Florida Interim Assessment Item Bank and Test Platform

Item Specifications

Algebra 1
Grades 9–12

FLORIDA DEPARTMENT OF EDUCATION
www.fldoe.org

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I. Introduction

In July 2010 the Florida Department of Education (FDOE) approved the adoption of the Common Core State Standards (CCSS) for Mathematics to support its pursuit of improved outcomes for all Florida mathematics students and participation in national educational initiatives, such as Race to the Top. The U.S. Department of Education awarded a Race to the Top grant to Florida in August 2010. An important component of this grant focused on the development of high-quality assessment items and balanced assessments for use by districts, schools, and teachers. The assessment items will be stored in the Florida Interim Assessment Item Bank and Test Platform (IBTP), a statewide secure system that allows Florida educators to search the item bank, export test items, and generate customized high-quality assessments for computer-based delivery or paper-and-pencil delivery. The IBTP allows Florida educators to determine what students know and are able to do relative to instruction based on Florida's Next Generation Sunshine State Standards and the Common Core State Standards.

A. Purpose of the Item Specifications

The *Item Specifications* define the expectations for content, standards alignment, and format of assessment items for the Item Bank and Test Platform. The *Item Specifications* are intended for use by item writers and reviewers in the development of high-quality assessment items.

B. Scope

The *Item Specifications* provide general and grade-specific guidelines for the development of all Mathematics assessment items available in the Florida Interim Assessment Item Bank.

C. Standards Alignment

Items developed for the Florida Interim Assessment Item Bank and Test Platform will align to the Common Core State Standards for Mathematics. The Common Core State Standards for Mathematics are structured into three levels of specificity: Domains, Clusters, and Standards. These define what mathematics students should know and be able to do at every grade level/course, kindergarten through high school.

II. Criteria for Item Development

Mathematics item writers for the Florida Interim Assessment Item Bank must have a comprehensive knowledge of mathematics curriculum based on the Common Core State Standards and an understanding of the range of cognitive abilities of the target student population. Item writers should understand and consistently apply the guidelines established in this document. Item writers are expected to use their best judgment in writing items that measure the Mathematics standards of the CCSS without introducing extraneous elements that reflect bias for or against a group of students.

A. Overall Considerations for Item Development

These guidelines are provided to ensure the development of high-quality assessment items for the Florida Interim Assessment Item Bank.

1. Each item should be written to measure primarily one Common Core State Standard; however, other standards may also be addressed for some item types. In addition to the content standard alignment, each item should align to at least one Mathematical Practice Standard. Some items should be written reflecting the ELA Literacy standards cited in the course descriptions.
2. Items should be appropriate for students in terms of grade-level/course instruction, experience and difficulty, cognitive development, and reading level. The reading level of the test items should be on grade level.
3. Items should be written at or above the cognitive level (DOK) of the standard unless otherwise noted in the Individual Standard Specifications sections.
4. Each item should be written clearly and unambiguously to elicit the desired response.
5. Items should not disadvantage or exhibit disrespect to anyone in regard to age, gender, race, ethnicity, language, religion, socioeconomic status, disability, occupation, or geographic region.
6. At grades kindergarten through 5, items should be able to be answered without using a calculator. For grades 6 through 7, a four-function calculator may be used. For grade 8, a scientific calculator may be used. For Algebra 1, Geometry, and Algebra 2, both a scientific calculator and a graphing calculator (with functionalities similar to that of a TI-84) may be used. For all grades, calculators should not be used for items where computational skills or fluency are being assessed.

B. Item Contexts

The context in which an item is presented is called the item context or scenario. These guidelines are provided to assist item writers with development of items within an appropriate context.

1. The item context should be designed to interest students at the targeted level. Scenarios should be appropriate for students in terms of grade-level experience and difficulty, cognitive development, and reading level.
2. The context should be directly related to the question asked. The context should lead the student cognitively to the question. Every effort should be made to keep items as concise as possible without losing cognitive flow or missing the overall idea or concept.
3. Item contexts should include subject areas other than mathematics. Specifically, topics from grade-level/course Next Generation Sunshine State Standards for Science and Social Studies, and Common Core State Standards for English Language Arts may be used where appropriate.
4. Items including specific information or data must be accurate and verified against reliable sources. Source documentation must accompany these types of items.
5. Mathematics item stimuli should include written text and/or visual material, such as graphs, tables, diagrams, maps, models, and/or other illustrations.

6. All item scenarios, graphics, diagrams, and illustrations must be age-, grade-, and experience-appropriate.

7. All graphs used in item stems or answer options must be complete with title, scale, and labeled axes, except when these components are to be completed by the student.

8. Any graphics in items should be uncluttered and should clearly depict the necessary information. Graphics should contain relevant details that contribute to the students' understanding of the item or that support the context of the item. Graphics should not introduce bias to the item.

9. Item content should be timely but not likely to become dated too quickly.

C. Use of Media

Media can be used to provide either necessary or supplemental information—that is, some media contain information that is necessary for answering the question, while other media support the context of the question. Items may include diagrams, illustrations, charts, tables, audio files, or video files unless otherwise noted in the Individual Standard Specifications. Some standards require a heavier use of graphics than others. Geometry, for example, relies heavily on graphics to convey information.

1. Items should not begin with media. Media in items are always preceded by text.

2. All visual media (tables, charts, graphs, photographs, etc.) should be titled. Titles should be in all caps, boldfaced, and centered, and may be placed above or below the visual media.

D. Item Style and Format

This section presents stylistic guidelines and formatting directions that should be followed while developing items.

1. Items should be clear and concise, and they should use vocabulary and sentence structure appropriate for the assessed grade level.

2. The words *most likely* or *best* should be used only when appropriate to the question.

3. Items using the word *not* should emphasize the word *not* using all uppercase letters (e.g., Which of the following is NOT an example of . . .). The word *not* should be used sparingly.

4. For items that refer to an estimate (noun), lowercase letters should be used.

5. As appropriate, boldface type should be used to emphasize key words in the item (e.g., least, most, greatest, percent, mode, median, mean, range).

6. Masculine pronouns should NOT be used to refer to both sexes. Plural forms should be used whenever possible to avoid gender-specific pronouns (e.g., instead of "The student will make changes so that he . . . ," use "The students will make changes so that they . . .").

7. An equal balance of male and female names should be used, including names representing different ethnic groups appropriate for Florida.

8. For clarity, operation symbols, equality signs, and ordinates should be preceded and followed by one space.
9. Decimal numbers between –1 and 1 (including currency) should have a leading zero.
10. Metric numbers should be expressed in a single unit when possible (e.g., 1.4 kilograms instead of 1 kilogram 400 grams).
11. Decimal notation should be used for numbers with metric units (e.g., 1.2 grams instead of 151 grams).
12. Commas should be used within numbers greater than or equal to 1,000. Commas may be omitted within an equation or expression.
13. Units of measure should be spelled out, except in graphics, where an abbreviation may be used (e.g., *ft* or *yd*). Abbreviations that also spell a word must be followed by a period to avoid confusion. For example, to avoid confusion with the preposition *in*, the abbreviation *in.* should be used for the unit of measure *inches* and should include a period. If an abbreviation is used in a graphic, an explanation of the meaning of the abbreviation should be included in the stem.
14. In titles for tables and charts and in labels for axes, the units of measure should be included, preferably in lowercase letters and in parentheses, e.g., *height* (*in inches*).
15. Fractions should be typed with a horizontal fraction bar. The numerator and denominator should be centered with respect to each other. The bar should cover all portions (superscripts, parentheses, etc.) of the numerator and denominator. In a mixed number, a half space should appear between the whole number and the fraction. If a variable appears before or after a fraction bar, the variable should be centered with respect to the fraction bar. If a stimulus, stem, or set of responses contains a fraction in fractional notation, that portion of the item should be 1.5-spaced.
16. In general, numbers zero through nine should be presented as words and numbers 10 and above should be presented as numerals. In the item stem, any numbers needed to compute answers should be presented as numerals.

E. Item Types

This section presents guidelines for development of the following types of items:

1. Selected Response (SR) Items (1 point)
2. Gridded Response (GR) and Short Response (SHR) Items (1 point)
3. Constructed Response and Extended Response Items
   a. Constructed Response (CR) Items (2 points)
   b. Extended Response (ER) Items (4 points)
4. Essay Response (ESR) Items (6 points)
5. Performance Task (PT) Items (1–10 points)

1. **Selected Response (SR) Items (1 point)**

   Selected response items require students to choose an answer from the choices given. Each item consists of a stem and either three or four answer options, depending on the grade level/course (see c below). One of the answer options is the correct answer, and the remaining options are called distractors. Selected response items may include a stimulus and/or passage.

   a. SR items should take an average of 1 minute per item to solve.

   b. SR items are worth 1 point each.

   c. SR items in grades K, 1, and 2 should have three answer choices (A, B, and C). SR items for all other grades and courses should have four answer choices (A, B, C, and D).

   d. Answer options that are single words should be arranged in alphabetical or reverse alphabetical order.

   e. Answer options that are phrases or sentences should be arranged from shortest to longest or longest to shortest.

   f. Numerical answer options should be arranged in ascending or descending order.

   g. Numerical answer options that represent relative magnitude or size should be arranged as they are shown in the stem or some other logical order.

   h. When the item requires the identification of a choice from the item stem, table, chart, or illustration, the options should be arranged as they are presented in the item stem, table, chart, or illustration.

   i. If the answer options for an item are neither strictly numerical nor denominate numbers, the options should be arranged by the logic presented in the item, by alphabetical order, or by length.

   j. Distractor rationales should represent computational or conceptual errors or misconceptions commonly made by students who have not mastered the assessed concepts.

   k. Outliers (i.e., answer choices that are longer phrases or sentences than the other choices, or choices with significantly more/fewer digits than the other choices) should NOT be used.

   l. Options such as none of the above, all of the above, not here, not enough information, or cannot be determined should not be used as answer options.

