Florida Interim Assessment Item Bank and Test Platform

Item Specifications

Mathematics
Grades 6–8

FLORIDA DEPARTMENT OF EDUCATION
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I. Introduction

In July 2010 the Florida Department of Education (FDOE) approved the adoption of the Common Core State Standards (CCSS) for Mathematics to support its pursuit of improved outcomes for all Florida mathematics students and participation in national educational initiatives, such as Race to the Top. The U.S. Department of Education awarded a Race to the Top grant to Florida in August 2010. An important component of this grant focused on the development of high-quality assessment items and balanced assessments for use by districts, schools, and teachers. The assessment items will be stored in the Florida Interim Assessment Item Bank and Test Platform (IBTP), a statewide secure system that allows Florida educators to search the item bank, export test items, and generate customized high-quality assessments for computer-based delivery or paper-and-pencil delivery. The IBTP allows Florida educators to determine what students know and are able to do relative to instruction based on Florida's Next Generation Sunshine State Standards and the Common Core State Standards.

A. Purpose of the Item Specifications

The *Item Specifications* define the expectations for content, standards alignment, and format of assessment items for the Item Bank and Test Platform. The *Item Specifications* are intended for use by item writers and reviewers in the development of high-quality assessment items.

B. Scope

The *Item Specifications* provide general and grade-specific guidelines for the development of all Mathematics assessment items available in the Florida Interim Assessment Item Bank.

C. Standards Alignment

Items developed for the Florida Interim Assessment Item Bank and Test Platform will align to the Common Core State Standards for Mathematics. The Common Core State Standards for Mathematics are structured into three levels of specificity: Domains, Clusters, and Standards. These define what mathematics students should know and be able to do at every grade level/course, kindergarten through high school.

II. Criteria for Item Development

Mathematics item writers for the Florida Interim Assessment Item Bank must have a comprehensive knowledge of mathematics curriculum based on the Common Core State Standards and an understanding of the range of cognitive abilities of the target student population. Item writers should understand and consistently apply the guidelines established in this document. Item writers are expected to use their best judgment in writing items that measure the Mathematics standards of the CCSS without introducing extraneous elements that reflect bias for or against a group of students.

A. Overall Considerations for Item Development

These guidelines are provided to ensure the development of high-quality assessment items for the Florida Interim Assessment Item Bank.

1. Each item should be written to measure primarily one Common Core State Standard; however, other standards may also be addressed for some item types. In addition to the content standard alignment, each item should align to at least one Mathematical Practice Standard.
2. Items should be appropriate for students in terms of grade-level/course instruction, experience and difficulty, cognitive development, and reading level. The reading level of the test items should be on grade level.
3. Items should be written at or above the cognitive level (DOK) of the standard unless otherwise noted in the Individual Standard Specifications sections.
4. Each item should be written clearly and unambiguously to elicit the desired response.
5. Items should not disadvantage or exhibit disrespect to anyone in regard to age, gender, race, ethnicity, language, religion, socioeconomic status, disability, occupation, or geographic region.
6. At grades kindergarten through 5, items should be able to be answered without using a calculator. For grades 6 through 7, a four-function calculator may be used. For grade 8, a scientific calculator may be used. For Algebra 1, Geometry, and Algebra 2, both a scientific calculator and a graphing calculator (with functionalities similar to that of a TI-84) may be used. For all grades, calculators should not be used for items where computational skills or fluency are being assessed.

B. Item Contexts

The context in which an item is presented is called the item context or scenario. These guidelines are provided to assist item writers with development of items within an appropriate context.

1. The item context should be designed to interest students at the targeted level. Scenarios should be appropriate for students in terms of grade-level experience and difficulty, cognitive development, and reading level.
2. The context should be directly related to the question asked. The context should lead the student cognitively to the question. Every effort should be made to keep items as concise as possible without losing cognitive flow or missing the overall idea or concept.
3. Item contexts should include subject areas other than mathematics. Specifically, topics from grade-level/course Next Generation Sunshine State Standards for Science and Social Studies, and Common Core State Standards for English Language Arts may be used where appropriate.
4. Items including specific information or data must be accurate and verified against reliable sources. Source documentation must accompany these types of items.
5. Mathematics item stimuli should include written text and/or visual material, such as graphs, tables, diagrams, maps, models, and/or other illustrations.
6. All item scenarios, graphics, diagrams, and illustrations must be age-, grade-, and experience-appropriate.

7. All graphs used in item stems or answer options must be complete with title, scale, and labeled axes, except when these components are to be completed by the student.
8. Any graphics in items should be uncluttered and should clearly depict the necessary information. Graphics should contain relevant details that contribute to the students' understanding of the item or that support the context of the item. Graphics should not introduce bias to the item.
9. Item content should be timely but not likely to become dated too quickly.

C. Use of Media

Media can be used to provide either necessary or supplemental information—that is, some media contain information that is necessary for answering the question, while other media support the context of the question. Items may include diagrams, illustrations, charts, tables, audio files, or video files unless otherwise noted in the Individual Standard Specifications. Some standards require a heavier use of graphics than others. Geometry, for example, relies heavily on graphics to convey information.

1. Items should not begin with media. Media in items are always preceded by text.
2. All visual media (tables, charts, graphs, photographs, etc.) should be titled. Titles should be in all caps, boldfaced, and centered, and may be placed above or below the visual media.

D. Item Style and Format

This section presents stylistic guidelines and formatting directions that should be followed while developing items.

1. Items should be clear and concise, and they should use vocabulary and sentence structure appropriate for the assessed grade level.
2. The words *most likely* or *best* should be used only when appropriate to the question.
3. Items using the word *not* should emphasize the word *not* using all uppercase letters (e.g., Which of the following is NOT an example of . . .). The word *not* should be used sparingly.
4. For items that refer to an estimate (noun), lowercase letters should be used.
5. As appropriate, boldface type should be used to emphasize key words in the item (e.g., least, most, greatest, percent, mode, median, mean, range).
6. Masculine pronouns should NOT be used to refer to both sexes. Plural forms should be used whenever possible to avoid gender-specific pronouns (e.g., instead of "The student will make changes so that he . . . ," use "The students will make changes so that they . . .").
7. An equal balance of male and female names should be used, including names representing different ethnic groups appropriate for Florida.
8. For clarity, operation symbols, equality signs, and ordinates should be preceded and followed by one space.

9. Decimal numbers between –1 and 1 (including currency) should have a leading zero.
10. Metric numbers should be expressed in a single unit when possible (e.g., 1.4 kilograms instead of 1 kilogram 400 grams).
11. Decimal notation should be used for numbers with metric units (e.g., 1.2 grams instead of 151 grams).
12. Commas should be used within numbers greater than or equal to 1,000. Commas may be omitted within an equation or expression.
13. Units of measure should be spelled out, except in graphics, where an abbreviation may be used (e.g., *ft* or *yd*). Abbreviations that also spell a word must be followed by a period to avoid confusion. For example, to avoid confusion with the preposition *in*, the abbreviation *in*. should be used for the unit of measure *inches* and should include a period. If an abbreviation is used in a graphic, an explanation of the meaning of the abbreviation should be included in the stem.
14. In titles for tables and charts and in labels for axes, the units of measure should be included, preferably in lowercase letters and in parentheses, e.g., *height* (*in inches*).
15. Fractions should be typed with a horizontal fraction bar. The numerator and denominator should be centered with respect to each other. The bar should cover all portions (superscripts, parentheses, etc.) of the numerator and denominator. In a mixed number, a half space should appear between the whole number and the fraction. If a variable appears before or after a fraction bar, the variable should be centered with respect to the fraction bar. If a stimulus, stem, or set of responses contains a fraction in fractional notation, that portion of the item should be 1.5-spaced.
16. In general, numbers zero through nine should be presented as words and numbers 10 and above should be presented as numerals. In the item stem, any numbers needed to compute answers should be presented as numerals.

E. Item Types

This section presents guidelines for development of the following types of items:

1. Selected Response (SR) Items (1 point)
2. Gridded Response (GR) and Short Response (SHR) Items (1 point)
3. Constructed Response and Extended Response Items
   a. Constructed Response (CR) Items (2 points)
   b. Extended Response (ER) Items (4 points)
4. Essay Response (ESR) Items (6 points)
5. Performance Task (PT) Items (1–10 points)

1. **Selected Response (SR) Items (1 point)**

   Selected response items require students to choose an answer from the choices given. Each item consists of a stem and either three or four answer options, depending on the grade level/course (see c below). One of the answer options is the correct answer, and the remaining options are called distractors. Selected response items may include a stimulus and/or passage.

   a. SR items should take an average of 1 minute per item to solve.

   b. SR items are worth 1 point each.

   c. SR items in grades K, 1, and 2 should have three answer choices (A, B, and C). SR items for all other grades and courses should have four answer choices (A, B, C, and D).

   d. Answer options that are single words should be arranged in alphabetical or reverse alphabetical order.

   e. Answer options that are phrases or sentences should be arranged from shortest to longest or longest to shortest.

   f. Numerical answer options should be arranged in ascending or descending order.

   g. Numerical answer options that represent relative magnitude or size should be arranged as they are shown in the stem or some other logical order.

   h. When the item requires the identification of a choice from the item stem, table, chart, or illustration, the options should be arranged as they are presented in the item stem, table, chart, or illustration.

   i. If the answer options for an item are neither strictly numerical nor denominate numbers, the options should be arranged by the logic presented in the item, by alphabetical order, or by length.

   j. Distractor rationales should represent computational or conceptual errors or misconceptions commonly made by students who have not mastered the assessed concepts.

   k. Outliers (i.e., answer choices that are longer phrases or sentences than the other choices, or choices with significantly more/fewer digits than the other choices) should NOT be used.

   l. Options such as none of the above, all of the above, not here, not enough information, or cannot be determined should not be used as answer options.

2. **Gridded Response (GR) and Short Response (SHR) Items (1 point)**

   a. Gridded response and short response items are worth 1 point.

   b. The GR format is designed for items that require a positive numeric solution (whole numbers, decimals, percents, or fractions).

   c. The bubble grids used with GR items should contain eight columns. Each column will contain the digits 0 through 9, decimal point (.), and fraction bar (/) enclosed in bubbles.

   d. Gridded response items should include instructions that specify the unit in which the answer is to be provided (e.g., inches). If several units of measure are in the item (e.g., in an item involving a conversion), the final unit needed for the answer should be written in boldface.

   e. The short response format is designed for items that result in a value or answer that cannot be answered in the gridded response format (negative numbers, expressions, etc.).

3. **Constructed Response and Extended Response Items**

   Mathematics constructed response and extended response items require students to produce a response in words, pictures, diagrams, and/or numbers. As such, these items are especially suited to assessing many of the more complex tasks and high-level thinking skills demanded by the Common Core State Standards for Mathematics. The Florida Interim Assessment Item Bank will include 2-point constructed response items (CR) and 4-point extended response items (ER).

