>

**Standards Clarifications/
Content Limits for Course**

As an explanation for some of the standards listed above, in Algebra 2, students will

- distinguish between linear, exponential, quadratic, radical, and simple rational relationships given multiple representations (verbal, numeric, or graphic) and then create the appropriate equation or inequality using the given information; create linear, exponential, and quadratic equations in one variable, including the creation of simple rational functions using all available types of expressions, including simple root functions; use the equations and inequalities to solve problems
- create and graph equations in two variables to represent relationships between quantities, which include appropriate scales and labels to accurately display all aspects of the relation; distinguish between relationships using all available types of expressions, including simple root functions, given multiple representations (numeric or verbal); determine unknown parameters needed to create an equation that accurately models a given situation using all available types of expressions, including simple root functions
- represent constraints in a modeling context using equations and inequalities using all available types of expressions, including simple root functions
- recognize or create equivalent forms of literal equations using all available types of expressions, including simple root functions

Conceptual Category	ALGEBRA
Domain	Reasoning with Equations and Inequalities
Cluster	Understand solving equations as a process of reasoning and explain the reasoning.
Standards	<p>MACC.912.A-REI.1.1—Explain each step in solving a simple equation as following from the equality of numbers asserted at the previous step, starting from the assumption that the original equation has a solution. Construct a viable argument to justify a solution method.</p> <p>MACC.912.A-REI.1.2—Solve simple rational and radical equations in one variable, and give examples showing how extraneous solutions may arise.</p>
Standards Clarifications/ Content Limits for Course	<p>As an explanation for some of the standards listed above, in Algebra 2, students will</p> <ul style="list-style-type: none"> • solve simple rational and radical equations in one variable; make connections between the domain of a function and extraneous solutions; identify extraneous solutions

Conceptual Category	ALGEBRA
Domain	Reasoning with Equations and Inequalities
Cluster	Solve equations and inequalities in one variable.
Standards	<p>MACC.912.A-REI.2.4—Solve quadratic equations in one variable.</p> <p>MACC.912.A-REI.2.4.b—Solve quadratic equations by inspection (e.g., for $x^2 = 49$), taking square roots, completing the square, the quadratic formula, and factoring as appropriate to the initial form of the equation. Recognize when the quadratic formula gives complex solutions and write them as $a \pm bi$ for real numbers a and b.</p>
Standards Clarifications/ Content Limits for Course	<p>As an explanation for some of the standards listed above, in Algebra 2, students will</p> <ul style="list-style-type: none"> • solve quadratic equations over the set of real or complex numbers using various methods and recognize the most efficient method as well as use the value of the discriminant to determine whether a quadratic equation has one double solution, two unique solutions, or no real solutions

Conceptual Category	ALGEBRA
Domain	Reasoning with Equations and Inequalities
Cluster	Solve systems of equations.
Standards	<p>MACC.912.A-REI.3.6—Solve systems of linear equations exactly and approximately (e.g., with graphs), focusing on pairs of linear equations in two variables.</p> <p>MACC.912.A-REI.3.7—Solve a simple system consisting of a linear equation and a quadratic equation in two variables algebraically and graphically. <i>For example, find the points of intersection between the line $y = -3x$ and the circle $x^2 + y^2 = 3$.</i></p>
Standards Clarifications/ Content Limits for Course	<p>As an explanation for some of the standards listed above, in Algebra 2, students will</p> <ul style="list-style-type: none"> • for standard MACC.912.A-REI.3.6, choose the most efficient method to solve systems of equations • for standard MACC.912.A-REI.3.7, solve systems of equations containing one linear and one quadratic equation in two variables algebraically and graphically

Conceptual Category	ALGEBRA
Domain	Reasoning with Equations and Inequalities
Cluster	Represent and solve equations and inequalities graphically.
Standards	<p>MACC.912.A-REI.4.11—Explain why the x-coordinates of the points where the graphs of the equations $y = f(x)$ and $y = g(x)$ intersect are the solutions of the equation $f(x) = g(x)$; find the solutions approximately, e.g., using technology to graph the functions, make tables of values, or find successive approximations. Include cases where $f(x)$ and/or $g(x)$ are linear, polynomial, rational, absolute value, exponential, and logarithmic functions.</p>
Standards Clarifications/ Content Limits for Course	<p>In Algebra 2, students will</p> <ul style="list-style-type: none"> • explain the equality of two functions using multiple representations; approximate solutions to systems of equations represented graphically <p>Course limit: Combine polynomial, rational, radical, absolute value, and exponential function</p>

Conceptual Category	FUNCTIONS
Domain	Interpreting Functions
Cluster	Interpret functions that arise in applications in terms of the context.
Standards	<p>MACC.912.F-IF.2.4—For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. <i>Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity.</i></p> <p>MACC.912.F-IF.2.5—Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. <i>For example, if the function $h(n)$ gives the number of person-hours it takes to assemble n engines in a factory, then the positive integers would be an appropriate domain for the function.</i></p> <p>MACC.912.F-IF.2.6—Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph.</p>
Standards Clarifications/ Content Limits for Course	<p>As an explanation for some of the standards listed above, in Algebra 2, students will</p> <ul style="list-style-type: none"> • describe the restrictions on the domain of all functions based on real-world context • calculate the rate of change from multiple representations; interpret, in context, the average rate of change of a function over a specified interval; determine that the rate of change of a function can be positive, negative, or zero <p>Course limit: Emphasize selection of appropriate models</p>

