Florida Interim Assessment Item Bank and Test Platform

Item Specifications

Mathematics
Grades 3–5

FLORIDA DEPARTMENT OF EDUCATION
www.fldoe.org

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I. Introduction

In July 2010 the Florida Department of Education (FDOE) approved the adoption of the Common Core State Standards (CCSS) for Mathematics to support its pursuit of improved outcomes for all Florida mathematics students and participation in national educational initiatives, such as Race to the Top. The U.S. Department of Education awarded a Race to the Top grant to Florida in August 2010. An important component of this grant focused on the development of high-quality assessment items and balanced assessments for use by districts, schools, and teachers. The assessment items will be stored in the Florida Interim Assessment Item Bank and Test Platform (IBTP), a statewide secure system that allows Florida educators to search the item bank, export test items, and generate customized high-quality assessments for computer-based delivery or paper-and-pencil delivery. The IBTP allows Florida educators to determine what students know and are able to do relative to instruction based on Florida's Next Generation Sunshine State Standards and the Common Core State Standards.

A. Purpose of the Item Specifications

The *Item Specifications* define the expectations for content, standards alignment, and format of assessment items for the Item Bank and Test Platform. The *Item Specifications* are intended for use by item writers and reviewers in the development of high-quality assessment items.

B. Scope

The *Item Specifications* provide general and grade-specific guidelines for the development of all Mathematics assessment items available in the Florida Interim Assessment Item Bank.

C. Standards Alignment

Items developed for the Florida Interim Assessment Item Bank and Test Platform will align to the Common Core State Standards for Mathematics. The Common Core State Standards for Mathematics are structured into three levels of specificity: Domains, Clusters, and Standards. These define what mathematics students should know and be able to do at every grade level/course, kindergarten through high school.

II. Criteria for Item Development

Mathematics item writers for the Florida Interim Assessment Item Bank must have a comprehensive knowledge of mathematics curriculum based on the Common Core State Standards and an understanding of the range of cognitive abilities of the target student population. Item writers should understand and consistently apply the guidelines established in this document. Item writers are expected to use their best judgment in writing items that measure the Mathematics standards of the CCSS without introducing extraneous elements that reflect bias for or against a group of students.

A. Overall Considerations for Item Development

These guidelines are provided to ensure the development of high-quality assessment items for the Florida Interim Assessment Item Bank.

1. Each item should be written to measure primarily one Common Core State Standard; however, other standards may also be addressed for some item types. In addition to the content standard alignment, each item should align to at least one Mathematical Practice Standard.
2. Items should be appropriate for students in terms of grade-level/course instruction, experience and difficulty, cognitive development, and reading level. The reading level of the test items should be on grade level.
3. Items should be written at or above the cognitive level (DOK) of the standard unless otherwise noted in the Individual Standard Specifications sections.
4. Each item should be written clearly and unambiguously to elicit the desired response.
5. Items should not disadvantage or exhibit disrespect to anyone in regard to age, gender, race, ethnicity, language, religion, socioeconomic status, disability, occupation, or geographic region.
6. At grades kindergarten through 5, items should be able to be answered without using a calculator. For grades 6 through 7, a four-function calculator may be used. For grade 8, a scientific calculator may be used. For Algebra 1, Geometry, and Algebra 2, both a scientific calculator and a graphing calculator (with functionalities similar to that of a TI-84) may be used. For all grades, calculators should not be used for items where computational skills or fluency are being assessed.

B. Item Contexts

The context in which an item is presented is called the item context or scenario. These guidelines are provided to assist item writers with development of items within an appropriate context.

1. The item context should be designed to interest students at the targeted level. Scenarios should be appropriate for students in terms of grade-level experience and difficulty, cognitive development, and reading level.
2. The context should be directly related to the question asked. The context should lead the student cognitively to the question. Every effort should be made to keep items as concise as possible without losing cognitive flow or missing the overall idea or concept.
3. Item contexts should include subject areas other than mathematics. Specifically, topics from grade-level/course Next Generation Sunshine State Standards for Science and Social Studies, and Common Core State Standards for English Language Arts may be used where appropriate.
4. Items including specific information or data must be accurate and verified against reliable sources. Source documentation must accompany these types of items.
5. Mathematics item stimuli should include written text and/or visual material, such as graphs, tables, diagrams, maps, models, and/or other illustrations.
6. All item scenarios, graphics, diagrams, and illustrations must be age-, grade-, and experience-appropriate.

7. All graphs used in item stems or answer options must be complete with title, scale, and labeled axes, except when these components are to be completed by the student.
8. Any graphics in items should be uncluttered and should clearly depict the necessary information. Graphics should contain relevant details that contribute to the students' understanding of the item or that support the context of the item. Graphics should not introduce bias to the item.
9. Item content should be timely but not likely to become dated too quickly.

C. Use of Media

Media can be used to provide either necessary or supplemental information—that is, some media contain information that is necessary for answering the question, while other media support the context of the question. Items may include diagrams, illustrations, charts, tables, audio files, or video files unless otherwise noted in the Individual Standard Specifications. Some standards require a heavier use of graphics than others. Geometry, for example, relies heavily on graphics to convey information.

1. Items should not begin with media. Media in items are always preceded by text.
2. All visual media (tables, charts, graphs, photographs, etc.) should be titled. Titles should be in all caps, boldfaced, and centered, and may be placed above or below the visual media.

D. Item Style and Format

This section presents stylistic guidelines and formatting directions that should be followed while developing items.