   Overall characteristics for mathematics CRs and ERs are as follows:

   a. The item should measure understanding and insight of mathematical concepts rather than rote memory or factual recall.

   b. Real-world, factual stimulus material (charts, graphs, tables, etc.) must cite the source used.

   c. Items requiring students to produce responses as pictures, diagrams, graphs, tables, etc., should provide workspace and/or templates where appropriate.

a. Constructed Response (CR) Items (2 points)

Constructed response items usually include a scenario and instructions on how to respond. The recommended time allotment for a student to respond is 5 minutes. A complete answer is worth 2 points, and a partial answer is worth 1 point. The constructed response holistic rubric and exemplar specific to each item are used for scoring as follows:

SCORING RUBRIC	
2	Work demonstrates a **clear and complete** understanding of the mathematical concepts and/or procedures required by the task. Appropriate strategy is shown with clear and complete explanations and interpretations.
1	Response demonstrates a **partial** understanding of the mathematical concepts and/or procedures. Appropriate strategy is shown, but explanation or interpretation has minor flaws. OR Response is incorrect because of calculation errors. Work and strategy indicate a **clear** understanding of the mathematical concepts and/or procedures required by the task.
0	Response is irrelevant, inappropriate, or not provided.

Exemplars: A specific exemplar should be developed for each constructed response item. Exemplars will be used as scoring guides and should be specific to the item, but not so specific as to discount multiple correct answers. Exemplars should include a clear and defensible description of the top score point, and contain straightforward language that is accurate, complete, and easy to interpret.

b. Extended Response (ER) Items (4 points)

Extended response items include a scenario and instructions on how to respond and are worth 4 points. However, ER items are usually more complex than SHR and 2-point CR items. The recommended time allotment for a student to respond is 10–15 minutes. The extended response holistic rubric and exemplar specific to each item are used for scoring as follows:

SCORING RUBRIC	
4	Work demonstrates a **clear and complete** understanding of the mathematical concepts and/or procedures required by the task. Appropriate strategy is shown with clear and complete explanations and interpretations.
3	Work demonstrates a **clear** understanding of the mathematical concepts and/or procedures but is not complete. Appropriate strategy is shown, but explanation or interpretation has minor flaws. OR Response is incorrect because of calculation errors. Work and strategy indicate a **clear** demonstration of the problem.
2	Response demonstrates a **partial** understanding of the mathematical concepts and/or procedures. Appropriate strategy is shown, but explanation or interpretation has minor flaws.
1	Response shows **minimal** understanding of the mathematical concepts and/or procedures or provides no explanation or interpretation for the solution or shows major flaws.
0	Response is irrelevant, inappropriate, or not provided.

Exemplars: A specific exemplar should be developed for each extended response item. Exemplars will be used as scoring guides and should be specific to the item, but not so specific as to discount multiple correct answers. Exemplars should include a clear and defensible description of the top score point, and contain straightforward language that is accurate, complete, and easy to interpret.

4. **Essay Response (ESR) Items (6 points)**

   The essay response item consists of asking a general question or providing a stimulus (such as an article or research paper on a relevant topic), and asking students to express their thoughts or provide facts about the topic using logic and reason. Essay response items encompass a higher level of thinking and a broader range of skills that includes CCSS literacy standards, which is critical to future success in higher education and the workforce.

   In most cases, essay responses will go beyond a single paragraph in length, with a distinct introduction, body, and conclusion. An essay response will be worth a total of 6 points, with a rubric structure similar to that of the 4-point extended response. Students should be given about 20 to 30 minutes to complete each item.

   Exemplars: A specific exemplar should be developed for each essay response item. Exemplars will be used as scoring guides and should be specific to the item, but not so specific as to discount multiple correct answers. Exemplars should include a clear and defensible description of the top score point, and contain straightforward language that is accurate, complete, and easy to interpret.

5. **Performance Tasks (PT) (1–10 points)**

   Performance tasks are used to measure students' ability to *demonstrate* knowledge and skills from one or more CCSS. Specifically, performance tasks may require students to create a product, demonstrate a process, or perform an activity that demonstrates proficiency in Mathematics. They are evaluated using customized scoring exemplars, and each task may be worth 1–10 points.

   Performance tasks may have the following characteristics:

   a. Performance tasks may cover a short time period or may cover an extended period.
   b. Performance tasks must contain clear and explicit directions for understanding and completing the required component tasks and producing the objective output.
   c. All tasks, skills, and/or behaviors required by the performance tasks must be objective, observable, and measurable.
   d. All necessary equipment, materials, and resources should be referenced within the text of the performance task.
   e. Performance tasks should elicit a range of score points.
   f. Performance tasks generally require students to organize, apply, analyze, synthesize, and/or evaluate concepts.
   g. Performance tasks may measure performance in authentic situations and outside the classroom, where appropriate and practical.
   h. Typical response formats include demonstrations, laboratory performance, oral presentations, exhibits, or other products.

i. Every performance task requires a companion exemplar to be used for scoring purposes. Exemplars should meet the following criteria.

i The exemplars and performance tasks should be developed in tandem to ensure compatibility.

ii Exemplars must be specific to the individual requirements of each performance task; generic rubrics are not acceptable.

iii The exemplar must allow for efficient and consistent scoring.

iv Each part of the performance task must have a clearly stated score point in the exemplar and when a part of the task is divided into sections or requirements, each of those must have a maximum score indicated.

v The exemplar descriptors consist of an ideal response exemplar and should allow for all foreseeable methods of correctly and thoroughly completing all requirements of the performance task.

F. Readability

Items must be written with readability in mind. In addition, vocabulary must be appropriate for the grade level being tested. The following sources provide information about the reading level of individual words:

Taylor, Stanford E. *EDL Core Vocabularies: Reading, Mathematics, Science, and Social Studies*. Austin, TX: Steck-Vaughn-EDL, 1989.

Mogilner, Alijandra. *Children's Writer's Word Book*. Cincinnati, OH: Writer's Digest Books, 1992.

G. Cognitive Complexity

1. Overview

Florida's adoption of the Common Core State Standards (CCSS) for Mathematics and English Language Arts & Literacy in History/Social Studies, Science, and Technical Subjects presents Florida with an opportunity to revise its current Depth of Knowledge (DOK) Model of Cognitive Complexity. More information about Florida's Depth of Knowledge levels is available online at http://www.cpalms.org/cpalms/dok.aspx.

2. Levels of Depth of Knowledge for Mathematics

Level 1 (Recall) includes the recall of information such as a fact, definition, term, or a simple procedure, as well as performing a simple algorithm or applying a formula. That is, in mathematics a one-step, well-defined, or straight algorithmic procedure should be included at this lowest level.

Some examples that represent but do not constitute all of Level 1 performance are:

- Count to 100 by ones and by tens.
- Fluently multiply and divide within 100, using strategies such as the relationship between multiplication and division (e.g., knowing that $8 \times 5 = 40$, one knows $40 \div 5 = 8$).

- Write simple expressions that record calculations with numbers, and interpret numerical expressions without evaluating them. For example, express the calculation "add 8 and 7 and then multiply by 2" as 2 × (8 + 7). Recognize that 3 × (18932 + 921) is three times as large as 18932 + 921 without having to calculate the indicated sum or product.
- Enter measurement data into a data table.
- Identify the variables indicated in a two-dimensional graph.

Level 2 (Basic Application of Concepts & Skills) includes the engagement of some mental processing beyond a habitual response. A Level 2 standard or assessment item requires students to make some decisions as to how to approach the problem or activity, whereas Level 1 requires students to demonstrate a rote response, perform a well-known algorithm, follow a set procedure (like a recipe), or perform a clearly defined series of steps. For example, to compare data requires first identifying characteristics of the objects or phenomenon and then grouping or ordering the objects. Interpreting information from a simple graph, requiring reading information from the graph, also is a Level 2. Interpreting information from a complex graph that requires some decisions on what features of the graph need to be considered and how information from the graph can be aggregated is a Level 3. Caution is warranted in interpreting Level 2 as only skills because some reviewers will interpret skills very narrowly as primarily numerical skills, and such interpretation excludes from this level other skills such as visualization skills and probability skills, which may be more complex simply because they are less common and require more mental processing.

Some examples that represent but do not constitute all of Level 2 performance are:

- Understand that the last number name said tells the number of objects counted. The number of objects is the same regardless of their arrangement or the order in which they were counted.
- Express the length of an object as a whole number of length units by laying multiple copies of a shorter object (the length unit) end to end.
- Measure and estimate liquid volumes and masses of objects using standard units of grams (g), kilograms (kg), and liters (l).
- Apply properties of operations as strategies to add and subtract rational numbers.
- Measure and record data and produce graphs of relevant variables.
- Graph proportional relationships, interpreting the unit rate as the slope of the graph.

Level 3 (Strategic Thinking & Complex Reasoning) requires reasoning, planning, using evidence, and a higher level of thinking than the previous two levels. In most instances, requiring students to explain their thinking is a Level 3. Activities that require students to make conjectures are also at this level. The cognitive demands at Level 3 are complex and abstract. The complexity does not result from the fact that there are multiple answers, a possibility for

both levels 1 and 2, but because the task requires more demanding reasoning. However, an activity that has more than one possible answer and requires students to justify the response they give would most likely be a Level 3.

Some examples that represent but do not constitute all of Level 3 performance are:

- Explain why addition and subtraction strategies work, using place value and the properties of operations.
- Interpret the rate of change and initial value of a linear function in terms of the situation it models, and in terms of its graph or a table of values.
- Given a real-world situation, formulate a problem.
- Organize, represent, and interpret data obtained through experiments or observations.
- Formulate a mathematical model to describe a complex phenomenon.
- Justify a solution to a problem.
- Analyze a deductive argument.

Level 4 (Extended Thinking & Complex Reasoning) in mathematics involves the application of Level 3 processes and skills over an extended period. This is likely to incorporate demands from other content areas (e.g., English language arts, science) in the development and support of mathematical arguments that describe some real-world phenomenon or situation.

Some examples that represent but do not constitute all of Level 4 performance are:

- Derive a mathematical model to explain a complex phenomenon or make a prediction.
- Complete a project requiring the formulation of questions, devising a plan, collecting data, analyzing the data, and preparing a written report describing the justification of the conclusions reached.

H. Item Difficulty

Item writers will not be expected to make a prediction of difficulty for each item created. However, item writers should develop items that reflect a range of difficulty.

I. Universal Design

The application of universal design principles helps develop assessments that are usable to the greatest number of students, including students with disabilities and nonnative speakers of English. To support the goal of providing access to all students, the items in the Florida Interim Assessment Item Bank maximize readability, legibility, and compatibility with accommodations, and item development includes a review for potential bias and sensitivity issues.

Items must allow for the widest possible range of student participation. Item writers must attend to the best practices suggested by universal design, including, but not limited to,

1. reduction in wordiness
2. avoidance of ambiguity
3. selection of reader-friendly construction and terminology
4. consistently applied concept names and graphic conventions

Universal design principles also inform decisions about item layout and design, including, but not limited to, type size, line length, spacing, and graphics.