2. **Gridded Response (GR) and Short Response (SHR) Items (1 point)**

    a. Gridded response and short response items are worth 1 point.

    b. The GR format is designed for items that require a positive numeric solution (whole numbers, decimals, percents, or fractions).

    c. The bubble grids used with GR items should contain eight columns. Each column will contain the digits 0 through 9, decimal point (.), and fraction bar (/) enclosed in bubbles.

    d. Gridded response items should include instructions that specify the unit in which the answer is to be provided (e.g., inches). If several units of measure are in the item (e.g., in an item involving a conversion), the final unit needed for the answer should be written in boldface.

    e. The short response format is designed for items that result in a value or answer that cannot be answered in the gridded response format (negative numbers, expressions, etc.).

3. **Constructed Response and Extended Response Items**

    Mathematics constructed response and extended response items require students to produce a response in words, pictures, diagrams, and/or numbers. As such, these items are especially suited to assessing many of the more complex tasks and high-level thinking skills demanded by the Common Core State Standards for Mathematics. The Florida Interim Assessment Item Bank will include 2-point constructed response items (CR) and 4-point extended response items (ER).

    Overall characteristics for mathematics CRs and ERs are as follows:

    a. The item should measure understanding and insight of mathematical concepts rather than rote memory or factual recall.

    b. Real-world, factual stimulus material (charts, graphs, tables, etc.) must cite the source used.

    c. Items requiring students to produce responses as pictures, diagrams, graphs, tables, etc., should provide workspace and/or templates where appropriate.

a. Constructed Response (CR) Items (2 points)

Constructed response items usually include a scenario and instructions on how to respond. The recommended time allotment for a student to respond is 5 minutes. A complete answer is worth 2 points, and a partial answer is worth 1 point. The constructed response holistic rubric and exemplar specific to each item are used for scoring as follows:

SCORING RUBRIC	
2	Work demonstrates a **clear and complete** understanding of the mathematical concepts and/or procedures required by the task. Appropriate strategy is shown with clear and complete explanations and interpretations.
1	Response demonstrates a **partial** understanding of the mathematical concepts and/or procedures. Appropriate strategy is shown, but explanation or interpretation has minor flaws. OR Response is incorrect because of calculation errors. Work and strategy indicate a **clear** understanding of the mathematical concepts and/or procedures required by the task.
0	Response is irrelevant, inappropriate, or not provided.

Exemplars: A specific exemplar should be developed for each constructed response item. Exemplars will be used as scoring guides and should be specific to the item, but not so specific as to discount multiple correct answers. Exemplars should include a clear and defensible description of the top score point, and contain straightforward language that is accurate, complete, and easy to interpret.

b. Extended Response (ER) Items (4 points)

Extended response items include a scenario and instructions on how to respond and are worth 4 points. However, ER items are usually more complex than SHR and 2-point CR items. The recommended time allotment for a student to respond is 10–15 minutes. The extended response holistic rubric and exemplar specific to each item are used for scoring as follows:

SCORING RUBRIC	
4	Work demonstrates a **clear and complete** understanding of the mathematical concepts and/or procedures required by the task. Appropriate strategy is shown with clear and complete explanations and interpretations.
3	Work demonstrates a **clear** understanding of the mathematical concepts and/or procedures but is not complete. Appropriate strategy is shown, but explanation or interpretation has minor flaws. OR Response is incorrect because of calculation errors. Work and strategy indicate a **clear** demonstration of the problem.
2	Response demonstrates a **partial** understanding of the mathematical concepts and/or procedures. Appropriate strategy is shown, but explanation or interpretation has minor flaws.
1	Response shows **minimal** understanding of the mathematical concepts and/or procedures or provides no explanation or interpretation for the solution or shows major flaws.
0	Response is irrelevant, inappropriate, or not provided.

Exemplars: A specific exemplar should be developed for each extended response item. Exemplars will be used as scoring guides and should be specific to the item, but not so specific as to discount multiple correct answers. Exemplars should include a clear and defensible description of the top score point, and contain straightforward language that is accurate, complete, and easy to interpret.

4. **Essay Response (ESR) Items (6 points)**

   The essay response item consists of asking a general question or providing a stimulus (such as an article or research paper on a relevant topic), and asking students to express their thoughts or provide facts about the topic using logic and reason. Essay response items encompass a higher level of thinking and a broader range of skills that includes CCSS literacy standards, which is critical to future success in higher education and the workforce.

   In most cases, essay responses will go beyond a single paragraph in length, with a distinct introduction, body, and conclusion. An essay response will be worth a total of 6 points, with a rubric structure similar to that of the 4-point extended response. Students should be given about 20 to 30 minutes to complete each item.

   Exemplars: A specific exemplar should be developed for each essay response item. Exemplars will be used as scoring guides and should be specific to the item, but not so specific as to discount multiple correct answers. Exemplars should include a clear and defensible description of the top score point, and contain straightforward language that is accurate, complete, and easy to interpret.

5. **Performance Tasks (PT) (1–10 points)**

   Performance tasks are used to measure students' ability to *demonstrate* knowledge and skills from one or more CCSS. Specifically, performance tasks may require students to create a product, demonstrate a process, or perform an activity that demonstrates proficiency in Mathematics. They are evaluated using customized scoring exemplars, and each task may be worth 1–10 points.

   Performance tasks may have the following characteristics:

   a. Performance tasks may cover a short time period or may cover an extended period.

   b. Performance tasks must contain clear and explicit directions for understanding and completing the required component tasks and producing the objective output.

   c. All tasks, skills, and/or behaviors required by the performance tasks must be objective, observable, and measurable.

   d. All necessary equipment, materials, and resources should be referenced within the text of the performance task.

   e. Performance tasks should elicit a range of score points.

   f. Performance tasks generally require students to organize, apply, analyze, synthesize, and/or evaluate concepts.

   g. Performance tasks may measure performance in authentic situations and outside the classroom, where appropriate and practical.

   h. Typical response formats include demonstrations, laboratory performance, oral presentations, exhibits, or other products.

i. Every performance task requires a companion exemplar to be used for scoring purposes. Exemplars should meet the following criteria.

i The exemplars and performance tasks should be developed in tandem to ensure compatibility.

ii Exemplars must be specific to the individual requirements of each performance task; generic rubrics are not acceptable.

iii The exemplar must allow for efficient and consistent scoring.

iv Each part of the performance task must have a clearly stated score point in the exemplar and when a part of the task is divided into sections or requirements, each of those must have a maximum score indicated.

v The exemplar descriptors consist of an ideal response exemplar and should allow for all foreseeable methods of correctly and thoroughly completing all requirements of the performance task.

F. Readability

Items must be written with readability in mind. In addition, vocabulary must be appropriate for the grade level being tested. The following sources provide information about the reading level of individual words:

Taylor, Stanford E. *EDL Core Vocabularies: Reading, Mathematics, Science, and Social Studies*. Austin, TX: Steck-Vaughn-EDL, 1989.

Mogilner, Alijandra. *Children's Writer's Word Book*. Cincinnati, OH: Writer's Digest Books, 1992.

G. Cognitive Complexity

1. Overview

Florida's adoption of the Common Core State Standards (CCSS) for Mathematics and English Language Arts & Literacy in History/Social Studies, Science, and Technical Subjects presents Florida with an opportunity to revise its current Depth of Knowledge (DOK) Model of Cognitive Complexity. More information about Florida's Depth of Knowledge levels is available online at http://www.cpalms.org/cpalms/dok.aspx.

2. Levels of Depth of Knowledge for Mathematics

Level 1 (Recall) includes the recall of information such as a fact, definition, term, or a simple procedure, as well as performing a simple algorithm or applying a formula. That is, in mathematics a one-step, well-defined, or straight algorithmic procedure should be included at this lowest level.

Some examples that represent but do not constitute all of Level 1 performance are:

- Count to 100 by ones and by tens.
- Fluently multiply and divide within 100, using strategies such as the relationship between multiplication and division (e.g., knowing that $8 \times 5 = 40$, one knows $40 \div 5 = 8$).

- Write simple expressions that record calculations with numbers, and interpret numerical expressions without evaluating them. For example, express the calculation "add 8 and 7 and then multiply by 2" as $2 \times (8 + 7)$. Recognize that $3 \times (18932 + 921)$ is three times as large as $18932 + 921$ without having to calculate the indicated sum or product.
- Enter measurement data into a data table.
- Identify the variables indicated in a two-dimensional graph.