Conceptual Category	FUNCTIONS
Domain	Interpreting Functions
Cluster	Analyze functions using different representations.
Standards	<p>MACC.912.F-IF.3.7—Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.</p> <p>MACC.912.F-IF.3.7.c—Graph polynomial functions, identifying zeros when suitable factorizations are available, and showing end behavior.</p> <p>MACC.912.F-IF.3.7.e—Graph exponential and logarithmic functions, showing intercepts and end behavior, and trigonometric functions, showing period, midline, and amplitude.</p> <p>MACC.912.F-IF.3.8—Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function.</p> <p>MACC.912.F-IF.3.8.b—Use the properties of exponents to interpret expressions for exponential functions. <i>For example, identify percent rate of change in functions such as $y = (1.02)^t$, $y = (0.97)^t$, $y = (1.01)^{12t}$, $y = (1.2)^{\frac{t}{6}}$, and classify them as representing exponential growth or decay.</i></p> <p>MACC.912.F-IF.3.9—Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). <i>For example, given a graph of one quadratic function and an algebraic expression for another, say which has the larger maximum.</i></p>

**Standards Clarifications/
Content Limits for Course**

As an explanation for some of the standards listed above, in Algebra 2, students will

- connect experience with graphing linear, exponential, and quadratic functions from Algebra 1 to graphing polynomial functions (for example, students should be able to recognize and graph parent functions); identify the zeros of the polynomial function on a graph when given the function in equation form and algebraically finding the zeros (by using the appropriate form of factorization and showing the relationship between zeros of quadratic functions and their factored forms); graph polynomial functions, identifying zeros and showing end behavior; identify the end behaviors of a polynomial function (represented as an equation)
- focus on using key features to guide selection of appropriate types of model functions; graph exponential and logarithmic functions, showing intercepts and end behavior; graph trigonometric functions, showing period, midline, and amplitude
- recognize common attributes of a quadratic function represented in context and identify key features of a quadratic function using factoring or completing the square
- use the properties of exponents to interpret expressions for exponential functions; identify common attributes of an exponential function from various representations (algebraically, graphically, numerically in tables, or by verbal descriptions); classify exponential functions as representing exponential growth or decay
- compare and identify key features of a pair of functions (exponential, polynomial, logarithmic, and trigonometric functions), such as their maxima, minima, intercepts, zeros, asymptotes, and end behaviors; recognize common attributes of a function from various representations (algebraically, graphically, numerically in tables, or by verbal descriptions)

Course limit: Focus on using key features to guide selection of the appropriate type of model function

Conceptual Category	FUNCTIONS
Domain	Building Functions
Cluster	Build a function that models a relationship between two quantities.
Standards	<p>MACC.912.F-BF.1.1—Write a function that describes a relationship between two quantities.</p> <p>MACC.912.F-BF.1.1.a—Determine an explicit expression, a recursive process, or steps for calculation from a context.</p> <p>MACC.912.F-BF.1.1.b—Combine standard function types using arithmetic operations. <i>For example, build a function that models the temperature of a cooling body by adding a constant function to a decaying exponential, and relate these functions to the model.</i></p> <p>MACC.912.F-BF.1.2—Write arithmetic and geometric sequences both recursively and with an explicit formula, use them to model situations, and translate between the two forms.</p>
Standards Clarifications/ Content Limits for Course	<p>As an explanation for some of the standards listed above, in Algebra 2, students will</p> <ul style="list-style-type: none"> analyze a given problem to determine the function expressed by identifying patterns in the function’s rate of change; specify intervals of increase, decrease, and constancy and, if possible, relate them to the function’s description in words or graphically; add, subtract, multiply, and divide functions

Conceptual Category	FUNCTIONS
Domain	Building Functions
Cluster	Build new functions from existing functions.
Standards	<p>MACC.912.F-BF.2.3—Identify the effect on the graph of replacing $f(x)$ by $f(x) + k$, $kf(x)$, $f(kx)$, and $f(x + k)$ for specific values of k (both positive and negative); find the value of k given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. Include recognizing even and odd functions from their graphs and algebraic expressions for them.</p> <p>MACC.912.F-BF.2.4—Find inverse functions.</p> <p>MACC.912.F-BF.2.4.a—Solve an equation of the form $f(x) = c$ for a simple function f that has an inverse and write an expression for the inverse. <i>For example, $f(x) = 2x^3$ for $x > 0$ or $f(x) = \frac{(x+1)}{(x-1)}$ for $x \neq 1$.</i></p>
Standards Clarifications/ Content Limits for Course	<p>As an explanation for some of the standards listed above, in Algebra 2, students will</p> <ul style="list-style-type: none"> • apply transformations to functions and recognize functions as even and odd; identify, through experimenting with technology, the effect on the graph of a function by replacing $f(x)$ with $f(x) + k$, $kf(x)$, $f(kx)$, and $f(x + k)$ for specific values of k (both positive and negative); compare and describe the shape and position of graphs and explain the differences; make generalizations about the changes that will take place in the graph of any function as a result of making a particular change to the algebraic representation of the function • solve a function for the dependent variable and write the inverse of a function by interchanging the values of the dependent and independent variables <p>Course limit: Include simple radical, rational, and exponential functions; emphasize the common effect of each transformation across function types</p>

Conceptual Category	FUNCTIONS
Domain	Linear, Quadratic, and Exponential Models
Cluster	Construct and compare linear, quadratic, and exponential models and solve problems.
Standards	MACC.912.F-LE.1.4—For exponential models, express as a logarithm the solution to $abct = d$ where a , c , and d are numbers and the base b is 2, 10, or e ; evaluate the logarithm using technology.
Standards Clarifications/ Content Limits for Course	In Algebra 2, students will <ul style="list-style-type: none"> • express logarithms as solutions to exponential functions using bases 2, 10, and e; use technology to evaluate a logarithm; analyze exponential models and evaluate logarithms; understand the properties of logarithms and exponents and their connection to one another; understand that logarithmic functions are inverses of exponential functions