J. Sample Items

Appendix A of this document contains a selection of sample items. The sample items represent a range of cognitive complexities and item types.

III. Review Procedures for Florida Interim Assessment Item Bank Items

Prior to being included in the Florida Interim Assessment Item Bank, all mathematics items must pass several levels of review as part of the item development process.

A. Review for Item Quality

Assessment items developed for the Florida Interim Assessment Item Bank will be reviewed by Florida educators, the FDOE, and the contractors to ensure the quality of the items, including grade-level/course appropriateness, alignment to the standard, accuracy, and other criteria for overall item quality.

B. Review for Bias and Sensitivity

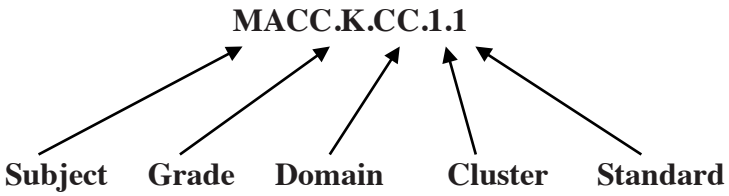
Items are reviewed by groups of Florida educators generally representative of Florida's geographic regions and culturally diverse population. Items are reviewed for the following kinds of bias: gender, racial, ethnic, linguistic, religious, geographic, and socioeconomic. Item reviews also include consideration of issues related to individuals with disabilities.

This review is to ensure that the primary purpose of assessing student achievement is not undermined by inadvertently including in the item bank any material that students, parents, or other stakeholders may deem inappropriate. Reviewers are asked to consider the variety of cultural, regional, philosophical, political, and religious backgrounds throughout Florida and to determine whether the subject matter will be acceptable to Florida students, their parents, and other members of Florida communities.

IV. Guide to the Individual Standard Specifications

A. CCSS Mathematics Standards Classification System

The graphic below demonstrates the coding schema for the Common Core State Standards for Mathematics.



Using this schema:

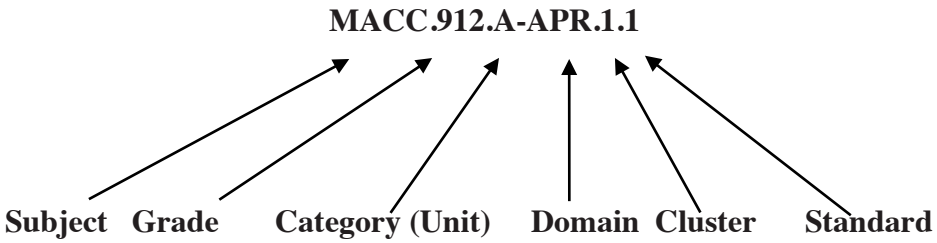
Subject Code MACC: Mathematics Common Core

Grade: Kindergarten

Domain CC: Counting and Cardinality

Cluster 1: Know number names and the count sequence.

Standard 1: Count to 100 by ones and by tens.



Using the schema, the bottom row refers to:

Subject Code MACC: Mathematics Common Core

Grade: High school Grades 9–12

Category A: Algebra

Domain APR: Arithmetic with Polynomials and Rational Expressions

Cluster 1: Perform arithmetic operations on polynomials.

Standard 1: Understand that polynomials form a system analogous to the integers, namely, they are closed under the operations of addition, subtraction, and multiplication; add, subtract, and multiply polynomials.

B. Definitions of Cluster and Standard Specifications

The *Item Specifications* identify how the standards in the CCSS are assessed by items in the Florida Interim Assessment Item Bank. For each assessed standard, the following information is provided in the Individual Standards Specifications section.

Domain	refers to larger groups of related standards. Standards from different domains may sometimes be closely related.
Cluster	refers to groups of related standards. Note that standards from different clusters may sometimes be closely related because mathematics is a connected subject.
Standards	define what students should understand and be able to do.
Standards Clarifications/ Content Limits	Standards clarifications, when needed as an explanation for some of the standards listed above, explain the type of behavior that the student should exhibit for mastery of the standard. The clarification statements explain what students are expected to do when responding to the question. Content limits define the range of content knowledge and degree of difficulty that should be assessed in the items for the standard. Content limits may be used to identify content beyond the scope of the targeted standard if the content is more appropriately assessed by another standard. These statements help to provide validity by ensuring the test items are clearly aligned to the targeted standard.

V. Individual Standards Specifications for Florida Interim Assessment Item Bank Mathematics Items

This section of the *Item Specifications* provides standard-specific guidance for assessment item development for the Florida Interim Assessment Item Bank based on the Common Core State Standards.

Each item developed for the Florida Interim Assessment Item Bank and Test Platform should assess one or more of the Mathematical Practice Standards listed in Appendix B.

A. Grade 6 Item Specifications

Domain	**RATIOS AND PROPORTIONAL RELATIONSHIPS**
Cluster	**Understand ratio concepts and use ratio reasoning to solve problems.**
Standards	MACC.6.RP.1.1—Understand the concept of a ratio and use ratio language to describe a ratio relationship between two quantities. *For example, "The ratio of wings to beaks in the bird house at the zoo was 2:1, because for every 2 wings there was 1 beak." "For every vote candidate A received, candidate C received nearly three votes."* MACC.6.RP.1.2—Understand the concept of a unit rate $\frac{a}{b}$ associated with a ratio a:b with $b \neq 0$, and use rate language in the context of a ratio relationship. *For example, "This recipe has a ratio of 3 cups of flour to 4 cups of sugar, so there is $\frac{3}{4}$ cup of flour for each cup of sugar." "We paid $75 for 15 hamburgers, which is a rate of $5 per hamburger."* MACC.6.RP.1.3—Use ratio and rate reasoning to solve real-world and mathematical problems, e.g., by reasoning about tables of equivalent ratios, tape diagrams, double number line diagrams, or equations. MACC.6.RP.1.3.a—Make tables of equivalent ratios relating quantities with whole-number measurements, find missing values in the tables, and plot the pairs of values on the coordinate plane. Use tables to compare ratios. MACC.6.RP.1.3.b—Solve unit rate problems including those involving unit pricing and constant speed. *For example, if it took 7 hours to mow 4 lawns, then at that rate, how many lawns could be mowed in 35 hours? At what rate were lawns being mowed?* MACC.6.RP.1.3.c—Find a percent of a quantity as a rate per 100 (e.g., 30% of a quantity means $\frac{30}{100}$ times the quantity); solve problems involving finding the whole, given a part and the percent. MACC.6.RP.1.3.d—Use ratio reasoning to convert measurement units; manipulate and transform units appropriately when multiplying or dividing quantities.

Standards Clarifications/ Content Limits	As an explanation for some of the standards listed above, in Grade 6, students will • understand and write ratios that represent a particular situation • understand equivalent ratios, find missing values in tables, express percents as ratio quantities out of 100, and determine unit rates by simplifying ratio quantities • apply properties of ratios to solve real-world problems • work with unit pricing and constant speed

Domain	**THE NUMBER SYSTEM**
Cluster	**Apply and extend previous understandings of multiplication and division to divide fractions by fractions.**
Standards	MACC.6.NS.1.1—Interpret and compute quotients of fractions, and solve word problems involving division of fractions by fractions, e.g., by using visual fraction models and equations to represent the problem. *For example, create a story context for* $(\frac{2}{3}) \div (\frac{3}{4})$ *and use a visual fraction model to show the quotient; use the relationship between multiplication and division to explain that* $(\frac{2}{3}) \div (\frac{3}{4}) = \frac{8}{9}$ *because* $\frac{3}{4}$ *of* $\frac{8}{9}$ *is* $\frac{2}{3}$*. (In general,* $(\frac{a}{b}) \div (\frac{c}{d}) = \frac{ad}{bc}$*.) How much chocolate will each person get if 3 people share* $\frac{1}{2}$ *lb of chocolate equally? How many* $\frac{3}{4}$*-cup servings are in* $\frac{2}{3}$ *of a cup of yogurt? How wide is a rectangular strip of land with length* $\frac{3}{4}$ *mi and area* $\frac{1}{2}$ *square mi?*
Standards Clarifications/ Content Limits	• N/A

Domain	THE NUMBER SYSTEM
Cluster	**Compute fluently with multi-digit numbers and find common factors and multiples.**
Standards	MACC.6.NS.2.2—Fluently divide multi-digit numbers using the standard algorithm. MACC.6.NS.2.3—Fluently add, subtract, multiply, and divide multi-digit decimals using the standard algorithm for each operation. MACC.6.NS.2.4—Find the greatest common factor of two whole numbers less than or equal to 100 and the least common multiple of two whole numbers less than or equal to 12. Use the distributive property to express a sum of two whole numbers 1−100 with a common factor as a multiple of a sum of two whole numbers with no common factor. *For example, express 36 + 8 as 4 (9 + 2).*
Standards Clarifications/ Content Limits	As an explanation for some of the standards listed above, in Grade 6, students will • solve problems involving addition, subtraction, multiplication, and division of multi-digit numbers, including whole numbers and decimals • identify and apply the greatest common factor and/or the least common multiple for pairs of whole numbers

Domain	THE NUMBER SYSTEM
Cluster	**Apply and extend previous understandings of numbers to the system of rational numbers.**
Standards	MACC.6.NS.3.5—Understand that positive and negative numbers are used together to describe quantities having opposite directions or values (e.g., temperature above/below zero, elevation above/below sea level, credits/debits, positive/negative electric charge); use positive and negative numbers to represent quantities in real-world contexts, explaining the meaning of 0 in each situation. MACC.6.NS.3.6—Understand a rational number as a point on the number line. Extend number line diagrams and coordinate axes familiar from previous grades to represent points on the line and in the plane with negative number coordinates. MACC.6.NS.3.6.a—Recognize opposite signs of numbers as indicating locations on opposite sides of 0 on the number line; recognize that the opposite of the opposite of a number is the number itself, e.g., $-(-3) = 3$, and that 0 is its own opposite. MACC.6.NS.3.6.b—Understand signs of numbers in ordered pairs as indicating locations in quadrants of the coordinate plane; recognize that when two ordered pairs differ only by signs, the locations of the points are related by reflections across one or both axes. MACC.6.NS.3.6.c—Find and position integers and other rational numbers on a horizontal or vertical number line diagram; find and position pairs of integers and other rational numbers on a coordinate plane. MACC.6.NS.3.7—Understand ordering and absolute value of rational numbers. MACC.6.NS.3.7.a—Interpret statements of inequality as statements about the relative position of two numbers on a number line diagram. *For example, interpret $-3 > -7$ as a statement that -3 is located to the right of -7 on a number line oriented from left to right*. MACC.6.NS.3.7.b—Write, interpret, and explain statements of order for rational numbers in real-world contexts. *For example, write $-3°C > -7°C$ to express the fact that $-3°C$ is warmer than $-7°C$.*