Level 2 (Basic Application of Concepts & Skills) includes the engagement of some mental processing beyond a habitual response. A Level 2 standard or assessment item requires students to make some decisions as to how to approach the problem or activity, whereas Level 1 requires students to demonstrate a rote response, perform a well-known algorithm, follow a set procedure (like a recipe), or perform a clearly defined series of steps. For example, to compare data requires first identifying characteristics of the objects or phenomenon and then grouping or ordering the objects. Interpreting information from a simple graph, requiring reading information from the graph, also is a Level 2. Interpreting information from a complex graph that requires some decisions on what features of the graph need to be considered and how information from the graph can be aggregated is a Level 3. Caution is warranted in interpreting Level 2 as only skills because some reviewers will interpret skills very narrowly as primarily numerical skills, and such interpretation excludes from this level other skills such as visualization skills and probability skills, which may be more complex simply because they are less common and require more mental processing.

Some examples that represent but do not constitute all of Level 2 performance are:

- Understand that the last number name said tells the number of objects counted. The number of objects is the same regardless of their arrangement or the order in which they were counted.
- Express the length of an object as a whole number of length units, by laying multiple copies of a shorter object (the length unit) end to end.
- Measure and estimate liquid volumes and masses of objects using standard units of grams (g), kilograms (kg), and liters (l).
- Apply properties of operations as strategies to add and subtract rational numbers.
- Measure and record data and produce graphs of relevant variables.
- Graph proportional relationships, interpreting the unit rate as the slope of the graph.

Level 3 (Strategic Thinking & Complex Reasoning) requires reasoning, planning, using evidence, and a higher level of thinking than the previous two levels. In most instances, requiring students to explain their thinking is a Level 3. Activities that require students to make conjectures are also at this level. The cognitive demands at Level 3 are complex and abstract. The complexity does not result from the fact that there are multiple answers, a possibility for both levels 1 and 2, but because the task requires more demanding reasoning. However, an activity that has more than one possible answer and requires students to justify the response they give would most likely be a Level 3.

Some examples that represent but do not constitute all of Level 3 performance are:

- Explain why addition and subtraction strategies work, using place value and the properties of operations.
- Interpret the rate of change and initial value of a linear function in terms of the situation it models, and in terms of its graph or a table of values.
- Given a real-world situation, formulate a problem.
- Organize, represent, and interpret data obtained through experiments or observations.
- Formulate a mathematical model to describe a complex phenomenon.
- Justify a solution to a problem.
- Analyze a deductive argument.

Level 4 (Extended Thinking & Complex Reasoning) in mathematics involves the application of Level 3 processes and skills over an extended period. This is likely to incorporate demands from other content areas (e.g., English language arts, science) in the development and support of mathematical arguments that describe some real-world phenomenon or situation.

Some examples that represent but do not constitute all of Level 4 performance are:

- Derive a mathematical model to explain a complex phenomenon or make a prediction.
- Complete a project requiring the formulation of questions, devising a plan, collecting data, analyzing the data, and preparing a written report describing the justification of the conclusions reached.

H. Item Difficulty

Item writers will not be expected to make a prediction of difficulty for each item created. However, item writers should develop items that reflect a range of difficulty.

I. Universal Design

The application of universal design principles helps develop assessments that are usable to the greatest number of students, including students with disabilities and nonnative speakers of English. To support the goal of providing access to all students, the items in the Florida Interim Assessment Item Bank maximize readability, legibility, and compatibility with accommodations, and item development includes a review for potential bias and sensitivity issues.

Items must allow for the widest possible range of student participation. Item writers must attend to the best practices suggested by universal design, including, but not limited to,

1. reduction in wordiness
2. avoidance of ambiguity
3. selection of reader-friendly construction and terminology
4. consistently applied concept names and graphic conventions

Universal design principles also inform decisions about item layout and design, including, but not limited to, type size, line length, spacing, and graphics.

J. Sample Items

Appendix A of this document contains a selection of sample items. The sample items represent a range of cognitive complexities and item types.

III. Review Procedures for Florida Interim Assessment Item Bank Items

Prior to being included in the Florida Interim Assessment Item Bank, all mathematics items must pass several levels of review as part of the item development process.

A. Review for Item Quality

Assessment items developed for the Florida Interim Assessment Item Bank will be reviewed by Florida educators, the FDOE, and the contractors to ensure the quality of the items, including grade-level/course appropriateness, alignment to the standard, accuracy, and other criteria for overall item quality.

B. Review for Bias and Sensitivity

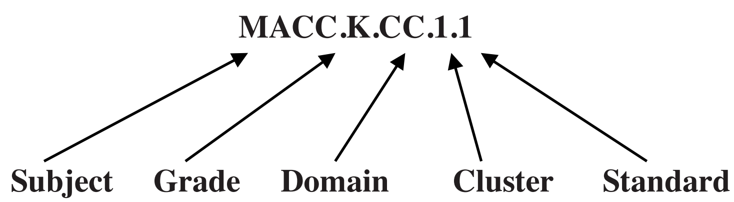
Items are reviewed by groups of Florida educators generally representative of Florida's geographic regions and culturally diverse population. Items are reviewed for the following kinds of bias: gender, racial, ethnic, linguistic, religious, geographic, and socioeconomic. Item reviews also include consideration of issues related to individuals with disabilities.

This review is to ensure that the primary purpose of assessing student achievement is not undermined by inadvertently including in the item bank any material that students, parents, or other stakeholders may deem inappropriate. Reviewers are asked to consider the variety of cultural, regional, philosophical, political, and religious backgrounds throughout Florida and to determine whether the subject matter will be acceptable to Florida students, their parents, and other members of Florida communities.

IV. Guide to the Individual Standard Specifications

A. CCSS Mathematics Standards Classification System

The graphic below demonstrates the coding schema for the Common Core State Standards for Mathematics.



Using this schema:

Subject Code MACC: Mathematics Common Core

Grade: Kindergarten

Domain CC: Counting and Cardinality

Cluster 1: Know number names and the count sequence.

Standard 1: Count to 100 by ones and by tens.

MACC.912.A-APR.1.1

Subject Grade Category (Unit) Domain Cluster Standard

Using the schema, the bottom row refers to:

Subject Code MACC: Mathematics Common Core

Grade: High school Grades 9–12

Category A: Algebra

Domain APR: Arithmetic with Polynomials and Rational Expressions

Cluster 1: Perform arithmetic operations on polynomials.

Standard 1: Understand that polynomials form a system analogous to the integers, namely, they are closed under the operations of addition, subtraction, and multiplication; add, subtract, and multiply polynomials.

B. Definitions of Cluster and Standard Specifications

The *Item Specifications* identify how the standards in the CCSS are assessed by items in the Florida Interim Assessment Item Bank. For each assessed standard, the following information is provided in the Individual Standards Specifications section.

Domain	refers to larger groups of related standards. Standards from different domains may sometimes be closely related.
Cluster	refers to groups of related standards. Note that standards from different clusters may sometimes be closely related because mathematics is a connected subject.
Standards	define what students should understand and be able to do.
Standards Clarifications/ Content Limits for Course	Standards clarifications, when needed as an explanation for some of the standards listed above, explain the type of behavior that the student should exhibit for mastery of the standard. The clarification statements explain what students are expected to do when responding to the question. Course limits define the range of content knowledge and degree of difficulty that should be assessed in the items for the standard. Course limits may be used to identify content beyond the scope of the targeted standard if the content is more appropriately assessed by another standard. These statements help to provide validity by ensuring the test items are clearly aligned to the targeted standard.

V. Individual Standards Specifications for Florida Interim Assessment Item Bank Mathematics Items

This section of the *Item Specifications* provides standard-specific guidance for assessment item development for the Florida Interim Assessment Item Bank based on the Common Core State Standards.

Each item developed for the Florida Interim Assessment Item Bank and Test Platform should assess one or more of the Mathematical Practice Standards listed in Appendix B.

A. Algebra 1 Item Specifications

Conceptual Category	NUMBER AND QUANTITY
Domain	**The Real Number System**
Cluster	**Extend the properties of exponents to rational exponents.**
Standards	MACC.912.N-RN.1.1—Explain how the definition of the meaning of rational exponents follows from extending the properties of integer exponents to those values, allowing for a notation for radicals in terms of rational exponents. *For example, we define* $5^{\frac{1}{3}}$ *to be the cube root of 5 because we want* $(5^{\frac{1}{3}})^3 = 5^{(\frac{1}{3})3}$ *to hold, so* $5^{(\frac{1}{3})3}$ *must equal 5.* MACC.912.N-RN.1.2—Rewrite expressions involving radicals and rational exponents using the properties of exponents.
Standards Clarifications/ Content Limits for Course	As an explanation for some of the standards listed above, in Algebra 1, students will • simplify and perform operations on expressions with rational exponents in radical or exponential form and explain how rational exponents can be defined by extending the laws of integer exponents, including sum law, product law, and distributive law • translate between radical and exponential notation

Conceptual Category	NUMBER AND QUANTITY
Domain	**The Real Number System**
Cluster	**Use properties of rational and irrational numbers.**
Standards	MACC.912.N-RN.2.3—Explain why the sum or product of two rational numbers is rational; that the sum of a rational number and an irrational number is irrational; and that the product of a nonzero rational number and an irrational number is irrational.