Conceptual Category	FUNCTIONS
Domain	Linear, Quadratic, and Exponential Models
Cluster	Interpret expressions for functions in terms of the situation they model.
Standards	MACC.912.F-LE.2.5—Interpret the parameters in a linear or exponential function in terms of a context.
Standards Clarifications/ Content Limits for Course	In Algebra 2, students will <ul style="list-style-type: none"> • model real world context and interpret parameters in exponential functions with domains not in the integers

Conceptual Category	FUNCTIONS
Domain	Trigonometric Functions
Cluster	Extend the domain of trigonometric functions using the unit circle.
Standards	<p>MACC.912.F-TF.1.1—Understand radian measure of an angle as the length of the arc on the unit circle subtended by the angle.</p> <p>MACC.912.F-TF.1.2—Explain how the unit circle in the coordinate plane enables the extension of trigonometric functions to all real numbers, interpreted as radian measures of angles traversed counterclockwise around the unit circle.</p>
Standards Clarifications/ Content Limits for Course	<p>As an explanation for some of the standards listed above, in Algebra 2, students will</p> <ul style="list-style-type: none"> • understand that angle measures in radians may be determined by a ratio of intercepted arc to radius; explain how radian measures of angles rotated counterclockwise in a unit circle are in a one-to-one correspondence with the nonnegative real numbers and that angles rotated clockwise in a unit circle are in a one-to-one correspondence with the nonpositive real numbers; extend the domain of trigonometric functions to angles beyond the unit circle using counterclockwise as the positive direction of rotation

Conceptual Category	FUNCTIONS
Domain	Trigonometric Functions
Cluster	Model periodic phenomena with trigonometric functions.
Standards	MACC.912.F-TF.2.5—Choose trigonometric functions to model periodic phenomena with specified amplitude, frequency, and midline.
Standards Clarifications/ Content Limits for Course	<p>In Algebra 2, students will</p> <ul style="list-style-type: none"> • model periodic phenomena with trigonometric functions and connect contextual situations to appropriate trigonometric functions (e.g., using sine or cosine to model cyclical behavior, such as the ocean’s tide or the rotation of a Ferris wheel); given the amplitude, frequency, and midline in situations or graphs, determine a trigonometric function used to model the situation

Conceptual Category	FUNCTIONS
Domain	Trigonometric Functions
Cluster	Prove and apply trigonometric identities.
Standards	MACC.912.F-TF.3.8—Prove the Pythagorean identity $\sin^2(\theta) + \cos^2(\theta) = 1$ and use it to find $\sin(\theta)$, $\cos(\theta)$, or $\tan(\theta)$ given $\sin(\theta)$, $\cos(\theta)$, or $\tan(\theta)$ and the quadrant of the angle.
Standards Clarifications/ Content Limits for Course	N/A

Conceptual Category	GEOMETRY
Domain	Expressing Geometric Properties with Equations
Cluster	Translate between the geometric description and the equation for a conic section.
Standards	MACC.912.G-GPE.1.2—Derive the equation of a parabola given a focus and directrix.
Standards Clarifications/ Content Limits for Course	In Algebra 2, students will <ul style="list-style-type: none"> • use the focus and directrix to determine the equation of a parabola or, given an equation, determine the focus and directrix

Conceptual Category	STATISTICS AND PROBABILITY
Domain	Making Inferences and Justifying Conclusions
Cluster	Understand and evaluate random processes underlying statistical experiments.
Standards	<p>MACC.912.S-IC.1.1—Understand statistics as a process for making inferences about population parameters based on a random sample from that population.</p> <p>MACC.912.S-IC.1.2—Decide if a specified model is consistent with results from a given data-generating process, e.g., using simulation. <i>For example, a model says a spinning coin falls heads up with probability 0.5. Would a result of 5 tails in a row cause you to question the model?</i></p>
Standards Clarifications/ Content Limits for Course	<p>As an explanation for some of the standards listed above, in Algebra 2, students will</p> <ul style="list-style-type: none"> • explain in context the difference between values describing a population and a sample and explain how well and why a sample represents the variable of interest from a population; demonstrate an understanding of the different kinds of sampling methods (random, self-selected, convenience, stratified, etc.)

Conceptual Category	STATISTICS AND PROBABILITY
Domain	Making Inferences and Justifying Conclusions
Cluster	Make inferences and justify conclusions from sample surveys, experiments, and observational studies.
Standards	<p>MACC.912.S-IC.2.3—Recognize the purposes of and differences among sample surveys, experiments, and observational studies; explain how randomization relates to each.</p> <p>MACC.912.S-IC.2.4—Use data from a sample survey to estimate a population mean or proportion; develop a margin of error through the use of simulation models for random sampling.</p> <p>MACC.912.S-IC.2.5—Use data from a randomized experiment to compare two treatments; use simulations to decide if differences between parameters are significant.</p> <p>MACC.912.S-IC.2.6—Evaluate reports based on data.</p>
Standards Clarifications/ Content Limits for Course	<p>As an explanation for some of the standards listed above, in Algebra 2, students will</p> <ul style="list-style-type: none"> • identify, design, and/or evaluate situations as sample surveys, experiments, and observational studies; understand the limitations of each technique in order to recognize and avoid bias • use sample means and sample proportions to estimate population values; conduct simulations of random sampling to gather sample means and sample proportions, explain what the results mean about variability in a population, and use the results to calculate margins of error for these estimates • evaluate effectiveness and differences in two treatments based on data from randomized experiments; use simulations to generate data for two treatments and use results to evaluate the significance of differences • read and explain, in context, data from outside reports