	MACC.6.NS.3.7.c—Understand the absolute value of a rational number as its distance from 0 on the number line; interpret absolute value as magnitude for a positive or negative quantity in a real-world situation. *For example, for an account balance of −30 dollars, write |−30| = 30 to describe the size of the debt in dollars.* MACC.6.NS.3.7.d—Distinguish comparisons of absolute value from statements about order. *For example, recognize that an account balance less than −30 dollars represents a debt greater than 30 dollars.* MACC.6.NS.3.8—Solve real-world and mathematical problems by graphing points in all four quadrants of the coordinate plane. Include use of coordinates and absolute value to find distances between points with the same first coordinate or the same second coordinate.
Standards Clarifications/ Content Limits	As an explanation for some of the standards listed above, in Grade 6, students will • apply characteristics of rational numbers, opposite numbers, and absolute values of rational numbers to solve real-world problems

Domain	EXPRESSIONS AND EQUATIONS
Cluster	**Apply and extend previous understandings of arithmetic to algebraic expressions.**
Standards	MACC.6.EE.1.1—Write and evaluate numerical expressions involving whole-number exponents. MACC.6.EE.1.2—Write, read, and evaluate expressions in which letters stand for numbers. MACC.6.EE.1.2.a—Write expressions that record operations with numbers and with letters standing for numbers. *For example, express the calculation "Subtract y from 5" as* $5 - y$. MACC.6.EE.1.2.b—Identify parts of an expression using mathematical terms (sum, term, product, factor, quotient, coefficient); view one or more parts of an expression as a single entity. *For example, describe the expression* $2(8 + 7)$ *as a product of two factors; view* $(8 + 7)$ *as both a single entity and a sum of two terms.* MACC.6.EE.1.2.c—Evaluate expressions at specific values of their variables. Include expressions that arise from formulas used in real-world problems. Perform arithmetic operations, including those involving whole-number exponents, in the conventional order when there are no parentheses to specify a particular order (Order of Operations). *For example, use the formulas* $V = s^3$ *and* $A = 6s^2$ *to find the volume and surface area of a cube with sides of length* $s = \frac{1}{2}$. MACC.6.EE.1.3—Apply the properties of operations to generate equivalent expressions. *For example, apply the distributive property to the expression* $3(2 + x)$ *to produce the equivalent expression* $6 + 3x$*; apply the distributive property to the expression* $24x + 18y$ *to produce the equivalent expression* $6(4x + 3y)$*; apply properties of operations to* $y + y + y$ *to produce the equivalent expression* $3y$. MACC.6.EE.1.4—Identify when two expressions are equivalent (i.e., when the two expressions name the same number regardless of which value is substituted into them). *For example, the expressions* $y + y + y$ *and* $3y$ *are equivalent because they name the same number regardless of which number y stands for.*

Standards Clarifications/ Content Limits	As an explanation for some of the standards listed above, in Grade 6, students will • identify and demonstrate whole-number exponents as shorthand for repeated multiplication and apply the order of operations to simplify mathematical statements with exponents • for standards MACC.6.EE.1.3 and MACC.6.EE.1.4, identify and apply properties of operation to simplify mathematical expressions to find equivalent expressions

Domain	**EQUATIONS AND EXPRESSIONS**
Cluster	**Reason about and solve one-variable equations and inequalities.**
Standards	MACC.6.EE.2.5—Understand solving an equation or inequality as a process of answering a question: Which values from a specified set, if any, make the equation or inequality true? Use substitution to determine whether a given number in a specified set makes an equation or inequality true. MACC.6.EE.2.6—Use variables to represent numbers and write expressions when solving a real-world or mathematical problem; understand that a variable can represent an unknown number, or, depending on the purpose at hand, any number in a specified set. MACC.6.EE.2.7—Solve real-world and mathematical problems by writing and solving equations of the form $x + p = q$ and $px = q$ for cases in which p, q and x are all non-negative rational numbers. MACC.6.EE.2.8—Write an inequality of the form $x > c$ or $x < c$ to represent a constraint or condition in a real-world or mathematical problem. Recognize that inequalities of the form $x > c$ or $x < c$ have infinitely many solutions; represent solutions of such inequalities on number line diagrams.
Standards Clarifications/ Content Limits	As an explanation for some of the standards listed above, in Grade 6, students will • identify and create models and/or solve problems involving equations and inequalities • develop, identify, and solve one-variable equations and inequalities that represent real-world or mathematical problems

Domain	EQUATIONS AND EXPRESSIONS
Cluster	**Represent and analyze quantitative relationships between dependent and independent variables.**
Standards	MACC.6.EE.3.9—Use variables to represent two quantities in a real-world problem that change in relationship to one another; write an equation to express one quantity, thought of as the dependent variable, in terms of the other quantity, thought of as the independent variable. Analyze the relationship between the dependent and independent variables using graphs and tables, and relate these to the equation. *For example, in a problem involving motion at constant speed, list and graph ordered pairs of distances and times, and write the equation $d = 65t$ to represent the relationship between distance and time.*
Standards Clarifications/ Content Limits	In Grade 6, students will • be able to differentiate between dependent and independent variables • demonstrate knowledge of variables by graphing ordered pairs and writing equations based on graphs and tables • solve real-world problems by identifying the relationships between independent and dependent variables

Domain	GEOMETRY
Cluster	**Solve real-world and mathematical problems involving area, surface area, and volume.**
Standards	MACC.6.G.1.1—Find the area of right triangles, other triangles, special quadrilaterals, and polygons by composing into rectangles or decomposing into triangles and other shapes; apply these techniques in the context of solving real-world and mathematical problems. MACC.6.G.1.2—Find the volume of a right rectangular prism with fractional edge lengths by packing it with unit cubes of the appropriate unit fraction edge lengths, and show that the volume is the same as would be found by multiplying the edge lengths of the prism. Apply the formulas $V = lwh$ and $V = bh$ to find volumes of right rectangular prisms with fractional edge lengths in the context of solving real-world and mathematical problem. MACC.6.G.1.3—Draw polygons in the coordinate plane given coordinates for the vertices; use coordinates to find the length of a side joining points with the same first coordinate or the same second coordinate. Apply these techniques in the context of solving real-world and mathematical problems. MACC.6.G.1.4—Represent three-dimensional figures using nets made up of rectangles and triangles, and use the nets to find the surface area of these figures. Apply these techniques in the context of solving real-world and mathematical problems.
Standards Clarifications/ Content Limits	As an explanation for some of the standards listed above, in Grade 6, students will • demonstrate knowledge of finding the area of polygons by composing and decomposing them into various shapes • apply the formula for the volume of right rectangular prisms • use a coordinate plane to identify and draw polygons and determine their side lengths • identify and create nets of three-dimensional figures and evaluate the surface area of the figures

Domain	STATISTICS AND PROBABILITY
Cluster	**Develop understanding of statistical variability.**
Standards	MACC.6.SP.1.1—Recognize a statistical question as one that anticipates variability in the data related to the question and accounts for it in the answers. *For example, "How old am I?" is not a statistical question, but "How old are the students in my school?" is a statistical question because one anticipates variability in students' ages.* MACC.6.SP.1.2—Understand that a set of data collected to answer a statistical question has a distribution which can be described by its center, spread, and overall shape. MACC.6.SP.1.3—Recognize that a measure of center for a numerical data set summarizes all of its values with a single number, while a measure of variation describes how its values vary with a single number.
Standards Clarifications/ Content Limits	As an explanation for some of the standards listed above, in Grade 6, students will • identify and develop statistical questions that demonstrate variability • work with collected data in tables, graphs, and charts • use the data to determine measures of center • evaluate measures of center, including mean, median, and mode, and identify measures of variation, including range

Domain	STATISTICS AND PROBABILITY
Cluster	**Summarize and describe distributions.**
Standards	MACC.6.SP.2.4—Display numerical data in plots on a number line, including dot plots, histograms, and box plots. MACC.6.SP.2.5—Summarize numerical data sets in relation to their context, such as by: MACC.6.SP.2.5.a—Reporting the number of observations. MACC.6.SP.2.5.b—Describing the nature of the attribute under investigation, including how it was measured and its units of measurement. MACC.6.SP.2.5.c—Giving quantitative measures of center (median and/or mean) and variability (interquartile range and/or mean absolute deviation), as well as describing any overall pattern and any striking deviations from the overall pattern with reference to the context in which the data were gathered. MACC.6.SP.2.5.d—Relating the choice of measures of center and variability to the shape of the data distribution and the context in which the data were gathered.
Standards Clarifications/ Content Limits	As an explanation for some of the standards listed above, in Grade 6, students will • create number lines, dot plots, histograms, and box plots to display data • identify various presentations of the same data • determine the appropriate methods for representing and analyzing data • compare measures of center and variability of shape to the distribution of the collected data

B. Grade 7 Item Specifications -

Domain	RATIOS AND PROPORTIONAL RELATIONSHIP
Cluster	**Analyze proportional relationships and use them to solve real-world and mathematical problems.**
Standards	MACC.7.RP.1.1—Compute unit rates associated with ratios of fractions, including ratios of lengths, areas, and other quantities measured in like or different units. *For example, if a person walks* $\frac{1}{2}$ *mile in each* $\frac{1}{4}$ *hour, compute the unit rate as the complex fraction* $\frac{\frac{1}{2}}{\frac{1}{4}}$ *miles per hour, equivalently 2 miles per hour.* MACC.7.RP.1.2—Recognize and represent proportional relationships between quantities. MACC.7.RP.1.2.a—Decide whether two quantities are in a proportional relationship, e.g., by testing for equivalent ratios in a table or graphing on a coordinate plane and observing whether the graph is a straight line through the origin. MACC.7.RP.1.2.b—Identify the constant of proportionality (unit rate) in tables, graphs, equations, diagrams, and verbal descriptions of proportional relationships. MACC.7.RP.1.2.c—Represent proportional relationships by equations. *For example, if total cost t is proportional to the number n of items purchased at a constant price p, the relationship between the total cost and the number of items can be expressed as* $t = pn$. MACC.7.RP.1.2.d—Explain what a point (x, y) on the graph of a proportional relationship means in terms of the situation, with special attention to the points (0, 0) and (1, r) where r is the unit rate. MACC.7.RP.1.3—Use proportional relationships to solve multi-step ratio and percent problems. *Examples: simple interest, tax, markups and markdowns, gratuities and commissions, fees, percent increase and decrease, percent error.*

Standards Clarifications/ Content Limits	As an explanation for some of the standards listed above, in Grade 7, students will • determine unit rates when both quantities are presented as fractions • determine whether two quantities are proportional and find their constant of proportionality using tables, graphs, equations, diagrams, and verbal descriptions • represent proportions using equations • analyze the graph of a line and find the rate of change and identify graphic representations of a proportional relationship • use proportions to solve complex problems involving ratios or percents • explain how a change in one variable affects the other variable or how a change in the slope of the line affects one of the variables

Domain	**THE NUMBER SYSTEM**
Cluster	**Apply and extend previous understandings of operations with fractions to add, subtract, multiply, and divide rational numbers.**
Standards	MACC.7.NS.1.1—Apply and extend previous understandings of addition and subtraction to add and subtract rational numbers; represent addition and subtraction on a horizontal or vertical number line diagram. MACC.7.NS.1.1.a—Describe situations in which opposite quantities combine to make 0. *For example, a hydrogen atom has 0 charge because its two constituents are oppositely charged.* MACC.7.NS.1.1.b—Understand $p + q$ as the number located a distance $\|q\|$ from p, in the positive or negative direction depending on whether q is positive or negative. Show that a number and its opposite have a sum of 0 (are additive inverses). Interpret sums of rational numbers by describing real-world contexts. MACC.7.NS.1.1.c—Understand subtraction of rational numbers as adding the additive inverse, $p - q = p + (-q)$. Show that the distance between two rational numbers on the number line is the absolute value of their difference, and apply this principle in real-world contexts. MACC.7.NS.1.1.d—Apply properties of operations as strategies to add and subtract rational numbers.