Standards Clarifications/ Content Limits for Course	As an explanation for some of the standards listed above, in Algebra 1, students will • explain why the four operations on rational numbers produce rational numbers • explain why the sum of a rational and an irrational number is irrational or why the product is irrational, which includes reasoning about the inverse relationship between addition and subtraction or between multiplication and addition • perform operations on combinations of both rational and irrational numbers

Conceptual Category	**NUMBER AND QUANTITY**
Domain	**Quantities**
Cluster	**Reason quantitatively and use units to solve problems.**
Standards	MACC.912.N-Q.1.1—Use units as a way to understand problems and to guide the solution of multi-step problems; choose and interpret units consistently in formulas; choose and interpret the scale and the origin in graphs and data displays. MACC.912.N-Q.1.2—Define appropriate quantities for the purpose of descriptive modeling. MACC.912.N-Q.1.3—Choose a level of accuracy appropriate to limitations on measurement when reporting quantities.
Standards Clarifications/ Content Limits for Course	As an explanation for some of the standards listed above, in Algebra 1, students will • select the appropriate units of measure to represent the context of the problem and use dimensional analysis to guide and check a method of solution • select the appropriate degree of accuracy or determine the level of precision needed within a mathematical model • select the most appropriate level of accuracy or precision for a given set of measurements after performing operations with the measurements • Graphics should be used for items as appropriate. Graphic representations and data displays include, but are not limited to, line graphs, circle graphs, histograms, multiline graphs, scatter plots, and multibar graphs. Course limit: Foundation for work with expressions, equations, and functions

Conceptual Category	ALGEBRA
Domain	**Seeing Structure in Expressions**
Cluster	**Interpret the structure of expressions.**
Standards	MACC.912.A-SSE.1.1—Interpret expressions that represent a quantity in terms of its context. MACC.912.A-SSE.1.1.a—Interpret parts of an expression, such as terms, factors, and coefficients. MACC.912.A-SSE.1.1.b—Interpret complicated expressions by viewing one or more of their parts as a single entity. *For example, interpret* $P(1 + r)^n$ *as the product of* P *and a factor not depending on* P. MACC.912.A-SSE.1.2—Use the structure of an expression to identify ways to rewrite it. *For example, see* $x^4 - y^4$ *as* $(x^2)^2 - (y^2)^2$, *thus recognizing it as a difference of squares that can be factored as* $(x^2 - y^2)(x^2 + y^2)$.
Standards Clarifications/ Content Limits for Course	As an explanation for some of the standards listed above, in Algebra 1, students will • for standard MACC.912.A-SSE.1.1, recognize equivalent forms of an expression as determined by its structure, interpret expressions or parts of expressions in the context of a problem, and make connections between symbolic representations and proper mathematics vocabulary • for standard MACC.912.A-SSE.1.2, use properties of mathematics to alter the structure of an expression and select and use an appropriate factoring technique Course limit: Linear, exponential, and quadratic

Conceptual Category	ALGEBRA
Domain	Seeing Structure in Expressions
Cluster	Write expressions in equivalent forms to solve problems.
Standards	MACC.912.A-SSE.2.3—Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression. MACC.912.A-SSE.2.3.a—Factor a quadratic expression to reveal the zeros of the function it defines. MACC.912.A-SSE.2.3.b—Complete the square in a quadratic expression to reveal the maximum or minimum value of the function it defines. MACC.912.A-SSE.2.3.c—Use the properties of exponents to transform expressions for exponential functions. *For example, the expression* 1.15^t *can be rewritten as* $(1.15^{\frac{1}{12}})^{12t} \approx 1.012^{12t}$ *to reveal the approximate equivalent monthly interest rate if the annual rate is 15%.*
Standards Clarifications/ Content Limits for Course	As an explanation for some of the standards listed above, in Algebra 1, students will • choose or produce an equivalent form of an expression, including factoring a quadratic expression, completing the square, and using properties of exponents • connect the factors, zeros, and *x*-intercepts of a graph • use the Zero-Product Property to solve quadratic equations • connect the form of the expression to a property of the quantity represented by the expression (for example, recognize key features of a quadratic model given in vertex form) Course limit: Quadratic and exponential

Conceptual Category	ALGEBRA
Domain	Arithmetic with Polynomials and Rational Expressions
Cluster	Perform arithmetic operations on polynomials.
Standards	MACC.912.A-APR.1.1—Understand that polynomials form a system analogous to the integers; namely, they are closed under the operations of addition, subtraction, and multiplication; add, subtract, and multiply polynomials.
Standards Clarifications/ Content Limits for Course	In Algebra 1, students will • simplify polynomials using addition, subtraction, and multiplication Course limit: Linear and quadratic

Conceptual Category	ALGEBRA
Domain	Arithmetic with Polynomials and Rational Expressions
Cluster	Understand the relationship between zeros and factors of polynomials.
Standards	MACC.912.A-APR.2.3—Identify zeros of polynomials when suitable factorizations are available, and use the zeros to construct a rough graph of the function defined by the polynomial.
Standards Clarifications/ Content Limits for Course	N/A

Conceptual Category	ALGEBRA
Domain	Creating Equations
Cluster	Create equations that describe numbers or relationships.
Standards	MACC.912.A-CED.1.1—Create equations and inequalities in one variable and use them to solve problems. Include equations arising from linear and quadratic functions, and simple rational and exponential functions. MACC.912.A-CED.1.2—Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales. MACC.912.A-CED.1.3—Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or nonviable options in a modeling context. *For example, represent inequalities describing nutritional and cost constraints on combinations of different foods.* MACC.912.A-CED.1.4—Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations. *For example, rearrange Ohm's law $V = IR$ to highlight resistance R.*

Standards Clarifications/ Content Limits for Course	As an explanation for some of the standards listed above, in Algebra 1, students will • for standard MACC.912.A-CED.1.1, create and graph equations and inequalities in one variable to solve problems; distinguish between linear, exponential, quadratic, radical, and simple rational relationships given multiple representations (verbal, numeric, or graphic) and then create the appropriate equation or inequality using the given information; and create linear, exponential, and quadratic equations in one variable, including the creation of simple rational functions • for standard MACC.912.A-CED.1.2, create and graph equations in two variables to represent relationships between quantities, which would include appropriate scales and labels to accurately display all aspects of the relation; distinguish between linear, exponential, quadratic, and simple rational relationships given multiple representations (numeric or verbal); and determine unknown parameters needed to create an equation that accurately models a given situation • for standard MACC.912.A-CED.1.4, recognize or create equivalent forms of literal equations and literal equations that are linear in the variables being solved for Course limit: Linear, quadratic, and exponential (integer inputs only)

Conceptual Category	**ALGEBRA**
Domain	**REASONING WITH EQUATIONS AND INEQUALITIES**
Cluster	**Understand solving equations as a process of reasoning and explain the reasoning**
Standards	MACC.912.A-REI.1.1—Explain each step in solving a simple equation as following from the equality of numbers asserted at the previous step, starting from the assumption that the original equation has a solution. Construct a viable argument to justify a solution method.
Standards Clarifications/ Content Limits for Course	As an explanation for some of the standards listed above, in Algebra 1, students will • identify the mathematical property (distributive property, equality properties, etc.) used at each step in the solution process as a means of justifying a step using linear and quadratic equations

Conceptual Category	ALGEBRA
Domain	Reasoning with Equations and Inequalities
Cluster	Solve equations and inequalities in one variable.
Standards	MACC.912.A-REI.2.3—Solve linear equations and inequalities in one variable, including equations with coefficients represented by letters. MACC.912.A-REI.2.4—Solve quadratic equations in one variable. MACC.912.A-REI.2.4.a—Use the method of completing the square to transform any quadratic equation in x into an equation of the form $(x - p)^2 = q$ that has the same solutions. Derive the quadratic formula from this form. MACC.912.A-REI.2.4.b—Solve quadratic equations by inspection (e.g., for $x^2 = 49$), taking square roots, completing the square, the quadratic formula, and factoring as appropriate to the initial form of the equation. Recognize when the quadratic formula gives complex solutions and write them as $a \pm bi$ for real numbers a and b.
Standards Clarifications/ Content Limits for Course	As an explanation for some of the standards listed above, in Algebra 1, students will • for standard MACC.912.A-REI.2.4, solve quadratic equations over the set of real or complex numbers using various methods and recognize the most efficient method as well as use the value of the discriminant to determine whether a quadratic equation has one double solution, two unique solutions, or no real solutions Course limit: Linear inequalities, literal equations that are linear in the variables being solved for, and quadratics with real solutions