Conceptual Category	STATISTICS AND PROBABILITY
Domain	Conditional Probability and the Rules of Probability
Cluster	Understand independence and conditional probability and use them to interpret data.
Standards	<p>MACC.912.S-CP.1.1—Describe events as subsets of a sample space (the set of outcomes) using characteristics (or categories) of the outcomes, or as unions, intersections, or complements of other events (“or,” “and,” “not”).</p> <p>MACC.912.S-CP.1.2—Understand that two events A and B are independent if the probability of A and B occurring together is the product of their probabilities, and use this characterization to determine if they are independent.</p> <p>MACC.912.S-CP.1.3—Understand the conditional probability of A given B as $\frac{P(A \text{ and } B)}{P(B)}$, and interpret independence of A and B as saying that the conditional probability of A given B is the same as the probability of A, and the conditional probability of B given A is the same as the probability of B.</p> <p>MACC.912.S-CP.1.4—Construct and interpret two-way frequency tables of data when two categories are associated with each object being classified. Use the two-way table as a sample space to decide if events are independent and to approximate conditional probabilities. <i>For example, collect data from a random sample of students in your school on their favorite subject among math, science, and English. Estimate the probability that a randomly selected student from your school will favor science given that the student is in tenth grade. Do the same for other subjects and compare the results.</i></p> <p>MACC.912.S-CP.1.5—Recognize and explain the concepts of conditional probability and independence in everyday language and everyday situations. <i>For example, compare the chance of having lung cancer if you are a smoker with the chance of being a smoker if you have lung cancer.</i></p>
Standards Clarifications/ Content Limits for Course	<p>As an explanation for some of the standards listed above, in Algebra 2, students will</p> <ul style="list-style-type: none"> • define and calculate conditional probabilities and use the multiplication principle to decide whether two events are independent • construct and interpret two-way frequency tables to approximate conditional probabilities and use probabilities from a frequency table to evaluate the independence of the two variables

Conceptual Category	STATISTICS AND PROBABILITY
Domain	Conditional Probability and the Rules of Probability
Cluster	Use the rules of probability to compute probabilities of compound events in a uniform probability model.
Standards	<p>MACC.912.S-CP.2.6—Find the conditional probability of A given B as the fraction of B’s outcomes that also belong to A, and interpret the answer in terms of the model.</p> <p>MACC.912.S-CP.2.7—Apply the Addition Rule, $P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$, and interpret the answer in terms of the model.</p> <p>+MACC.912.S-CP.2.8—Apply the general Multiplication Rule in a uniform probability model, $P(A \text{ and } B) = P(A)P(B A) = P(B)P(A B)$, and interpret the answer in terms of the model.</p> <p>+MACC.912.S-CP.2.9—Use permutations and combinations to compute probabilities of compound events and solve problems.</p>
Standards Clarifications/ Content Limits for Course	<p>As an explanation for some of the standards listed above, in Algebra 2, students will</p> <ul style="list-style-type: none"> • analyze a situation to determine the conditional probability of a described event given that another event occurs • identify two events as mutually exclusive and calculate probabilities using the Addition Rule • analyze a situation to determine the probability of a described event using the Multiplication Rule • use formulas containing factorial notation and analyze a situation to determine the probability of a described event using permutations or combinations in conjunction with other probability methods

Appendices

Appendix A

Sample Items

Item Type: Selected Response

Correct Answer: C

Possible Points: 1

DOK: 1

Calculator Usage: Allowed but Not Required

CCSS Standard:

MACC.912.N-CN.1.2—Use the relation $i^2 = -1$ and the commutative, associative, and distributive properties to add, subtract, and multiply complex numbers.

Standards for Mathematical Practice:

1. Make sense of problems and persevere in solving them.

What is the product of $(4 + 5i)$ and $(-7 - 3i)$?

- A. -13
- B. $-90i$
- * C. $-13 - 47i$
- D. $-28 - 15i$

Distractor Rationales

- A. This response is the result of incorrectly using the distributive property when multiplying two binomials; the real parts are multiplied correctly and then the imaginary parts are multiplied and simplified correctly to get $-28 + 15$.
- B. This response is the result of combining the real and imaginary part in each binomial, $(9i)(-10i)$, and then multiplying both to get $-90i$.
- C. Correct answer
- D. This response is the result of multiplying only the like terms in each expression instead of multiplying each term in the first expression by each term in the second expression and then incorrectly stating that $i \cdot i = i$ instead of i^2 .

Item Type: Gridded Response

Correct Answer: 4

Possible Points: 1

DOK: 2

Calculator Usage: Allowed but Not Required

CCSS Standard:

MACC.912.A-REI.1.2—Solve simple rational and radical equations in one variable, and give examples showing how extraneous solutions may arise.