	MACC.7.NS.1.2—Apply and extend previous understandings of multiplication and division and of fractions to multiply and divide rational numbers. MACC.7.NS.1.2.a—Understand that multiplication is extended from fractions to rational numbers by requiring that operations continue to satisfy the properties of operations, particularly the distributive property, leading to products such as $(-1)(-1) = 1$ and the rules for multiplying signed numbers. Interpret products of rational numbers by describing real-world contexts. MACC.7.NS.1.2.b—Understand that integers can be divided, provided that the divisor is not zero, and every quotient of integers (with a non-zero divisor) is a rational number. If p and q are integers, then $-(p/q) = (-p)/q = p/(-q)$. Interpret quotients of rational numbers by describing real-world contexts. MACC.7.NS.1.2.c—Apply properties of operations as strategies to multiply and divide rational numbers. MACC.7.NS.1.2.d—Convert a rational number to a decimal using long division; know that the decimal form of a rational number terminates in 0s or eventually repeats. MACC.7.NS.1.3—Solve real-world and mathematical problems involving the four operations with rational numbers.
Standards Clarifications/ Content Limits	As an explanation for some of the standards listed above, in Grade 7, students will • add and subtract rational numbers, including representing addition or subtraction on a vertical or horizontal number line and being able to use the concept of additive inverse • multiply and divide rational numbers, including negative numbers • convert rational numbers to decimals

Domain	EXPRESSIONS AND EQUATIONS
Cluster	**Use properties of operations to generate equivalent expressions.**
Standards	MACC.7.EE.1.1—Apply properties of operations as strategies to add, subtract, factor, and expand linear expressions with rational coefficients. MACC.7.EE.1.2—Understand that rewriting an expression in different forms in a problem context can shed light on the problem and how the quantities in it are related. *For example, a + 0.05a = 1.05a means that "increase by 5%" is the same as "multiply by 1.05."*
Standards Clarifications/ Content Limits	As an explanation for some of the standards listed above, in Grade 7, students will • add, subtract, factor, and expand linear expressions and rewrite expressions in different forms using properties of operations

Domain	EXPRESSIONS AND EQUATIONS
Cluster	**Solve real-life and mathematical problems using numerical and algebraic expressions and questions.**
Standards	MACC.7.EE.2.3—Solve multi-step real-life and mathematical problems posed with positive and negative rational numbers in any form (whole numbers, fractions, and decimals), using tools strategically. Apply properties of operations to calculate with numbers in any form; convert between forms as appropriate; and assess the reasonableness of answers using mental computation and estimation strategies. *For example: If a woman making $25 an hour gets a 10% raise, she will make an additional $\frac{1}{10}$ of her salary an hour, or $2.50, for a new salary of $27.50. If you want to place a towel bar $9\frac{3}{4}$ inches long in the center of a door that is $27\frac{1}{2}$ inches wide, you will need to place the bar about 9 inches from each edge; this estimate can be used as a check on the exact computation.* MACC.7.EE.2.4—Use variables to represent quantities in a real-world or mathematical problem, and construct simple equations and inequalities to solve problems by reasoning about the quantities.

	MACC.7.EE.2.4.a—Solve word problems leading to equations of the form $px + q = r$ and $p(x + q) = r$, where p, q, and r are specific rational numbers. Solve equations of these forms fluently. Compare an algebraic solution to an arithmetic solution, identifying the sequence of the operations used in each approach. *For example, the perimeter of a rectangle is 54 cm. Its length is 6 cm. What is its width?* MACC.7.EE.2.4.b—Solve word problems leading to inequalities of the form $px + q > r$ or $px + q < r$, where p, q, and r are specific rational numbers. Graph the solution set of the inequality and interpret it in the context of the problem. *For example: As a salesperson, you are paid $50 per week plus $3 per sale. This week you want your pay to be at least $100. Write an inequality for the number of sales you need to make, and describe the solutions.*
Standards Clarifications/ Content Limits	As an explanation for some of the standards listed above, in Grade 7, students will • solve multi-step problems involving positive and negative whole numbers, fractions, and/or decimals and convert between rational numbers • solve word problems that involve equations and inequalities

Domain	GEOMETRY
Cluster	**Draw, construct, and describe geometrical figures and describe the relationships between them.**
Standards	MACC.7.G.1.1—Solve problems involving scale drawings of geometric figures, including computing actual lengths and areas from a scale drawing and reproducing a scale drawing at a different scale. MACC.7.G.1.2—Draw (freehand, with ruler and protractor, and with technology) geometric shapes with given conditions. Focus on constructing triangles from three measures of angles or sides, noticing when the conditions determine a unique triangle, more than one triangle, or no triangle. MACC.7.G.1.3—Describe the two-dimensional figures that result from slicing three-dimensional figures, as in plane sections of right rectangular prisms and right rectangular pyramids.
Standards Clarifications/ Content Limits	As an explanation for some of the standards listed above, in Grade 7, students will • solve problems involving scale drawings and use them to determine lengths and areas • draw geometric shapes that fit specific conditions, particularly triangles of given side lengths or angle measures • describe plane sections of three-dimensional shapes

Domain	GEOMETRY
Cluster	**Solve real-life and mathematical problems involving angle measure, area, surface area, and volume.**
Standards	MACC.7.G.2.4—Know the formulas for the area and circumference of a circle and use them to solve problems; give an informal derivation of the relationship between the circumference and area of a circle. MACC.7.G.2.5—Use facts about supplementary, complementary, vertical, and adjacent angles in a multi-step problem to write and solve simple equations for an unknown angle in a figure. MACC.7.G.2.6—Solve real-world and mathematical problems involving area, volume, and surface area of two- and three-dimensional objects composed of triangles, quadrilaterals, polygons, cubes, and right prisms.
Standards Clarifications/ Content Limits	As an explanation for some of the standards listed above, in Grade 7, students will • know and apply formulas for area and circumference of a circle • use facts about angles to determine unknown angles in a figure • solve problems involving the area of two-dimensional objects composed of polygons and three-dimensional figures composed of right prisms

Domain	STATISTICS AND PROBABILITY
Cluster	**Use random sampling to draw inferences about a population.**
Standards	MACC.7.SP.1.1—Understand that statistics can be used to gain information about a population by examining a sample of the population; generalizations about a population from a sample are valid only if the sample is representative of that population. Understand that random sampling tends to produce representative samples and support valid inferences. MACC.7.SP.1.2—Use data from a random sample to draw inferences about a population with an unknown characteristic of interest. Generate multiple samples (or simulated samples) of the same size to gauge the variation in estimates or predictions. *For example, estimate the mean word length in a book by randomly sampling words from the book; predict the winner of a school election based on randomly sampled survey data. Gauge how far off the estimate or prediction might be.*
Standards Clarifications/ Content Limits	As an explanation for some of the standards listed above, in Grade 7, students will • use random sampling to find representative samples and use them to make inferences about a population, including the use of multiple samples to determine variation • represent probabilities as fractions, ratios, decimals between 0 and 1 (inclusive), and/or percentages between 0 and 100 (inclusive)

Domain	STATISTICS AND PROBABILITY
Cluster	**Draw informal comparative inferences about two populations.**
Standards	MACC.7.SP.2.3—Informally assess the degree of visual overlap of two numerical data distributions with similar variabilities, measuring the difference between the centers by expressing it as a multiple of a measure of variability. *For example, the mean height of players on the basketball team is 10 cm greater than the mean height of players on the soccer team, about twice the variability (mean absolute deviation) on either team; on a dot plot, the separation between the two distributions of heights is noticeable.* MACC.7.SP.2.4—Use measures of center and measures of variability for numerical data from random samples to draw informal comparative inferences about two populations. *For example, decide whether the words in a chapter of a seventh-grade science book are generally longer than the words in a chapter of a fourth-grade science book.*
Standards Clarifications/ Content Limits	As an explanation for some of the standards listed above, in Grade 7, students will • use measures of center and variability to compare data sets and make inferences about populations

Domain	STATISTICS AND PROBABILITY
Cluster	**Investigate chance processes and develop, use, and evaluate probability models.**
Standards	MACC.7.SP.3.5—Understand that the probability of a chance event is a number between 0 and 1 that expresses the likelihood of the event occurring. Larger numbers indicate greater likelihood. A probability near 0 indicates an unlikely event, a probability around $\frac{1}{2}$ indicates an event that is neither unlikely nor likely, and a probability near 1 indicates a likely event. MACC.7.SP.3.6—Approximate the probability of a chance event by collecting data on the chance process that produces it and observing its long-run relative frequency, and predict the approximate relative frequency given the probability. *For example, when rolling a number cube 600 times, predict that a 3 or 6 would be rolled roughly 200 times, but probably not exactly 200 times.* MACC.7.SP.3.7—Develop a probability model and use it to find probabilities of events. Compare probabilities from a model to observed frequencies; if the agreement is not good, explain possible sources of the discrepancy. MACC.7.SP.3.7.a—Develop a uniform probability model by assigning equal probability to all outcomes, and use the model to determine probabilities of events. *For example, if a student is selected at random from a class, find the probability that Jane will be selected and the probability that a girl will be selected.* MACC.7.SP.3.7.b—Develop a probability model (which may not be uniform) by observing frequencies in data generated from a chance process. *For example, find the approximate probability that a spinning penny will land heads up or that a tossed paper cup will land open-end down. Do the outcomes for the spinning penny appear to be equally likely based on the observed frequencies?* MACC.7.SP.3.8—Find probabilities of compound events using organized lists, tables, tree diagrams, and simulation. MACC.7.SP.3.8.a—Understand that, just as with simple events, the probability of a compound event is the fraction of outcomes in the sample space for which the compound event occurs.