Conceptual Category	ALGEBRA
Domain	Reasoning with Equations and Inequalities
Cluster	Solve systems of equations.
Standards	MACC.912.A-REI.3.5—Prove that, given a system of two equations in two variables, replacing one equation by the sum of that equation and a multiple of the other produces a system with the same solutions. MACC.912.A-REI.3.6—Solve systems of linear equations exactly and approximately (e.g., with graphs), focusing on pairs of linear equations in two variables.
Standards Clarifications/ Content Limits for Course	As an explanation for some of the standards listed above, in Algebra 1, students will • for standard MACC.912.A-REI.3.5, use the elimination method to show a step in the process of solving systems of equations algebraically and identify the mathematical property (distributive property, equality properties, etc.) used at each step in the solution process as a means of justifying a step • for standard MACC.912.A-REI.3.6, choose the most efficient method to solve systems of two equations Course limit: Linear-linear and linear-quadratic

Conceptual Category	ALGEBRA
Domain	**Reasoning with Equations and Inequalities**
Cluster	**Represent and solve equations and inequalities graphically.**
Standards	MACC.912.A-REI.4.10—Understand that the graph of an equation in two variables is the set of all its solutions plotted in the coordinate plane, often forming a curve (which could be a line). MACC.912.A-REI.4.11—Explain why the x-coordinates of the points where the graphs of the equations $y = f(x)$ and $y = g(x)$ intersect are the solutions of the equation $f(x) = g(x)$; find the solutions approximately, e.g., using technology to graph the functions, make tables of values, or find successive approximations. Include cases where $f(x)$ and/or $g(x)$ are linear, polynomial, rational, absolute value, exponential, and logarithmic functions. MACC.912.A-REI.4.12—Graph the solutions to a linear inequality in two variables as a half plane (excluding the boundary in the case of a strict inequality), and graph the solution set to a system of linear inequalities in two variables as the intersection of the corresponding half-planes.
Standards Clarifications/ Content Limits for Course	As an explanation for some of the standards listed above, in Algebra 1, students will • for standard MACC.912.A-REI.4.10, interpret a line or curve as a solution set of an equation in two variables, including points beyond the displayed portion of a graph, and construct an argument as to how the points that make up a line or curve are the algebraic representation of the function that is being represented by the graph • for standard MACC.912.A-REI.4.12, graph solutions to linear inequalities and systems of linear inequalities in two variables, including whether indicated point(s) or regions are part of a solution set, and explain the mathematics behind the dotted versus solid boundary lines used when graphing the solutions to linear inequalities Course limit: Focus on linear and exponential

Conceptual Category	FUNCTIONS
Domain	**Interpreting Functions**
Cluster	**Understand the concept of a function and use function notation.**
Standards	MACC.912.F-IF.1.1—Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If f is a function and x is an element of its domain, then $f(x)$ denotes the output of f corresponding to the input x. The graph of f is the graph of the equation $y = f(x)$. MACC.912.F-IF.1.2—Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context. MACC.912.F-IF.1.3—Recognize that sequences are functions, sometimes defined recursively, whose domain is a subset of the integers. *For example, the Fibonacci sequence is defined recursively by* $f(0) = f(1) = 1$, $f(n + 1) = f(n) + f(n - 1)$ *for* $n \geq 1$.
Standards Clarifications/ Content Limits for Course	As an explanation for some of the standards listed above, in Algebra 1, students will • use the definition of a function to determine whether a relationship is a function given a table, graph, or words and use function notation (when a relation is determined to be a function, use $f(x)$ notation) as well as interpret statements that use function notation in terms of the context in which they are used • know that the graph of the function f is the graph of the equation $y = f(x)$ and given the function $f(x)$, identify x as an element of the domain, the input, and $f(x)$ as an element of the range, the output • identify the domain and range of a function from multiple representations and evaluate functions for inputs in their domains and make connections between context and algebraic representations that use function notation • find a specific or general term in a sequence, define a recursive rule for a sequence and recognize that sequences, sometimes defined recursively, are functions with a domain that is a subset of the set of integers • emphasize arithmetic and geometric sequences as examples of linear and exponential functions

Conceptual Category	FUNCTIONS
Domain	Interpreting Functions
Cluster	Interpret functions that arise in applications in terms of the context.
Standards	MACC.912.F-IF.2.4—For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. *Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity*. MACC.912.F-IF.2.5—Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. *For example, if the function $h(n)$ gives the number of person-hours it takes to assemble n engines in a factory, then the positive integers would be an appropriate domain for the function.* MACC.912.F-IF.2.6—Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph.
Standards Clarifications/ Content Limits for Course	As an explanation for some of the standards listed above, in Algebra 1, students will • given the key features of a function, sketch the graph • translate from algebraic representations to graphic or numeric representations and identify key features using the various representations • describe the restrictions on the domain of all functions based on real-world contexts • identify the rate of change from multiple representations • interpret, in context, the average rate of change of a function over a specified interval • determine that the rate of change of a function can be positive, negative, or zero Course limit: Emphasize selection of appropriate models

Conceptual Category	FUNCTIONS
Domain	**Interpreting Functions**
Cluster	**Analyze functions using different representations**
Standards	MACC.912.F-IF.3.7—Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases. MACC.912.F-IF.3.7.a—Graph linear and quadratic functions and show intercepts, maxima, and minima. MACC.912.F-IF.3.7.b—Graph square root, cube root, and piecewise-defined functions, including step functions and absolute value functions. MACC.912.F-IF.3.8—Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function. MACC.912.F-IF.3.8.a—Use the process of factoring and completing the square in a quadratic function to show zeros, extreme values, and symmetry of the graph, and interpret these in terms of a context. MACC.912.F-IF.3.9—Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). *For example, given a graph of one quadratic function and an algebraic expression for another, say which has the larger maximum.*
Standards Clarifications/ Content Limits for Course	As an explanation for some of the standards listed above, in Algebra 1, students will • recognize common attributes of a function from various representations (algebraically, graphically, numerically in tables, or by verbal descriptions) • graph linear and quadratic functions and show intercepts, maxima, and minima and identify key features of a function: maxima, minima, intercepts, and zeros • graph square root, cube root, and a variety of piecewise-defined functions • graph the parent function for each type of function and for each type of function, understand how parameters introduced into a function alter the shape of the graph of the parent function • recognize functions in various forms and write a function in equivalent forms to show different properties of the function Course limit: Linear, exponential, quadratic, absolute value, step, and piecewise-defined

Conceptual Category	**FUNCTIONS**
Domain	**Building Functions**
Cluster	**Build a function that models a relationship between two quantities.**
Standards	MACC.912.F-BF.1.1—Write a function that describes a relationship between two quantities. MACC.912.F-BF.1.1.a—Determine an explicit expression, a recursive process, or steps for calculation from a context. MACC.912.F-BF.1.1.b—Combine standard function types using arithmetic operations. *For example, build a function that models the temperature of a cooling body by adding a constant function to a decaying exponential, and relate these functions to the model.*
Standards Clarifications/ Content Limits for Course	As an explanation for some of the standards listed above, in Algebra 1, students will • write an explicit rule for the nth term of a sequence with a_n as an expression in the term's position n or a recursive rule defining the first term of a sequence • write a recursive equation relating a_n to the preceding term(s); present a sequence that describes a_n as a function of n • add, subtract, multiply, and divide functions Course limit: Linear, exponential, and quadratic

Conceptual Category	FUNCTIONS
Domain	**Building Functions**
Cluster	**Build new functions from existing functions.**
Standards	MACC.912.F-BF.2.3—Identify the effect on the graph of replacing $f(x)$ by $f(x) + k$, $kf(x)$, $f(kx)$, and $f(x + k)$ for specific values of k (both positive and negative); find the value of k given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. *Include recognizing even and odd functions from their graphs and algebraic expressions for them.*
Standards Clarifications/ Content Limits for Course	As an explanation for some of the standards listed above, in Algebra 1, students will • compare and describe the shape and position of graphs and explain the differences • identify, through experimenting with technology, the effect on the graph of a function by replacing $f(x)$ with $f(x) + k$, $kf(x)$, $f(kx)$, and $f(x + k)$ for specific values of k (both positive and negative) • solve a linear function for the dependent variable and write the inverse of a function by interchanging the values of the dependent and independent variables • make generalizations about the changes that will take place in the graph of any function as a result of making a particular change to the algebraic representation of the function Course limit: Linear, exponential, quadratic, and absolute value

Conceptual Category	FUNCTIONS
Domain	**Linear, Quadratic, and Exponential Models**
Cluster	**Construct and compare linear, quadratic, and exponential models and solve problems.**
Standards	MACC.912.F-LE.1.1—Distinguish between situations that can be modeled with linear functions and with exponential functions. MACC.912.F-LE.1.1.a—Prove that linear functions grow by equal differences over equal intervals, and that exponential functions grow by equal factors over equal intervals. MACC.912.F-LE.1.1.b—Recognize situations in which one quantity changes at a constant rate per unit interval relative to another. MACC.912.F-LE.1.1.c—Recognize situations in which a quantity grows or decays by a constant percent rate per unit interval relative to another. MACC.912.F-LE.1.2—Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table). MACC.912.F-LE.1.3—Observe using graphs and tables that a quantity increasing exponentially eventually exceeds a quantity increasing linearly, quadratically, or (more generally) as a polynomial function.