Standards for Mathematical Practice:

7. Look for and make use of structure.

What is the value of x in the equation below?

$$\sqrt{2x + 17} = x + 1$$

0	0	0	0	0	0	0	0	0
1	1	1	1	1	1	1	1	1
2	2	2	2	2	2	2	2	2
3	3	3	3	3	3	3	3	3
4	4	4	4	4	4	4	4	4
5	5	5	5	5	5	5	5	5
6	6	6	6	6	6	6	6	6
7	7	7	7	7	7	7	7	7
8	8	8	8	8	8	8	8	8
9	9	9	9	9	9	9	9	9
.
/	/	/	/	/	/	/	/	/

Item Type: Short Answer

Correct Answer: $x = \log 5 - 2\log 3$

Possible Points: 1

DOK: 2

Calculator Usage: Allowed but Not Required

CCSS Standard:

MACC.912.F-LE.1.4—For exponential models, express as a logarithm the solution to $ab^ct = d$ where a , c , and d are numbers and the base b is 2, 10, or e ; evaluate the logarithm using technology.

Standards for Mathematical Practice:

7. Look for and make use of structure.

What is the exact solution, expressed as a logarithm in simplest form, of the exponential equation below?

$$10^x = \frac{5}{9}$$

Item Type: Constructed Response

Correct Answer: See Scoring Exemplar

Possible Points: 2

DOK: 2

Calculator Usage: Allowed but Not Required

CCSS Standard:

MACC.912.A-APR.3.4—Prove polynomial identities and use them to describe numerical relationships. *For example, the polynomial identity $(x^2 + y^2)^2 = (x^2 - y^2)^2 + (2xy)^2$ can be used to generate Pythagorean triples.*

Standards for Mathematical Practice:

6. Attend to precision.
7. Look for and make use of structure.

Given: $ax^2 + bx + c = 0$

Prove that $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ by completing the square and simplifying.

SCORING RUBRIC	
2	Work demonstrates a clear and complete understanding of the mathematical concepts and/or procedures required by the task. Appropriate strategy is shown with clear and complete explanations and interpretations.
1	Response demonstrates a partial understanding of the mathematical concepts and/or procedures. Appropriate strategy is shown, but explanation or interpretation has minor flaws. OR Response is incorrect because of calculation errors. Work and strategy indicate a clear understanding of the mathematical concepts and/or procedures required by the task.
0	Response is irrelevant, inappropriate, or not provided.

Maximum Points—2

A complete and correct proof that includes completing the square is shown.

$$\begin{aligned}ax^2 + bx + c &= 0 \\x^2 + \frac{bx}{a} + \frac{c}{a} &= 0 \\x^2 + \frac{b}{a}x &= -\frac{c}{a} \\x^2 + \left(\frac{b}{a}\right)x + \left(\frac{b}{2a}\right)^2 &= \left(\frac{b}{2a}\right)^2 - \frac{c}{a} \\ \left(x + \frac{b}{2a}\right)^2 &= \frac{b^2}{4a^2} - \frac{c}{a} \\ \left(x + \frac{b}{2a}\right)^2 &= \frac{b^2 - 4ac}{4a^2} \\x + \frac{b}{2a} &= \pm \sqrt{\frac{b^2 - 4ac}{4a^2}} \\x &= -\frac{b}{2a} \pm \frac{\sqrt{b^2 - 4ac}}{2a} \\x &= \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}\end{aligned}$$

Item Type: Extended Response

Correct Answer: See Scoring Exemplar

Possible Points: 4

DOK: 2

Calculator Usage: Allowed but Not Required

CCSS Standard:

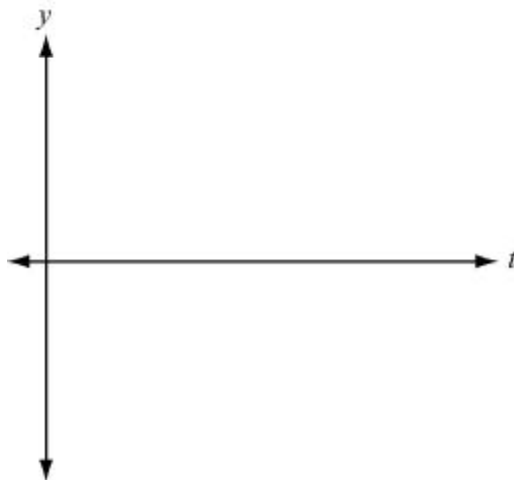
MACC.912.F-TF.2.5—Choose trigonometric functions to model periodic phenomena with specified amplitude, frequency, and midline.

Standards for Mathematical Practice:

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
4. Model with mathematics.

During a storm, a boat docked in a harbor moves up and down with the waves. The boat moves a total of 4 feet from its highest point to its lowest point. At its highest point, the boat is 21 feet above the bottom of the lake; it returns to this point every 6 seconds.

Part A. Sketch and label the graph that represents the boat's distance from the bottom of the lake, in y feet, at t seconds. The boat is at its highest point at $t = 0$.



Part B. Which trigonometric function, sine or cosine, best represents the movement of the boat shown in your graph? Explain your choice.

Part C. Use the trigonometric function selected in part B to write an equation that models the boat's distance from the bottom of the lake, in y feet at t seconds.

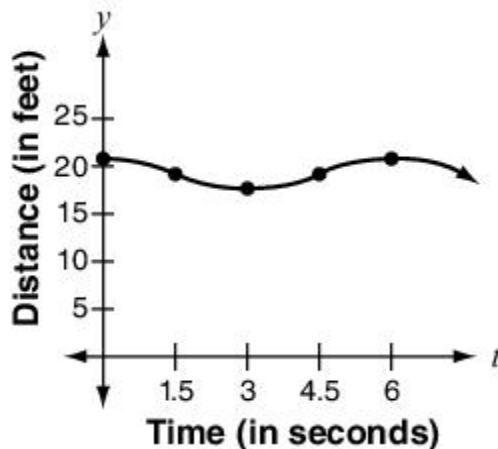
Part D. Use a phase shift to write an equation, using a different trigonometric function from the one used in part C, that also models the boat's distance from the bottom of the lake.