	MACC.7.SP.3.8.*b*—Represent sample spaces for compound events using methods such as organized lists, tables, and tree diagrams. For an event described in everyday language (e.g., "rolling double sixes"), identify the outcomes in the sample space which compose the event. MACC.7.SP.3.8.c—Design and use a simulation to generate frequencies for compound events. *For example, use random digits as a simulation tool to approximate the answer to the question: If 40% of donors have type A blood, what is the probability that it will take at least 4 donors to find one with type A blood?*
Standards Clarifications/ Content Limits	As an explanation for some of the standards listed above, in Grade 7, students will • understand the concept of probability and use it to make predictions • find probabilities of compound events and represent sample spaces for compound events through lists, tables, and diagrams

C. Grade 8 Item Specifications -

Domain	THE NUMBER SYSTEM
Cluster	**Know that there are numbers that are not rational and approximate them by rational numbers.**
Standards	MACC.8.NS.1.1—Know that numbers that are not rational are called irrational. Understand informally that every number has a decimal expansion; for rational numbers show that the decimal expansion repeats eventually, and convert a decimal expansion which repeats eventually into a rational number. MACC.8.NS.1.2—Use rational approximations of irrational numbers to compare the size of irrational numbers, locate them approximately on a number line diagram, and estimate the value of expressions (e.g., π^2). *For example, by truncating the decimal expansion of $\sqrt{2}$, show that $\sqrt{2}$ is between 1 and 2, then between 1.4 and 1.5, and explain how to continue on to get better approximations.*
Standards Clarifications/ Content Limits	As an explanation for some of the standards listed above, in Grade 8, students will • identify rational or irrational numbers by their definition and convert between a repeating decimal expansion and its rational number equivalent • approximate irrational numbers and/or manipulate them with expressions to produce approximations and note the approximate location of irrational numbers on a number line diagram

Domain	EXPRESSIONS AND EQUATIONS
Cluster	**Work with radicals and integer exponents.**
Standards	MACC.8.EE.1.1—Know and apply the properties of integer exponents to generate equivalent numerical expressions. *For example,* $3^2 \times 3^{-5} = 3^{-3} = \frac{1}{3^3} = \frac{1}{27}$. MACC.8.EE.1.2—Use square root and cube root symbols to represent solutions to equations of the form $x^2 = p$ and $x^3 = p$, where p is a positive rational number. Evaluate square roots of small perfect squares and cube roots of small perfect cubes. Know that $\sqrt{2}$ is irrational. MACC.8.EE.1.3—Use numbers expressed in the form of a single digit times an integer power of 10 to estimate very large or very small quantities, and to express how many times as much one is than the other. *For example, estimate the population of the United States as* 3×10^8 *and the population of the world as* 7×10^9*, and determine that the world population is more than 20 times larger.* MACC.8.EE.1.4—Perform operations with numbers expressed in scientific notation, including problems where both decimal and scientific notation are used. Use scientific notation and choose units of appropriate size for measurements of very large or very small quantities (e.g., use millimeters per year for seafloor spreading). Interpret scientific notation that has been generated by technology.
Standards Clarifications/ Content Limits	As an explanation for some of the standards listed above, in Grade 8, students will • generate and/or manipulate numerical expressions involving numbers with integer exponents • compare numbers expressed as single digits times an integer power of 10 and express their relative size as a multiplier • perform operations on numbers expressed in scientific notation, including conversion of very small or large quantities to appropriate scientific notation • interpret scientific notation as generated by technology • for standards MACC.8.EE.1.3 and MACC.8.EE.1.4, use items that are in a real-world context only

Domain	EXPRESSIONS AND EQUATIONS
Cluster	**Understand the connections between proportional relationships, lines, and linear equations.**
Standards	MACC.8.EE.2.5—Graph proportional relationships, interpreting the unit rate as the slope of the graph. Compare two different proportional relationships represented in different ways. *For example, compare a distance-time graph to a distance-time equation to determine which of two moving objects has greater speed.* MACC.8.EE.2.6—Use similar triangles to explain why the slope m is the same between any two distinct points on a non-vertical line in the coordinate plane; derive the equation $y = mx$ for a line through the origin and the equation $y = mx + b$ for a line intercepting the vertical axis at b.
Standards Clarifications/ Content Limits	As an explanation for some of the standards listed above, in Grade 8, students will • graph proportional relationships • compare two representations of linear relationships • use similar triangles to explain that the slope of a non-vertical line is the same throughout • make the connection between proportionality and linear equations and lines • derive the equation for $y = mx + b$

Domain	EXPRESSIONS AND EQUATIONS
Cluster	**Analyze and solve linear equations and pairs of simultaneous linear equations.**
Standards	MACC.8.EE.3.7—Solve linear equations in one variable. MACC.8.EE.3.7.a—Give examples of linear equations in one variable with one solution, infinitely many solutions, or no solutions. Show which of these possibilities is the case by successively transforming the given equation into simpler forms, until an equivalent equation of the form $x = a$, $a = a$, or $a = b$ results (where a and b are different numbers). MACC.8.EE.3.7.b—Solve linear equations with rational number coefficients, including equations whose solutions require expanding expressions using the distributive property and collecting like terms. MACC.8.EE.3.8—Analyze and solve pairs of simultaneous linear equations. MACC.8.EE.3.8.a—Understand that solutions to a system of two linear equations in two variables correspond to points of intersection of their graphs, because points of intersection satisfy both equations simultaneously. MACC.8.EE.3.8.b—Solve systems of two linear equations in two variables algebraically, and estimate solutions by graphing the equations. Solve simple cases by inspection. *For example, $3x + 2y = 5$ and $3x + 2y = 6$ have no solution because $3x + 2y$ cannot simultaneously be 5 and 6.* MACC.8.EE.3.8.c—Solve real-world and mathematical problems leading to two linear equations in two variables. *For example, given coordinates for two pairs of points, determine whether the line through the first pair of points intersects the line through the second pair.*

Standards Clarifications/ Content Limits	As an explanation for some of the standards listed above, in Grade 8, students will • solve linear equations • transform given equations to simpler forms and determine whether there is one solution, no solution, or infinite solutions to the equations • solve pairs of simultaneous equations algebraically and/or graphically • solve simple cases of simultaneous linear equations by inspection • solve problems of two linear equations algebraically • graph a system of two linear equations and estimate the solution

Domain	**FUNCTIONS**
Cluster	**Define, evaluate, and compare functions.**
Standards	MACC.8.F.1.1—Understand that a function is a rule that assigns to each input exactly one output. The graph of a function is the set of ordered pairs consisting of an input and the corresponding output. MACC.8.F.1.2—Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). *For example, given a linear function represented by a table of values and a linear function represented by an algebraic expression, determine which function has the greater rate of change.* MACC.8.F.1.3—Interpret the equation $y = mx + b$ as defining a linear function, whose graph is a straight line; give examples of functions that are not linear. *For example, the function $A = s^2$ giving the area of a square as a function of its side length is not linear because its graph contains the points (1, 1), (2, 4) and (3, 9), which are not on a straight line.*
Standards Clarifications/ Content Limits	As an explanation for some of the standards listed above, in Grade 8, students will • identify a function in written and in graphic forms, including points from all four quadrants • compare properties of two functions, each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions) • identify that a linear function is defined by $y = mx + b$ • give examples or identify examples of nonlinear functions

Domain	FUNCTIONS
Cluster	**Use functions to model relationships between quantities.**
Standards	MACC.8.F.2.4—Construct a function to model a linear relationship between two quantities. Determine the rate of change and initial value of the function from a description of a relationship or from two (x, y) values, including reading these from a table or from a graph. Interpret the rate of change and initial value of a linear function in terms of the situation it models, and in terms of its graph or a table of values. MACC.8.F.2.5—Describe qualitatively the functional relationship between two quantities by analyzing a graph (e.g., where the function is increasing or decreasing, linear or nonlinear). Sketch a graph that exhibits the qualitative features of a function that has been described verbally.
Standards Clarifications/ Content Limits	As an explanation for some of the standards listed above, in Grade 8, students will • identify a table, graph, or equation that represents a linear function or other simple relationship • analyze a table or graph to identify or describe the rate of change and/or initial value • sketch or otherwise identify different representations of the same relationship, including translating among graphs, equations, tables, and words

Domain	GEOMETRY
Cluster	**Understand congruence and similarity using physical models, transparencies, or geometry software.**
Standards	MACC.8.G.1.1—Verify experimentally the properties of rotations, reflections, and translations: MACC.8.G.1.1.a—Lines are taken to lines, and line segments to line segments of the same length. MACC.8.G.1.1.b—Angles are taken to angles of the same measure. MACC.8.G.1.1.c—Parallel lines are taken to parallel lines. MACC.8.G.1.2—Understand that a two-dimensional figure is congruent to another if the second can be obtained from the first by a sequence of rotations, reflections, and translations; given two congruent figures, describe a sequence that exhibits the congruence between them. MACC.8.G.1.3—Describe the effect of dilations, translations, rotations, and reflections on two-dimensional figures using coordinates. MACC.8.G.1.4—Understand that a two-dimensional figure is similar to another if the second can be obtained from the first by a sequence of rotations, reflections, translations, and dilations; given two similar two-dimensional figures, describe a sequence that exhibits the similarity between them. MACC.8.G.1.5—Use informal arguments to establish facts about the angle sum and exterior angle of triangles, about the angles created when parallel lines are cut by a transversal, and the angle-angle criterion for similarity of triangles. *For example, arrange three copies of the same triangle so that the sum of the three angles appears to form a line, and give an argument in terms of transversals why this is so.*

Standards Clarifications/ Content Limits	As an explanation for some of the standards listed above, in Grade 8, students will • verify experimentally that a property of rotations, reflections, and translations is that lines are taken to lines and line segments to line segments of the same length • verify experimentally that a property of rotations, reflections, and translations is that angles are taken to angles of the same measure • verify experimentally that a property of rotations, reflections, and translations is that parallel lines are taken to parallel lines • identify and demonstrate the congruency of two-dimensional figures through the use of rotations, reflections, and translations • describe a sequence of rotations, reflections, and translations that exhibits the congruence between 2 two-dimensional figures • describe the effects of dilations, translations, rotations, and reflections on two-dimensional figures using coordinates • identify and demonstrate the similarity of two-dimensional figures through the use of rotations, reflections, translations, and dilations • describe a sequence of rotations, reflections, translations, and dilations that exhibits the similarity between 2 two-dimensional figures • use informal arguments to establish the fact about angle sum in triangles • use informal arguments to establish the fact about exterior angles of triangles • use informal arguments to establish the fact about the angles created when parallel lines are cut by a transversal • use informal arguments to establish the fact about the angle-angle criterion for similarity of triangles

Domain	GEOMETRY
Cluster	**Understand and apply the Pythagorean Theorem.**
Standards	MACC.8.G.2.6—Explain a proof of the Pythagorean Theorem and its converse. MACC.8.G.2.7—Apply the Pythagorean Theorem to determine unknown side lengths in right triangles in real-world and mathematical problems in two and three dimensions. MACC.8.G.2.8—Apply the Pythagorean Theorem to find the distance between two points in a coordinate system.
Standards Clarifications/ Content Limits	As an explanation for some of the standards listed above, in Grade 8, students will • explain a proof of the Pythagorean Theorem • explain a proof of the converse of the Pythagorean Theorem • apply the Pythagorean Theorem to find distances in real-world and mathematical situations in two or three dimensions • apply the Pythagorean Theorem to find distances between points on the coordinate plane