Standards Clarifications/ Content Limits for Course	As an explanation for some of the standards listed above, in Algebra 1, students will • given a contextual situation, describe whether the situation in question has a linear pattern of change or an exponential pattern of change • describe situations where a quantity grows or decays at a constant percentage rate per unit interval as compared with another • show that linear functions change at the same rate over time and that exponential functions change by equal factors over time • describe situations where one quantity changes at a constant rate per unit interval as compared with another • make the connection, using graphs and tables, that a quantity increasing exponentially eventually exceeds a quantity increasing linearly, quadratically, or by any other polynomial function • investigate functions and graphs modeling different situations involving simple and compound interest • compare interest rates with different periods of compounding (monthly, daily) and compare them with the corresponding annual percentage rate

Conceptual Category	**FUNCTIONS**
Domain	**Linear, Quadratic, and Exponential Models**
Cluster	**Interpret expressions for functions in terms of the situation they model.**
Standards	MACC.912.F-LE.2.5—Interpret the parameters in a linear or exponential function in terms of a context.
Standards Clarifications/ Content Limits for Course	In Algebra 1, students will • model and interpret parameters in linear or exponential functions • interpret the slope and y-intercept of a linear model in terms of context • based on the context of a situation, explain the meaning of the coefficients, factors, exponents, and/or intercepts in a linear or exponential function • identify the initial amount present in an exponential model Course limit: Linear and exponential of form $f(x) = b^x + k$

Conceptual Category	STATISTICS AND PROBABILITY
Domain	Interpreting Categorical and Quantitative Data
Cluster	Summarize, represent, and interpret data on a single count or measurement variable.
Standards	MACC.912.S-ID.1.1—Represent data with plots on the real number line (dot plots, histograms, and box plots). MACC.912.S-ID.1.2—Use statistics appropriate to the shape of the data distribution to compare center (median, mean) and spread (interquartile range, standard deviation) of two or more different data sets. MACC.912.S-ID.1.3—Interpret differences in shape, center, and spread in the context of the data sets, accounting for possible effects of extreme data points (outliers). MACC.912.S-ID.1.4—Use the mean and standard deviation of a data set to fit it to a normal distribution and to estimate population percentages. Recognize that there are data sets for which such a procedure is not appropriate. Use calculators, spreadsheets, and tables to estimate areas under the normal curve.
Standards Clarifications/ Content Limits for Course	As an explanation for some of the standards listed above, in Algebra 1, students will • for standard MACC.912.S-ID.1.1, determine the best data representation to use for a given situation, correctly display given data in an appropriate plot, identify key features of plots, and analyze data given in different formats • for standard MACC.912.S-ID.1.2, describe a distribution using center and spread, use the correct measure of center and spread to describe a distribution that is symmetric or skewed, and compare two or more different data sets using the center and spread of each • for standard MACC.912.S-ID.1.3, recognize gaps, clusters, and trends in the data sets as well as extreme data points and their impact on center and effectively communicate what the data reveal in context • for standard MACC.912.S-ID.1.4, construct, interpret, and use normal curves based on mean and standard deviation, estimate and interpret area under curves using the Empirical Rule, calculators, spreadsheets, and/or tables, and identify data sets as approximately normal or not

Conceptual Category	STATISTICS AND PROBABILITY
Domain	Interpreting Categorical and Quantitative Data
Cluster	Summarize, represent, and interpret data on two categorical and quantitative variables.
Standards	MACC.912.S-ID.2.5—Summarize categorical data for two categories in two-way frequency tables. Interpret relative frequencies in the context of the data (including joint, marginal, and conditional relative frequencies). Recognize possible associations and trends in the data. MACC.912.S-ID.2.6—Represent data on two quantitative variables on a scatter plot, and describe how the variables are related. MACC.912.S-ID.2.6.a—Fit a function to the data; use functions fitted to data to solve problems in the context of the data. *Use given functions or choose a function suggested by the context. Emphasize linear, quadratic, and exponential models.* MACC.912.S-ID.2.6.b—Informally assess the fit of a function by plotting and analyzing residuals. MACC.912.S-ID.2.6.c—Fit a linear function for a scatter plot that suggests a linear association.
Standards Clarifications/ Content Limits for Course	As an explanation for some of the standards listed above, in Algebra 1, students will • for standard MACC.912.S-ID.2.5, create and interpret values from a two-way frequency table and compute joint, marginal, and conditional relative frequencies in context • for standard MACC.912.S-ID.2.6.a, recognize types of relationships that lend themselves to linear, quadratic, and exponential models and create and use regression models that represent a contextual situation using algebraic methods and/or technology • for standard MACC.912.S-ID.2.6.b, create a graphic display of residuals, recognize patterns in residual plots, and use residuals for the purpose of communicating the goodness of fit of a linear model for a given data set • for standard MACC.912.S-ID.2.6.c, recognize a linear relationship displayed in a scatter plot, determine an equation for the line of best fit for a set of data points using algebraic methods and/or technology, and use the function to predict values Course limit: Linear focus; discuss general principle

Conceptual Category	STATISTICS AND PROBABILITY
Domain	**Interpreting Categorical and Quantitative Data**
Cluster	**Interpret linear models.**
Standards	MACC.912.S-ID.3.7—Interpret the slope (rate of change) and the intercept (constant term) of a linear model in the context of the data. MACC.912.S-ID.3.8—Compute (using technology) and interpret the correlation coefficient of a linear fit. MACC.912.S-ID.3.9—Distinguish between correlation and causation.
Standards Clarifications/ Content Limits for Course	As an explanation for some of the standards listed above, in Algebra 1, students will • for standard MACC.912.S-ID.3.7, interpret the slope and intercepts of a linear model in the context of the data • for standard MACC.912.S-ID.3.8, know the range of the values for correlation coefficients and use a calculator or computer to find and analyze the correlation coefficient for the purpose of communicating a measure of the strength and direction of the linear relationship between two variables • for standard MACC.912.S-ID.3.9, identify and analyze examples of two variables that have a strong correlation but one does not cause the other

Appendices

Appendix A

Sample Items

Item Type: Selected Response

Correct Answer: C

Possible Points: 1

DOK: 1

Calculator Usage: Allowed but Not Required

CCSS Standard:

MACC.912.A-APR.1.1—Understand that polynomials form a system analogous to the integers; namely, they are closed under the operations of addition, subtraction, and multiplication; add, subtract, and multiply polynomials.

Standards for Mathematical Practice:

1. Make sense of problems and persevere in solving them.

Simplify the expression.

$$(3x^2 - 8x) + (-5x^2 + 3x) - (6x^2 - 4x)$$

A. $4x^2 - x$

B. $4x^2 - 9x$

* **C.** $-8x^2 - x$

D. $-8x^2 - 9x$

Distractor Rationales

A. The negative before $(6x^2 - 4x)$ was disregarded before the like terms were combined.

B. The negative before $(6x^2 - 4x)$ was distributed only to the second term before the like terms were combined.

C. Correct answer

D. The negative before $(6x^2 - 4x)$ was distributed only to the first term before the like terms were combined.

Item Type: Gridded Response

Correct Answer: 34.57

Possible Points: 1

DOK: 2

Calculator Usage: Required

CCSS Standard:

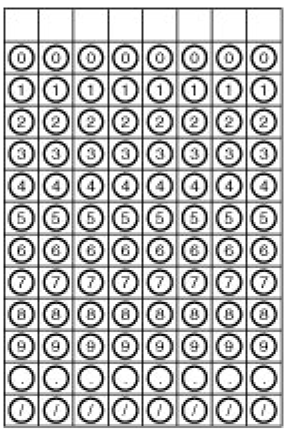
MACC.912.F-LE.1.1—Distinguish between situations that can be modeled with linear functions and with exponential functions.

> MACC.912.F-LE.1.1.c—Recognize situations in which a quantity grows or decays by a constant percent rate per unit interval relative to another.

Standards for Mathematical Practice:

1. Make sense of problems and persevere in solving them.

A patch of mold grows at a rate of 13% of its current size every 3 minutes. If the mold begins with a volume of 3 in.^{3}, how large, to the nearest hundredth of a cubic inch, will the mold be after 1 hour?



Item Type: Short Response

Correct Answer: $x = -7, -6$

Possible Points: 1

DOK: 1

Calculator Usage: Allowed but Not Required

CCSS Standard:

MACC.912.A-SSE.2.3—Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression.

MACC.912.A-SSE.2.3.a—Factor a quadratic expression to reveal the zeros of the function it defines.

Standards for Mathematical Practice:

7. Look for and make use of structure.

Factor this quadratic function.

$f(\text{x}) = x^2 + 13x + 42$

What are the zeros of the function?

____

Item Type: Constructed Response

Correct Answer: See Scoring Exemplar

Possible Points: 2

DOK: 2

Calculator Usage: Allowed but Not Required

CCSS Standard:

MACC.912.N-Q.1.2—Define appropriate quantities for the purpose of descriptive modeling.

Standards for Mathematical Practice:

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
6. Attend to precision.

A circular oil slick has a radius of 15 feet. Over a period of 11 seconds, enough oil is added to the oil slick to increase its radius to 18 feet. What is the average rate at which the area of the oil slick increases during this time? Use appropriate units of measure and explain why the units are correct.