SCORING RUBRIC

4	Work demonstrates a clear and complete understanding of the mathematical concepts and/or procedures required by the task. Appropriate strategy is shown with clear and complete explanations and interpretations.
3	Work demonstrates a clear understanding of the mathematical concepts and/or procedures but is not complete. Appropriate strategy is shown, but explanation or interpretation has minor flaws. OR Response is incorrect because of calculation errors. Work and strategy indicate a clear demonstration of the problem.
2	Response demonstrates a partial understanding of the mathematical concepts and/or procedures. Appropriate strategy is shown, but explanation or interpretation has minor flaws.
1	Response shows minimal understanding of the mathematical concepts and/or procedures or provides no explanation or interpretation for the solution or shows major flaws.
0	Response is irrelevant, inappropriate, or not provided.

Maximum Points—4

Part A—1 point



Part B—1 point

- The graph of the trigonometric function has a cosine curve. A normal, positive cosine curve begins at the highest point on the y -axis (or at the beginning of the cycle), then dips down to the lowest point halfway through the cycle, and then comes back up to the high point to end the cycle. The curve then repeats the cycle over and over.

Other correct explanations are possible.

Part C—1 point

$$y = 2\cos\left(\frac{\pi t}{3}\right) + 19$$

Part D—1 point

$$y = 2\sin\left(\frac{\pi t}{3} + 1.5\right) + 19$$

Item Type: Performance Task

Correct Answer: See Scoring Exemplar

Possible Points: 10

DOK: 3

Calculator Usage: Allowed but Not Required

CCSS Standard:

MACC.912.S-IC.1.2—Decide if a specified model is consistent with results from a given data-generating process, e.g., using simulation. *For example, a model says a spinning coin falls heads up with probability 0.5. Would a result of 5 tails in a row cause you to question the model?*

Standards for Mathematical Practice:

2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
5. Use appropriate tools strategically.

To Guess or Not to Guess

Teacher Directions:

Before administration, discuss how to design simulations of random sampling. Students should have experience assigning digits in appropriate proportion for events and carrying out the simulation using random number generators. (You may need to include directions on how to use the random number generator on a graphing calculator if this has not yet been covered.)

Read the problem aloud and respond to any questions.

Instruct students to use words, numbers, pictures, and/or models to show their work.

Allow 60 minutes for this task.

This task can be modified by using another method for generating random numbers (spinners, tables, spreadsheets, etc.).

Make all necessary materials available.

Guide students and answer questions, but encourage independent thinking.

After the task, discuss answers as time permits.

Suggested Materials: Graphing calculator (other data-generating devices are acceptable if calculators cannot be provided; a spinner with four equal sections or random number tables can be substituted)

TASK:

Suppose that you GUESS the answers to 10 questions of a multiple-choice test. Each question has four answer choices, with one correct answer choice.

Part A. Describe a trial and a success for this situation. How many trials are there in one simulation? What is the probability of success on any single trial?

Part B. Design and conduct a simulation to determine the expected number of questions you will get correct on the test. Use the random number generator on a calculator and create a table to record your results. Run the simulation 10 times.

Part C. Explain how you set up your table and what the different aspects of your table represent in terms of the situation. (What do the rows and columns represent? What do the entries within your table represent? Which values are considered a “success” and a “failure”?)

Part D. Based on your simulation, how many out of the 10 test questions are you likely to guess correctly and how did you arrive at this conclusion? Are the results in your simulation consistent with what you would have expected? Use theoretical probability to explain why or why not. If the results are not what you expected, give an explanation for why you believe this happened.

Part E. How would you adapt your simulation

- a. if the test were true/false
- b. if the test had 5 answer choices
- c. if the test had 25 questions? Would you expect the results of a 25-question simulation to be more or less consistent with the theoretical probability than the results of a 10-question simulation? Why or why not?

Part F. Should these results be used to predict the outcome on a multiple-choice test for an individual who has studied for the test or has prior knowledge of the subject matter? Why or why not?

SCORING EXEMPLAR

Maximum Points—10

Part A—1 point

Each guess, or test question, is one trial. Each correct answer is a success. There will be 10 trials, and the probability of success on each trial is 0.25.

Part B—3 points

	Numbers Generated for each Trial										# of Correct Guesses
	Trial #										
Simulation	1	2	3	4	5	6	7	8	9	10	
1	4	4	1	3	2	3	1	2	4	1	3
2	4	4	1	2	1	4	1	1	3	4	4
3	4	2	2	1	1	3	1	2	2	4	3
4	1	4	3	1	4	3	2	1	4	4	3
5	1	1	3	4	3	2	1	4	1	2	4
6	1	1	4	2	3	3	1	1	1	3	5
7	3	4	2	1	2	4	3	3	3	4	1
8	3	4	3	2	1	3	4	1	2	2	2
9	2	4	4	2	2	1	3	3	4	4	1
10	2	4	3	2	2	4	4	2	3	2	0

*Note: Results and the format of the table may vary.

Part C—1 point

An exemplary student response might resemble the following:

Each row represents a different simulation of the 10-question test. The columns represent each of the 10 questions. Each number entry within the table represents the answer that was selected. The answer choices include 1, 2, 3, and 4 to represent each of the 4 different answer choices. Answer choice 1 represents the correct answer (success), while 2, 3, and 4 represent incorrect answer choices (failure). The final column represents how many questions, out of 10, were guessed correctly in each simulation.