Domain	GEOMETRY
Cluster	**Solve real-world and mathematical problems involving volume of cylinders, cones, and spheres.**
Standards	MACC.8.G.3.9—Know the formulas for the volumes of cones, cylinders, and spheres and use them to solve real-world and mathematical problems.
Standards Clarifications/ Content Limits	In Grade 8, students will • apply formulas to solve problems related to volume of cones, spheres, and right circular cylinders • determine one or two dimension(s) of a three-dimensional figure given its volume

Domain	STATISTICS AND PROBABILITY
Cluster	Investigate patterns of association in bivariate data.
Standards	MACC.8.SP.1.1—Construct and interpret scatter plots for bivariate measurement data to investigate patterns of association between two quantities. Describe patterns such as clustering, outliers, positive or negative association, linear association, and nonlinear association. MACC.8.SP.1.2—Know that straight lines are widely used to model relationships between two quantitative variables. For scatter plots that suggest a linear association, informally fit a straight line, and informally assess the model fit by judging the closeness of the data points to the line. MACC.8.SP.1.3—Use the equation of a linear model to solve problems in the context of bivariate measurement data, interpreting the slope and intercept. *For example, in a linear model for a biology experiment, interpret a slope of 1.5 cm/hr as meaning that an additional hour of sunlight each day is associated with an additional 1.5 cm in mature plant height.* MACC.8.SP.1.4—Understand that patterns of association can also be seen in bivariate categorical data by displaying frequencies and relative frequencies in a two-way table. Construct and interpret a two-way table summarizing data on two categorical variables collected from the same subjects. Use relative frequencies calculated for rows or columns to describe possible association between the two variables. *For example, collect data from students in your class on whether or not they have a curfew on school nights and whether or not they have assigned chores at home. Is there evidence that those who have a curfew also tend to have chores?*

Standards Clarifications/ Content Limits	As an explanation for some of the standards listed above, in Grade 8, students will • construct and interpret scatter plots of bivariate data for association patterns, such as clustering, outliers, positive or negative association, and linear and nonlinear association • know that straight lines model many of the relationships between bivariate data • informally fit a line of best fit for data that suggest a linear relationship and informally assess the fit of the data to the model line • use the equation of a linear model to interpret the meaning of the slope and intercept • construct and interpret a two-way table of frequencies or relative frequencies for data collected from the same subjects • use relative frequencies for either rows or columns of a two-way table

Appendices

Appendix A

Sample Items

Grade: 8

Item Type: Selected Response

Correct Answer: C

Possible Points: 1

DOK: 3

Calculator Usage: No

CCSS Standard:

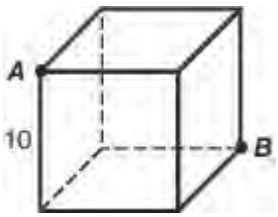
MACC.8.G.2.7—Apply the Pythagorean Theorem to determine unknown side lengths in right triangles in real-world and mathematical problems in two and three dimensions.

Standards for Mathematical Practice:

2. Reason abstractly and quantitatively.

6. Attend to precision.

The cube-shaped box below has side lengths that measure 10 centimeters.



What is the length of the box's diagonal, from point *A* to point *B*?

A. 10 centimeters

B. $10\sqrt{2}$ centimeters

* **C.** $10\sqrt{3}$ centimeters

D. 300 centimeters

Distractor Rationales

A. This answer makes the incorrect assumption that the length of the box's diagonal will be equal to the length of each side.

B. This answer is the length of the diagonal of the lateral faces of the cube.

C. Correct answer

D. This answer does not take the square root of both sides of the equation when solving for c^2.

Grade: 6

Item Type: Gridded Response

Correct Answer: 16

Possible Points: 1

DOK: 1

Calculator Usage: No

CCSS Standard:

MACC.6.EE.1.2—Write, read, and evaluate expressions in which letters stand for numbers.

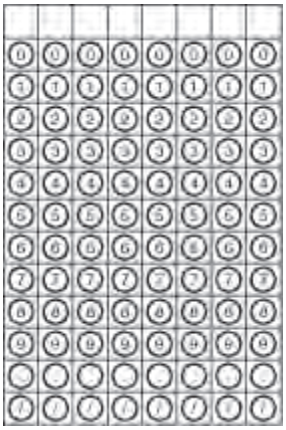
> MACC.6.EE.1.2.c—Evaluate expressions at specific values of their variables. Include expressions that arise from formulas used in real-world problems. Perform arithmetic operations, including those involving whole-number exponents, in the conventional order when there are no parentheses to specify a particular order (Order of Operations). *For example, use the formulas* $V = s^3$ *and* $A = 6\ s^2$ *to find the volume and surface area of a cube with sides of length* $s = \frac{1}{2}$.

Standards for Mathematical Practice:

2. Reason abstractly and quantitatively.

What is the value of the expression below when $x = 2$?

$$\frac{4x}{x^2} + x + 2 \cdot 3x$$



Grade: 7

Item Type: Short Answer

Correct Answer: (10 × 12.5) − (4.5 × 5) or other acceptable answers

Possible Points: 1

DOK: 2

Calculator Usage: No

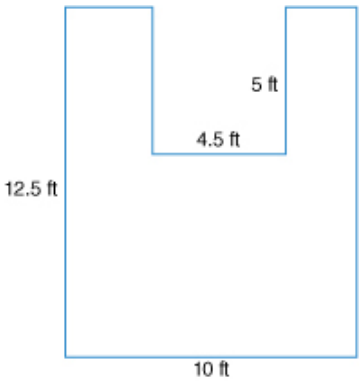
CCSS Standard:

MACC.7.G.2.6—Solve real-world and mathematical problems involving area, volume, and surface area of two- and three-dimensional objects composed of triangles, quadrilaterals, polygons, cubes, and right prisms.

Standards for Mathematical Practice:

1. Make sense of problems and persevere in solving them.

The diagram below shows a deck that James and his father are building in their backyard.



Write an expression that can be used to find the area, in square feet, of the entire deck.

_____

Grade: 6

Item Type: Constructed Response

Correct Answer: See Scoring Exemplar

Possible Points: 2

DOK: 2

Calculator Usage: No

CCSS Standard:

MACC.6.SP.2.5—Summarize numerical data sets in relation to their context, such as by:

> MACC.6.SP.2.5.c—Giving quantitative measures of center (median and/or mean) and variability (interquartile range and/or mean absolute deviation), as well as describing any overall pattern and any striking deviations from the overall pattern with reference to the context in which the data were gathered.

Standards for Mathematical Practice:

1. Make sense of problems and persevere in solving them.

2. Reason abstractly and quantitatively.

During the past 3 months, 11 sixth grade students read fiction books. They listed the number of fiction books they read below.

2, 7, 9, 3, 10, 4, 9, 7, 5, 8, 7

Part A. Find the median and the mean of the numbers of fiction books read.

Part B. If the student who read 2 books remembered that he had really read 5 books, would the median and the mean change? Explain how.

Use words, numbers, and/or pictures to show your work.

SCORING RUBRIC	
2	Work demonstrates a **clear and complete** understanding of the mathematical concepts and/or procedures required by the task. Appropriate strategy is shown with clear and complete explanations and interpretations.
1	Response demonstrates a **partial** understanding of the mathematical concepts and/or procedures. Appropriate strategy is shown, but explanation or interpretation has minor flaws OR Response is incorrect because of calculation errors. Work and strategy indicate a **clear** understanding of the mathematical concepts and/or procedures required by the task.
0	Response is irrelevant, inappropriate, or not provided.

SCORING EXEMPLAR

Maximum Points—2

Part A—1 point

- The median is 7 and the mean is 6.45.

Part B—1 point

- The median would stay the same, but the mean would rise to 6.7. The mean stays the same because there are still 5 numbers below the center and 5 above since the number of students stayed the same.

or equivalent explanation

Grade: 7

Item Type: Extended Response

Correct Answer: See Scoring Exemplar

Possible Points: 4

DOK: 3

Calculator Usage: No

CCSS Standard:

MACC.7.SP.3.6—Approximate the probability of a chance event by collecting data on the chance process that produces it and observing its long-run relative frequency, and predict the approximate relative frequency given the probability. *For example, when rolling a number cube 600 times, predict that a 3 or 6 would be rolled roughly 200 times, but probably not exactly 200 times.*

Standards for Mathematical Practice:

4. Model with mathematics.

6. Attend to precision.

Carol has a bag full of different colored marbles. She pulled out 20 marbles from the bag. After each pull, she recorded the color and returned the marble to the bag. She recorded 7 red marbles, 3 green marbles, and 10 blue marbles.

Part A. Write the fraction of marbles she pulled out of the bag that were red, the fraction that were green, and the fraction that were blue.

Part B. What would be a good prediction of how many times out of 100 Carol would pull out a blue marble?

Part C. Explain your prediction.

Use words, numbers, and/or pictures to show your work.

SCORING RUBRIC	
4	Work demonstrates a **clear and complete** understanding of the mathematical concepts and/or procedures required by the task. Appropriate strategy is shown with clear and complete explanations and interpretations.
3	Work demonstrates a **clear** understanding of the mathematical concepts and/or procedures but is not complete. Appropriate strategy is shown, but explanation or interpretation has minor flaws. OR Response is incorrect because of calculation errors. Work and strategy indicate a **clear** demonstration of the problem.
2	Response demonstrates a **partial** understanding of the mathematical concepts and/or procedures. Appropriate strategy is shown, but explanation or interpretation has minor flaws.
1	Response shows **minimal** understanding of the mathematical concepts and/or procedures or provides no explanation or interpretation for the solution or shows major flaws.
0	Response is irrelevant, inappropriate, or not provided.

SCORING EXEMPLAR

Maximum Points—4

Part A—1 point

- $\frac{7}{20}$ were red, $\frac{3}{20}$ were green, and $\frac{10}{20}$ or $\frac{1}{2}$ were blue.

Part B—1 point

- If Carol pulls 100 total marbles from the bag, a good prediction would be that she will pull about 50 blue marbles.

Part C—2 points

- Ten out of the 20 marbles originally pulled from the bag were blue, so $\frac{1}{2}$ of the marbles pulled from the bag are expected to be blue. Out of 100 pulls, a good prediction would be that about 50 marbles will be blue because half of 100 is 50.

or equivalent explanation

Grade: 8

Item Type: Performance Task

Correct Answer: See Scoring Exemplar

Possible Points: 10

DOK: 3

Calculator Usage: No

CCSS Standard:

MACC.8.EE.3.8—Analyze and solve pairs of simultaneous linear equations.