SCORING RUBRIC	
2	Work demonstrates a **clear and complete** understanding of the mathematical concepts and/or procedures required by the task. Appropriate strategy is shown with clear and complete explanations and interpretations.
1	Response demonstrates a **partial** understanding of the mathematical concepts and/or procedures. Appropriate strategy is shown, but explanation or interpretation has minor flaws. OR Response is incorrect because of calculation errors. Work and strategy indicate a **clear** understanding of the mathematical concepts and/or procedures required by the task.
0	Response is irrelevant, inappropriate, or not provided.

SCORING EXEMPLAR
Maximum Points—2 **Correct rate—1 point** **Correct units/explanation—1 point** • The rate of change for the area of the oil slick would be $9\pi \frac{\text{ft}^2}{\text{sec}}$. The rate of change for the area of the oil slick would be measured in square feet per second because area is measured in square units. Other appropriate answers are acceptable.

Item Type: Extended Response

Correct Answer: See Scoring Exemplar

Possible Points: 4

DOK: 3

Calculator Usage: Required

CCSS Standard:

MACC.912.F-IF.1.3—Recognize that sequences are functions, sometimes defined recursively, whose domain is a subset of the integers. *For example, the Fibonacci sequence is defined recursively by* $f(0) = f(1) = 1$, $f(n+1) = f(n) + f(n-1)$ *for* $n \geq 1$.

Standards for Mathematical Practice:

1. Make sense of problems and persevere in solving them.

4. Model with mathematics.

7. Look for and make use of structure.

In the 62-year period from 1949 through 2010, the total consumption of primary energy in the United States increased by a factor of about 1.01896 each year. Energy consumption is measured in British thermal units (Btu). During the first year of this period, 1949, the primary energy consumption in the United States was approximately 31.98 trillion Btu.

Part A. Write a function, $f(n)$, to represent the U.S. primary energy consumption for the nth year in this sequence.

Part B. What restrictions should be placed upon the domain and why?

Part C. Determine the approximate U.S. primary energy consumption for 1997, to the nearest trillion Btu.

SCORING RUBRIC	
4	Work demonstrates a **clear and complete** understanding of the mathematical concepts and/or procedures required by the task. Appropriate strategy is shown with clear and complete explanations and interpretations.
3	Work demonstrates a **clear** understanding of the mathematical concepts and/or procedures but is not complete. Appropriate strategy is shown, but explanation or interpretation has minor flaws. OR Response is incorrect because of calculation errors. Work and strategy indicate a **clear** demonstration of the problem.
2	Response demonstrates a **partial** understanding of the mathematical concepts and/or procedures. Appropriate strategy is shown, but explanation or interpretation has minor flaws.
1	Response shows **minimal** understanding of the mathematical concepts and/or procedures or provides no explanation or interpretation for the solution or shows major flaws.
0	Response is irrelevant, inappropriate, or not provided.

SCORING EXEMPLAR

Maximum Points—4

Part A—1 point

- $f(n) = 31.98(1.01896)^{n-1}$

Part B—2 points

- The domain of n is should be restricted to include integers from 1 to 62. The measurement begins in 1949 ($n = 1$) and is measured for 62 years (including 2010). Because the measurement takes place once per year, only integers are used as inputs. This restriction can also be described as restricting to domain $\mathbb{Z}:[1, 62]$.

Part C—1 point

- 79 trillion Btu

  For 1997, $n = 49$.

  $f(49) = 31.98(1.01896)^{49-1}$

  $f(49) = 31.98(1.01896)^{48}$

  $f(49) = 79$

Other appropriate strategies are acceptable.

Item Type: Performance Task

Correct Answer: See Scoring Exemplar

Possible Points: 6

DOK: 3

Calculator Usage: Required

CCSS Standards:

MACC.912.N-Q.1.1—Use units as a way to understand problems and to guide the solution of multi-step problems; choose and interpret units consistently in formulas; choose and interpret the scale and the origin in graphs and data displays.

MACC.912.A-CED.1.2—Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.

Standards for Mathematical Practice:

1. Make sense of problems and persevere in solving them.

2. Reason abstractly and quantitatively.

4. Model with mathematics.

5. Use appropriate tools strategically.

6. Attend to precision.

Laying Sod

Teacher Directions:

Before administration, discuss how to convert units and how to represent a contextual situation graphically and algebraically.

Read the problem aloud.

Instruct students to use words, numbers, equations, pictures, and/or models to show their work.

Give students the appropriate conversion. Instruct students to use words, numbers, and/or pictures to show their work.

Allow 30 to 40 minutes for this task.

This task can be modified by using different units of measure.

Make all necessary materials available.

Look for and express regularity in repeated reasoning.

Guide students and answer questions, but encourage independent thinking.

After the task, discuss different approaches that may have been used to solve the various aspects of this problem (tables, graphs, algebra, guess and check, etc.). Discuss the pros, cons, and limitations, if any, of each approach.

Suggested Materials: Paper, graphing paper, pencils, conversion chart

TASK:

Paulina is landscaping her yard. She will cover $\frac{1}{2}$ acre with sod. Sod is sold in large pieces that can be resized as needed. Follow these steps to help Paulina compare the prices of two nurseries to determine which one has the better price for the total amount of sod she needs. One **acre** equals 4,840 square yards, or 43,560 square feet.

Part A. Nursery A sells pieces of sod that are 2 feet wide by 5 feet long for $2 per piece. Find the area, in square feet, of each piece of sod. How many pieces will Paulina need to buy to cover her entire yard?

Part B. Nursery B sells pieces of sod that are 16 inches wide by 81 inches long for 18¢ per square foot. Find the area, in square feet, of each piece of sod. How many pieces will Paulina need to buy to cover her entire yard?

Part C. Graph the relationship between the price and the area of sod for each nursery on a coordinate plane. Include axis labels and appropriate scale. Which nursery offers Paulina the better value and why? Explain.

Part D. If Paulina must buy sod in units of 20 pieces, how much sod will she have remaining from the total amount she would need to buy from each nursery?

SCORING EXEMPLAR

Maximum Points—6

Part A—1point

- A common measurement is found, and the calculations are correct.
  The response indicates that each piece of sod is 10 square feet and Paulina will need 2,178 pieces of sod, and appropriate work is shown.

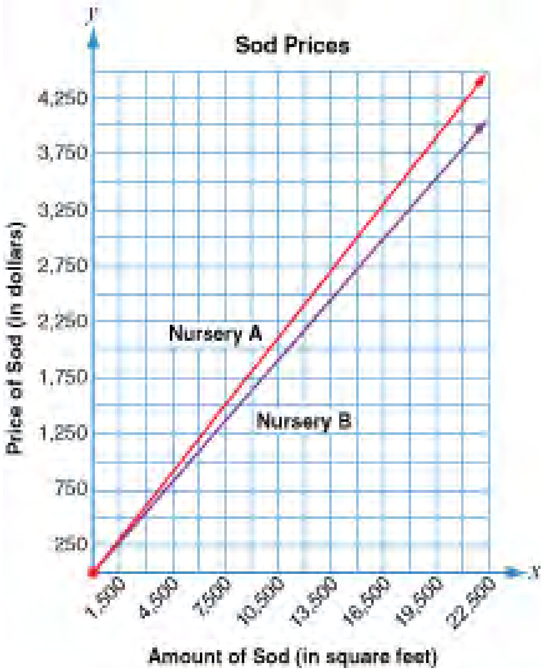
  An exemplary student response might include the following:
- Each piece of sod is 2(5) = 10 square feet. To determine the number of pieces, Paulina needs to convert from acres to square feet. Because $\frac{1}{2}$ acre = 21,780 square feet,Paulina will need 2,178 pieces of sod.

Part B—1 point

- A common measurement is found, and the calculations are correct.
  The response indicates that each piece of sod is 16 in. x 81 in. = 1,296 in.^{2} = $\frac{1{,}296}{144}$= 9 square feet and Paulina will need 2,420 pieces of sod, and appropriate work is shown.

Part C—2 points

- Student creates a graph similar to the one below that includes all appropriate labels, and an appropriate explanation is given.



- Nursery B offers a better value per square foot than nursery A because nursery B charges 18¢ and nursery A charges 20¢.

Part D—2 points

- The number of units and the square footage of sod remaining are determined for each nursery, and a correct analysis and/or justification is given.
- An exemplary student response might include the following:
  Nursery A: Paulina would need to buy 2,178 pieces, which is approximately 109 units. Because 109(20) = 2,180 and 2,180 − 2,178 = 2 pieces, she would have 2(10) = 20 ft^2 of sod remaining. Nursery B: Paulina would need to buy 2,420 pieces, which is 121 units. Because the total amount she must buy is divisible by 20, she would be able to use the entire amount of sod.

Other solutions are possible.