Part D—1 point

Based on the information from the simulation, 3 out of the 10 questions would be answered correctly.

Mean = 2.6

$$\bar{x} = \frac{\text{\# of correct guesses}}{\text{\# of simulations}} = \frac{3+4+3+3+4+5+1+2+1+0}{10} = \frac{26}{10} = 2.6$$

Mode = 3

Median = 3

An appropriate strategy with a correct explanation and analysis is given. An exemplary student response might resemble the following:

The average number correct of the 10 simulations is 2.6, and the median and mode are both 3. Theoretically, 2.5 out of 10 questions would be answered correctly; therefore, these results are consistent with the expected value. If the simulation result was far from the expected result, more simulations should be performed.

Part E—3 points

- There would be only 2 answer choices instead of 4. You would need to use a random number generator but have only the options of 1 and 2. A coin could also be used to generate random numbers for this circumstance. The expected value would be close to 5.
- There would be 5 answer choices instead of 4. You would need to use a random number generator, with numbers 1–5 instead of 1–4. The expected number of correct answer choices would most likely decrease.
- You would have to generate 25 random numbers each simulation instead of 10 to account for 25 test questions.

An exemplary student response might resemble the following:

- The results of this simulation would more closely match the expected value (theoretical) because there are more trials. As the number of trials increases, the accuracy is also expected to increase.

Part F—1 point

An exemplary student response might resemble the following:

- These results should not be used for a student with prior knowledge of the subject. These results and simulation are based on the fact that the individual has the same odds of choosing the correct answer as each of the incorrect answers. If the individual taking the test has studied or already knows information about the questions, he or she is more likely to choose the correct answer. This simulation does not account for the knowledge level of the individual taking the test.

or equivalent work

Appendix B

Standards for Mathematical Practice

The Standards for Mathematical Practice describe varieties of expertise that mathematics educators at all levels should seek to develop in their students. These practices rest on important “processes and proficiencies” with longstanding importance in mathematics education. The first of these are the NCTM process standards of problem solving, reasoning and proof, communication, representation, and connections. The second are the strands of mathematical proficiency specified in the National Research Council’s report *Adding It Up*: adaptive reasoning, strategic competence, conceptual understanding (comprehension of mathematical concepts, operations and relations), procedural fluency (skill in carrying out procedures flexibly, accurately, efficiently and appropriately), and productive disposition (habitual inclination to see mathematics as sensible, useful, and worthwhile, coupled with a belief in diligence and one’s own efficacy).¹

MACC.K12.MP.1.1 Make sense of problems and persevere in solving them.

Mathematically proficient students start by explaining to themselves the meaning of a problem and looking for entry points to its solution. They analyze givens, constraints, relationships, and goals. They make conjectures about the form and meaning of the solution and plan a solution pathway rather than simply jumping into a solution attempt. They consider analogous problems, and try special cases and simpler forms of the original problem in order to gain insight into its solution. They monitor and evaluate their progress and change course if necessary. Older students might, depending on the context of the problem, transform algebraic expressions or change the viewing window on their graphing calculator to get the information they need. Mathematically proficient students can explain correspondences between equations, verbal descriptions, tables, and graphs or draw diagrams of important features and relationships, graph data, and search for regularity or trends. Younger students might rely on using concrete objects or pictures to help conceptualize and solve a problem. Mathematically proficient students check their answers to problems using a different method, and they continually ask themselves, “Does this make sense?” They can understand the approaches of others to solving complex problems and identify correspondences between different approaches.

MACC.K12.MP.2.1 Reason abstractly and quantitatively.

Mathematically proficient students make sense of quantities and their relationships in problem situations. They bring two complementary abilities to bear on problems involving quantitative relationships: the ability to *decontextualize*—to abstract a given situation and represent it symbolically and manipulate the representing symbols as if they have a life of their own, without necessarily attending to their referents—and the ability to *contextualize*, to pause as needed during the manipulation process in order to probe into the referents for the symbols involved. Quantitative reasoning entails habits of creating a coherent representation of the problem at hand; considering the units involved; attending to the meaning of quantities, not just how to compute them; and knowing and flexibly using different properties of operations and objects.

¹ Common Core State Standards Initiative (CCSSI), 2010, Common Core State Standards for Mathematics. Washington, DC: National Governors Association Center for Best Practices and the Council of Chief State School Officers.
http://www.corestandards.org/assets/CCSSI_Math%20Standards.pdf

MACC.K12.MP.3.1 Construct viable arguments and critique the reasoning of others.

Mathematically proficient students understand and use stated assumptions, definitions, and previously established results in constructing arguments. They make conjectures and build a logical progression of statements to explore the truth of their conjectures. They are able to analyze situations by breaking them into cases, and can recognize and use counterexamples. They justify their conclusions, communicate them to others, and respond to the arguments of others. They reason inductively about data, making plausible arguments that take into account the context from which the data arose. Mathematically proficient students are also able to compare the effectiveness of two plausible arguments, distinguish correct logic or reasoning from that which is flawed, and—if there is a flaw in an argument—explain what it is. Elementary students can construct arguments using concrete referents such as objects, drawings, diagrams, and actions. Such arguments can make sense and be correct, even though they are not generalized or made formal until later grades. Later, students learn to determine domains to which an argument applies. Students at all grades can listen or read the arguments of others, decide whether they make sense, and ask useful questions to clarify or improve the arguments.

MACC.K12.MP.4.1 Model with mathematics.