MACC.8.EE.3.8.b—Solve systems of two linear equations in two variables algebraically, and estimate solutions by graphing the equations. Solve simple cases by inspection. *For example, $3x + 2y = 5$ and $3x + 2y = 6$ have no solution because $3x + 2y$ cannot simultaneously be 5 and 6.*

Standards for Mathematical Practice:

1. Make sense of problems and persevere in solving them.

3. Construct viable arguments and critique the reasoning of others.

4. Model with mathematics.

6. Attend to precision.

Painters

Teacher Directions:

Instruct students to use words, numbers, and/or pictures to show their work.

Encourage students to find exact values for their answers by solving the system of equations algebraically as well.

Allow 30 to 40 minutes for this task.

This task can be modified by using different multiples of each independent variable (number of hours) as long as the relationship between the two costs stays the same.

Make all necessary materials available.

TASK:

Painter A charges \$27 per hour plus a one-time fee of \$150 for materials to paint a building. Painter B charges $\frac{2}{3}$ of the hourly rate of painter A but charges a one-time fee that is \$100 more than painter A.

Part A. Create and solve a system of equations that models this situation.

Part B. Support your answer from part A with a graph showing the total cost for both painters, along with any other necessary additional work. Be sure to include all labels and scales.

Part C. Present a detailed explanation of which painter will charge the least amount of money, depending on the number of hours the project will take. Describe how the graph and/or system of equations support your explanation.

SCORING EXEMPLAR

Maximum Points—10

Part A—2 points

- System of equations:

  $y = 27x + 150$

  $y = 18x + 250$

  $18x + 250 = 27x + 150$

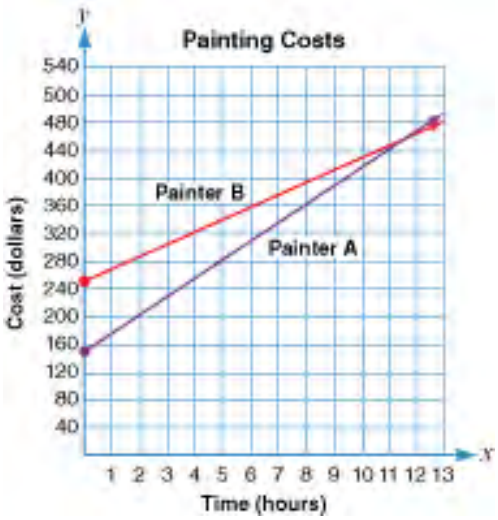
  $9x + 100$

  $x = 11\frac{1}{9}$ hours

Other appropriate equations or strategies may earn full points.

Part B—3 points

- Graph (2 points for graph and 1 point for all appropriate formatting):



Part C—5 points

Student includes the following information (1 point each):

- If the project takes exactly $11\frac{1}{9}$ hours, the cost of both painters will be the same. If the project takes fewer than $11\frac{1}{9}$ hours, painter A's total cost will be less than painter B's total cost. If the project takes more than $11\frac{1}{9}$ hours, painter B's total cost will be less than painter A's total cost.
- Because this graph does not show an exact intersection point, the system of equations must be solved to find a more accurate intersection point
- The solution to the system of equations is $(11\frac{1}{9}, 450)$, which means that at $11\frac{1}{9}$ hours the total cost of both painters is \$450.
- Before the intersection, the line representing painter A's total cost is below the line of representing painter B's total cost, which shows that painter A's cost is lower than painter B's cost for any project lasting fewer than $11\frac{1}{9}$ hours.
- After the intersection, the line representing painter A's total cost is above the line representing painter B's total cost, which shows that painter A's cost is higher than painter B's cost for any project lasting longer than $11\frac{1}{9}$ hours.

Appendix B

Standards for Mathematical Practice

The Standards for Mathematical Practice describe varieties of expertise that mathematics educators at all levels should seek to develop in their students. These practices rest on important "processes and proficiencies" with longstanding importance in mathematics education. The first of these are the NCTM process standards of problem solving, reasoning and proof, communication, representation, and connections. The second are the strands of mathematical proficiency specified in the National Research Council's report *Adding It Up*: adaptive reasoning, strategic competence, conceptual understanding (comprehension of mathematical concepts, operations and relations), procedural fluency (skill in carrying out procedures flexibly, accurately, efficiently and appropriately), and productive disposition (habitual inclination to see mathematics as sensible, useful, and worthwhile, coupled with a belief in diligence and one's own efficacy).[1]

MACC.K12.MP.1.1 Make sense of problems and persevere in solving them.

Mathematically proficient students start by explaining to themselves the meaning of a problem and looking for entry points to its solution. They analyze givens, constraints, relationships, and goals. They make conjectures about the form and meaning of the solution and plan a solution pathway rather than simply jumping into a solution attempt. They consider analogous problems, and try special cases and simpler forms of the original problem in order to gain insight into its solution. They monitor and evaluate their progress and change course if necessary. Older students might, depending on the context of the problem, transform algebraic expressions or change the viewing window on their graphing calculator to get the information they need. Mathematically proficient students can explain correspondences between equations, verbal descriptions, tables, and graphs or draw diagrams of important features and relationships, graph data, and search for regularity or trends. Younger students might rely on using concrete objects or pictures to help conceptualize and solve a problem. Mathematically proficient students check their answers to problems using a different method, and they continually ask themselves, "Does this make sense?" They can understand the approaches of others to solving complex problems and identify correspondences between different approaches.

MACC.K12.MP.2.1 Reason abstractly and quantitatively.

Mathematically proficient students make sense of quantities and their relationships in problem situations. They bring two complementary abilities to bear on problems involving quantitative relationships: the ability to *decontextualize*—to abstract a given situation and represent it symbolically and manipulate the representing symbols as if they have a life of their own, without necessarily attending to their referents—and the ability to *contextualize*, to pause as needed during the manipulation process in order to probe into the referents for the symbols involved. Quantitative reasoning entails habits of creating a coherent representation of the problem at hand; considering the units involved; attending to the meaning of quantities, not just how to compute them; and knowing and flexibly using different properties of operations and objects.

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[1] Common Core State Standards Initiative (CCSSI), 2010, Common Core State Standards for Mathematics. Washington, DC: National Governors Association Center for Best Practices and the Council of Chief State School Officers. http://www.corestandards.org/assets/CCSSI_Math%20Standards.pdf

MACC.K12.MP.3.1 Construct viable arguments and critique the reasoning of others.

Mathematically proficient students understand and use stated assumptions, definitions, and previously established results in constructing arguments. They make conjectures and build a logical progression of statements to explore the truth of their conjectures. They are able to analyze situations by breaking them into cases, and can recognize and use counterexamples. They justify their conclusions, communicate them to others, and respond to the arguments of others. They reason inductively about data, making plausible arguments that take into account the context from which the data arose. Mathematically proficient students are also able to compare the effectiveness of two plausible arguments, distinguish correct logic or reasoning from that which is flawed, and—if there is a flaw in an argument—explain what it is. Elementary students can construct arguments using concrete referents such as objects, drawings, diagrams, and actions. Such arguments can make sense and be correct, even though they are not generalized or made formal until later grades. Later, students learn to determine domains to which an argument applies. Students at all grades can listen or read the arguments of others, decide whether they make sense, and ask useful questions to clarify or improve the arguments.

MACC.K12.MP.4.1 Model with mathematics.

Mathematically proficient students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. In early grades, this might be as simple as writing an addition equation to describe a situation. In middle grades, a student might apply proportional reasoning to plan a school event or analyze a problem in the community. By high school, a student might use geometry to solve a design problem or use a function to describe how one quantity of interest depends on another. Mathematically proficient students who can apply what they know are comfortable making assumptions and approximations to simplify a complicated situation, realizing that these may need revision later. They are able to identify important quantities in a practical situation and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts and formulas. They can analyze those relationships mathematically to draw conclusions. They routinely interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.

MACC.K12.MP.5.1 Use appropriate tools strategically.

Mathematically proficient students consider the available tools when solving a mathematical problem. These tools might include pencil and paper, concrete models, a ruler, a protractor, a calculator, a spreadsheet, a computer algebra system, a statistical package, or dynamic geometry software. Proficient students are sufficiently familiar with tools appropriate for their grade or course to make sound decisions about when each of these tools might be helpful, recognizing both the insight to be gained and their limitations. For example, mathematically proficient high school students analyze graphs of functions and solutions generated using a graphing calculator. They detect possible errors by strategically using estimation and other mathematical knowledge. When making mathematical models, they know that technology can enable them to visualize the results of varying assumptions, explore consequences, and compare predictions with data. Mathematically proficient students at various grade levels are able to identify relevant external mathematical resources, such as digital content located on a website, and use them to pose or solve problems. They are able to use technological tools to explore and deepen their understanding of concepts.

MACC.K12.MP.6.1 Attend to precision.

Mathematically proficient students try to communicate precisely to others. They try to use clear definitions in discussion with others and in their own reasoning. They state the meaning of the symbols they choose, including using the equal sign consistently and appropriately. They are careful about specifying units of measure, and labeling axes to clarify the correspondence with quantities in a problem. They calculate accurately and efficiently, express numerical answers with a degree of precision appropriate for the problem context. In the elementary grades, students give carefully formulated explanations to each other. By the time they reach high school they have learned to examine claims and make explicit use of definitions.

MACC.K12.MP.7.1 Look for and make use of structure.

Mathematically proficient students look closely to discern a pattern or structure. Young students, for example, might notice that three and seven more is the same amount as seven and three more, or they may sort a collection of shapes according to how many sides the shapes have. Later, students will see 7×8 equals the well remembered $7 \times 5 + 7 \times 3$, in preparation for learning about the distributive property. In the expression $x^2 + 9x + 14$, older students can see the 14 as 2×7 and the 9 as $2 + 7$. They recognize the significance of an existing line in a geometric figure and can use the strategy of drawing an auxiliary line for solving problems. They also can step back for an overview and shift perspective. They can see complicated things, such as some algebraic expressions, as single objects or as being composed of several objects. For example, they can see $5 - 3(x - y)^2$ as 5 minus a positive number times a square and use that to realize that its value cannot be more than 5 for any real numbers x and y.

MACC.K12.MP.8.1 Look for and express regularity in repeated reasoning.

Mathematically proficient students notice if calculations are repeated, and look both for general methods and for shortcuts. Upper elementary students might notice when dividing 25 by 11 that they are repeating the same calculations over and over again, and conclude they have a repeating decimal. By paying attention to the calculation of slope as they repeatedly check whether points are on the line through (1, 2) with slope 3, middle school students might abstract the equation $(y - 2)/(x - 1) = 3$. Noticing the regularity in the way terms cancel when expanding $(x - 1)(x + 1)$, $(x - 1)(x^2 + x + 1)$, and $(x - 1)(x^3 + x^2 + x + 1)$ might lead them to the general formula for the sum of a geometric series. As they work to solve a problem, mathematically proficient students maintain oversight of the process, while attending to the details. They continually evaluate the reasonableness of their intermediate results.