Appendix B

Standards for Mathematical Practice

The Standards for Mathematical Practice describe varieties of expertise that mathematics educators at all levels should seek to develop in their students. These practices rest on important "processes and proficiencies" with longstanding importance in mathematics education. The first of these are the NCTM process standards of problem solving, reasoning and proof, communication, representation, and connections. The second are the strands of mathematical proficiency specified in the National Research Council's report *Adding It Up*: adaptive reasoning, strategic competence, conceptual understanding (comprehension of mathematical concepts, operations and relations), procedural fluency (skill in carrying out procedures flexibly, accurately, efficiently and appropriately), and productive disposition (habitual inclination to see mathematics as sensible, useful, and worthwhile, coupled with a belief in diligence and one's own efficacy).[1]

MACC.K12.MP.1.1 Make sense of problems and persevere in solving them.

Mathematically proficient students start by explaining to themselves the meaning of a problem and looking for entry points to its solution. They analyze givens, constraints, relationships, and goals. They make conjectures about the form and meaning of the solution and plan a solution pathway rather than simply jumping into a solution attempt. They consider analogous problems, and try special cases and simpler forms of the original problem in order to gain insight into its solution. They monitor and evaluate their progress and change course if necessary. Older students might, depending on the context of the problem, transform algebraic expressions or change the viewing window on their graphing calculator to get the information they need. Mathematically proficient students can explain correspondences between equations, verbal descriptions, tables, and graphs or draw diagrams of important features and relationships, graph data, and search for regularity or trends. Younger students might rely on using concrete objects or pictures to help conceptualize and solve a problem. Mathematically proficient students check their answers to problems using a different method, and they continually ask themselves, "Does this make sense?" They can understand the approaches of others to solving complex problems and identify correspondences between different approaches.

MACC.K12.MP.2.1 Reason abstractly and quantitatively.

Mathematically proficient students make sense of quantities and their relationships in problem situations. They bring two complementary abilities to bear on problems involving quantitative relationships: the ability to *decontextualize*—to abstract a given situation and represent it symbolically and manipulate the representing symbols as if they have a life of their own, without necessarily attending to their referents—and the ability to *contextualize*, to pause as needed during the manipulation process in order to probe into the referents for the symbols involved. Quantitative reasoning entails habits of creating a coherent representation of the problem at hand; considering the units involved; attending to the meaning of quantities, not just how to compute them; and knowing and flexibly using different properties of operations and objects.

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[1] Common Core State Standards Initiative (CCSSI), 2010, Common Core State Standards for Mathematics. Washington, DC: National Governors Association Center for Best Practices and the Council of Chief State School Officers. http://www.corestandards.org/assets/CCSSI_Math%20Standards.pdf

MACC.K12.MP.3.1 Construct viable arguments and critique the reasoning of others.

Mathematically proficient students understand and use stated assumptions, definitions, and previously established results in constructing arguments. They make conjectures and build a logical progression of statements to explore the truth of their conjectures. They are able to analyze situations by breaking them into cases, and can recognize and use counterexamples. They justify their conclusions, communicate them to others, and respond to the arguments of others. They reason inductively about data, making plausible arguments that take into account the context from which the data arose. Mathematically proficient students are also able to compare the effectiveness of two plausible arguments, distinguish correct logic or reasoning from that which is flawed, and—if there is a flaw in an argument—explain what it is. Elementary students can construct arguments using concrete referents such as objects, drawings, diagrams, and actions. Such arguments can make sense and be correct, even though they are not generalized or made formal until later grades. Later, students learn to determine domains to which an argument applies. Students at all grades can listen or read the arguments of others, decide whether they make sense, and ask useful questions to clarify or improve the arguments.

MACC.K12.MP.4.1 Model with mathematics.

Mathematically proficient students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. In early grades, this might be as simple as writing an addition equation to describe a situation. In middle grades, a student might apply proportional reasoning to plan a school event or analyze a problem in the community. By high school, a student might use geometry to solve a design problem or use a function to describe how one quantity of interest depends on another. Mathematically proficient students who can apply what they know are comfortable making assumptions and approximations to simplify a complicated situation, realizing that these may need revision later. They are able to identify important quantities in a practical situation and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts and formulas. They can analyze those relationships mathematically to draw conclusions. They routinely interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.

MACC.K12.MP.5.1 Use appropriate tools strategically.

Mathematically proficient students consider the available tools when solving a mathematical problem. These tools might include pencil and paper, concrete models, a ruler, a protractor, a calculator, a spreadsheet, a computer algebra system, a statistical package, or dynamic geometry software. Proficient students are sufficiently familiar with tools appropriate for their grade or course to make sound decisions about when each of these tools might be helpful, recognizing both the insight to be gained and their limitations. For example, mathematically proficient high school students analyze graphs of functions and solutions generated using a graphing calculator. They detect possible errors by strategically using estimation and other mathematical knowledge. When making mathematical models, they know that technology can enable them to visualize the results of varying assumptions, explore consequences, and compare predictions with data. Mathematically proficient students at various grade levels are able to identify relevant external mathematical resources, such as digital content located on a website, and use them to pose or solve problems. They are able to use technological tools to explore and deepen their understanding of concepts.

MACC.K12.MP.6.1 Attend to precision.

Mathematically proficient students try to communicate precisely to others. They try to use clear definitions in discussion with others and in their own reasoning. They state the meaning of the symbols they choose, including using the equal sign consistently and appropriately. They are careful about specifying units of measure, and labeling axes to clarify the correspondence with quantities in a problem. They calculate accurately and efficiently, express numerical answers with a degree of precision appropriate for the problem context. In the elementary grades, students give carefully formulated explanations to each other. By the time they reach high school they have learned to examine claims and make explicit use of definitions.

MACC.K12.MP.7.1 Look for and make use of structure.

Mathematically proficient students look closely to discern a pattern or structure. Young students, for example, might notice that three and seven more is the same amount as seven and three more, or they may sort a collection of shapes according to how many sides the shapes have. Later, students will see 7×8 equals the well remembered $7 \times 5 + 7 \times 3$, in preparation for learning about the distributive property. In the expression $x^2 + 9x + 14$, older students can see the 14 as 2×7 and the 9 as $2 + 7$. They recognize the significance of an existing line in a geometric figure and can use the strategy of drawing an auxiliary line for solving problems. They also can step back for an overview and shift perspective. They can see complicated things, such as some algebraic expressions, as single objects or as being composed of several objects. For example, they can see $5 - 3(x - y)^2$ as 5 minus a positive number times a square and use that to realize that its value cannot be more than 5 for any real numbers x and y.

MACC.K12.MP.8.1 Look for and express regularity in repeated reasoning.

Mathematically proficient students notice if calculations are repeated, and look both for general methods and for shortcuts. Upper elementary students might notice when dividing 25 by 11 that they are repeating the same calculations over and over again, and conclude they have a repeating decimal. By paying attention to the calculation of slope as they repeatedly check whether points are on the line through (1, 2) with slope 3, middle school students might abstract the equation $(y - 2)/(x - 1) = 3$. Noticing the regularity in the way terms cancel when expanding $(x - 1)(x + 1)$, $(x - 1)(x^2 + x + 1)$, and $(x - 1)(x^3 + x^2 + x + 1)$ might lead them to the general formula for the sum of a geometric series. As they work to solve a problem, mathematically proficient students maintain oversight of the process, while attending to the details. They continually evaluate the reasonableness of their intermediate results.

Appendix C

Literacy Standards for Algebra 1

LACC.910.RST.1.3—Key Ideas and Details

Follow precisely a complex multi-step procedure when carrying out experiments, taking measurements, or performing technical tasks, attending to special cases or exceptions defined in the text.

LACC.910.RST.2.4—Craft and Structure

Determine the meaning of symbols, key terms, and other domain-specific words and phrases as they are used in a specific scientific or technical context relevant to grades 9–10 texts and topics.

LACC.910.RST.3.7—Integration of Knowledge and Ideas

Translate quantitative or technical information expressed in words in a text into visual form (e.g., a table or chart) and translate information expressed visually or mathematically (e.g., in an equation) into words.

LACC.910.WHST.1.1—Text Types and Purposes

Write arguments focused on discipline-specific content.

> LACC.910.WHST.1.1.a—Introduce precise claim(s), distinguish the claim(s) from alternate or opposing claims, and create an organization that establishes clear relationships among the claim(s), counterclaims, reasons, and evidence.
>
> LACC.910.WHST.1.1.b—Develop claim(s) and counterclaims fairly, supplying data and evidence for each while pointing out the strengths and limitations of both claim(s) and counterclaims in a discipline-appropriate form and in a manner that anticipates the audience's knowledge level and concerns.
>
> LACC.910.WHST.1.1.c—Use words, phrases, and clauses to link the major sections of the text, create cohesion, and clarify the relationships between claim(s) and reasons, between reasons and evidence, and between claim(s) and counterclaims.
>
> LACC.910.WHST.1.1.d—Establish and maintain a formal style and objective tone while attending to the norms and conventions of the discipline in which they are writing. Provide a concluding statement or section that follows from or supports the argument presented.

LACC.910.WHST.2.4—Production and Distribution of Writing

Produce clear and coherent writing in which the development, organization, and style are appropriate to task, purpose, and audience.

LACC.910.WHST.3.9—Research to Build and Present Knowledge

Draw evidence from informational texts to support analysis, reflection, and research.