Mathematically proficient students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. In early grades, this might be as simple as writing an addition equation to describe a situation. In middle grades, a student might apply proportional reasoning to plan a school event or analyze a problem in the community. By high school, a student might use geometry to solve a design problem or use a function to describe how one quantity of interest depends on another. Mathematically proficient students who can apply what they know are comfortable making assumptions and approximations to simplify a complicated situation, realizing that these may need revision later. They are able to identify important quantities in a practical situation and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts and formulas. They can analyze those relationships mathematically to draw conclusions. They routinely interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.

MACC.K12.MP.5.1 Use appropriate tools strategically.

Mathematically proficient students consider the available tools when solving a mathematical problem. These tools might include pencil and paper, concrete models, a ruler, a protractor, a calculator, a spreadsheet, a computer algebra system, a statistical package, or dynamic geometry software. Proficient students are sufficiently familiar with tools appropriate for their grade or course to make sound decisions about when each of these tools might be helpful, recognizing both the insight to be gained and their limitations. For example, mathematically proficient high school students analyze graphs of functions and solutions generated using a graphing calculator. They detect possible errors by strategically using estimation and other mathematical knowledge. When making mathematical models, they know that technology can enable them to visualize the results of varying assumptions, explore consequences, and compare predictions with data. Mathematically proficient students at various grade levels are able to identify relevant external mathematical resources, such as digital content located on a website, and use them to pose or solve problems. They are able to use technological tools to explore and deepen their understanding of concepts.

MACC.K12.MP.6.1 Attend to precision.

Mathematically proficient students try to communicate precisely to others. They try to use clear definitions in discussion with others and in their own reasoning. They state the meaning of the symbols they choose, including using the equal sign consistently and appropriately. They are careful about specifying units of measure, and labeling axes to clarify the correspondence with quantities in a problem. They calculate accurately and efficiently, express numerical answers with a degree of precision appropriate for the problem context. In the elementary grades, students give carefully formulated explanations to each other. By the time they reach high school they have learned to examine claims and make explicit use of definitions.

MACC.K12.MP.7.1 Look for and make use of structure.

Mathematically proficient students look closely to discern a pattern or structure. Young students, for example, might notice that three and seven more is the same amount as seven and three more, or they may sort a collection of shapes according to how many sides the shapes have. Later, students will see 7×8 equals the well remembered $7 \times 5 + 7 \times 3$, in preparation for learning about the distributive property. In the expression $x^2 + 9x + 14$, older students can see the 14 as 2×7 and the 9 as $2 + 7$. They recognize the significance of an existing line in a geometric figure and can use the strategy of drawing an auxiliary line for solving problems. They also can step back for an overview and shift perspective. They can see complicated things, such as some algebraic expressions, as single objects or as being composed of several objects. For example, they can see $5 - 3(x - y)^2$ as 5 minus a positive number times a square and use that to realize that its value cannot be more than 5 for any real numbers x and y .

MACC.K12.MP.8.1 Look for and express regularity in repeated reasoning.

Mathematically proficient students notice if calculations are repeated, and look both for general methods and for shortcuts. Upper elementary students might notice when dividing 25 by 11 that they are repeating the same calculations over and over again, and conclude they have a repeating decimal. By paying attention to the calculation of slope as they repeatedly check whether points are on the line through (1, 2) with slope 3, middle school students might abstract the equation $(y - 2)/(x - 1) = 3$. Noticing the regularity in the way terms cancel when expanding $(x - 1)(x + 1)$, $(x - 1)(x^2 + x + 1)$, and $(x - 1)(x^3 + x^2 + x + 1)$ might lead them to the general formula for the sum of a geometric series. As they work to solve a problem, mathematically proficient students maintain oversight of the process, while attending to the details. They continually evaluate the reasonableness of their intermediate results.

Appendix C

Literacy Standards for Algebra 2

LACC.1112.RST.1.3—Key Ideas and Details

Follow precisely a complex multi-step procedure when carrying out experiments, taking measurements, or performing technical tasks; analyze the specific results based on explanations in the text.

LACC.1112.RST.2.4—Craft and Structure

Determine the meaning of symbols, key terms, and other domain-specific words and phrases as they are used in a specific scientific or technical context relevant to grades 11–12 texts and topics.

LACC.1112.RST.3.7—Integration of Knowledge and Ideas

Integrate and evaluate multiple sources of information presented in diverse formats and media (e.g., quantitative data, video, multimedia) in order to address a question or solve a problem.

LACC.1112.WHST.1.1—Text Types and Purposes

Write arguments focused on discipline-specific content.

- a. Introduce precise, knowledgeable claim(s), establish the significance of the claim(s), distinguish the claim(s) from alternate or opposing claims, and create an organization that logically sequences the claim(s), counterclaims, reasons, and evidence.
- b. Develop claim(s) and counterclaims fairly and thoroughly, supplying the most relevant data and evidence for each while pointing out the strengths and limitations of both claim(s) and counterclaims in a discipline-appropriate form that anticipates the audience’s knowledge level, concerns, values, and possible biases.
- c. Use words, phrases, and clauses as well as varied syntax to link the major sections of the text, create cohesion, and clarify the relationships between claim(s) and reasons, between reasons and evidence, and between claim(s) and counterclaims.
- d. Establish and maintain a formal style and objective tone while attending to the norms and conventions of the discipline in which they are writing. Provide a concluding statement or section that follows from or supports the argument presented.

LACC.1112.WHST.2.4—Production and Distribution of Writing

Produce clear and coherent writing in which the development, organization, and style are appropriate to task, purpose, and audience.

LACC.1112.WHST.3.9—Research to Build and Present Knowledge

Draw evidence from informational texts to support analysis, reflection, and research.