1. Items should be clear and concise, and they should use vocabulary and sentence structure appropriate for the assessed grade level.
2. The words *most likely* or *best* should be used only when appropriate to the question.
3. Items using the word *not* should emphasize the word *not* using all uppercase letters (e.g., Which of the following is NOT an example of . . .). The word *not* should be used sparingly.
4. For items that refer to an estimate (noun), lowercase letters should be used.
5. As appropriate, boldface type should be used to emphasize key words in the item (e.g., least, most, greatest, percent, mode, median, mean, range).
6. Masculine pronouns should NOT be used to refer to both sexes. Plural forms should be used whenever possible to avoid gender-specific pronouns (e.g., instead of "The student will make changes so that he . . . ," use "The students will make changes so that they . . .").
7. An equal balance of male and female names should be used, including names representing different ethnic groups appropriate for Florida.
8. For clarity, operation symbols, equality signs, and ordinates should be preceded and followed by one space.

9. Decimal numbers between –1 and 1 (including currency) should have a leading zero.
10. Metric numbers should be expressed in a single unit when possible (e.g., 1.4 kilograms instead of 1 kilogram 400 grams).
11. Decimal notation should be used for numbers with metric units (e.g., 1.2 grams instead of 151 grams).
12. Commas should be used within numbers greater than or equal to 1,000. Commas may be omitted within an equation or expression.
13. Units of measure should be spelled out, except in graphics, where an abbreviation may be used (e.g., *ft* or *yd*). Abbreviations that also spell a word must be followed by a period to avoid confusion. For example, to avoid confusion with the preposition *in*, the abbreviation *in*. should be used for the unit of measure *inches* and should include a period. If an abbreviation is used in a graphic, an explanation of the meaning of the abbreviation should be included in the stem.
14. In titles for tables and charts and in labels for axes, the units of measure should be included, preferably in lowercase letters and in parentheses, e.g., *height* (*in inches*).
15. Fractions should be typed with a horizontal fraction bar. The numerator and denominator should be centered with respect to each other. The bar should cover all portions (superscripts, parentheses, etc.) of the numerator and denominator. In a mixed number, a half space should appear between the whole number and the fraction. If a variable appears before or after a fraction bar, the variable should be centered with respect to the fraction bar. If a stimulus, stem, or set of responses contains a fraction in fractional notation, that portion of the item should be 1.5-spaced.
16. In general, numbers zero through nine should be presented as words and numbers 10 and above should be presented as numerals. In the item stem, any numbers needed to compute answers should be presented as numerals.

E. Item Types

This section presents guidelines for development of the following types of items:

1. Selected Response (SR) Items (1 point)
2. Gridded Response (GR) and Short Response (SHR) Items (1 point)
3. Constructed Response and Extended Response Items
    a. Constructed Response (CR) Items (2 points)
    b. Extended Response (ER) Items (4 points)
4. Essay Response (ESR) Items (6 points)
5. Performance Task (PT) Items (1–10 points)

1. **Selected Response (SR) Items (1 point)**

   Selected response items require students to choose an answer from the choices given. Each item consists of a stem and either three or four answer options, depending on the grade level/course (see c below). One of the answer options is the correct answer, and the remaining options are called distractors. Selected response items may include a stimulus and/or passage.

   a. SR items should take an average of 1 minute per item to solve.

   b. SR items are worth 1 point each.

   c. SR items in grades K, 1, and 2 should have three answer choices (A, B, and C). SR items for all other grades and courses should have four answer choices (A, B, C, and D).

   d. Answer options that are single words should be arranged in alphabetical or reverse alphabetical order.

   e. Answer options that are phrases or sentences should be arranged from shortest to longest or longest to shortest.

   f. Numerical answer options should be arranged in ascending or descending order.

   g. Numerical answer options that represent relative magnitude or size should be arranged as they are shown in the stem or some other logical order.

   h. When the item requires the identification of a choice from the item stem, table, chart, or illustration, the options should be arranged as they are presented in the item stem, table, chart, or illustration.

   i. If the answer options for an item are neither strictly numerical nor denominate numbers, the options should be arranged by the logic presented in the item, by alphabetical order, or by length.

   j. Distractor rationales should represent computational or conceptual errors or misconceptions commonly made by students who have not mastered the assessed concepts.

   k. Outliers (i.e., answer choices that are longer phrases or sentences than the other choices, or choices with significantly more/fewer digits than the other choices) should NOT be used.

   l. Options such as none of the above, all of the above, not here, not enough information, or cannot be determined should not be used as answer options.

2. **Gridded Response (GR) and Short Response (SHR) Items (1 point)**

    a. Gridded response and short response items are worth 1 point.

    b. The GR format is designed for items that require a positive numeric solution (whole numbers, decimals, percents, or fractions).

    c. The bubble grids used with GR items should contain eight columns. Each column will contain the digits 0 through 9, decimal point (.), and fraction bar (/) enclosed in bubbles.

    d. Gridded response items should include instructions that specify the unit in which the answer is to be provided (e.g., inches). If several units of measure are in the item (e.g., in an item involving a conversion), the final unit needed for the answer should be written in boldface.

    e. The short response format is designed for items that result in a value or answer that cannot be answered in the gridded response format (negative numbers, expressions, etc.).

3. **Constructed Response and Extended Response Items**

    Mathematics constructed response and extended response items require students to produce a response in words, pictures, diagrams, and/or numbers. As such, these items are especially suited to assessing many of the more complex tasks and high-level thinking skills demanded by the Common Core State Standards for Mathematics. The Florida Interim Assessment Item Bank will include 2-point constructed response items (CR) and 4-point extended response items (ER).

    Overall characteristics for mathematics CRs and ERs are as follows:

    a. The item should measure understanding and insight of mathematical concepts rather than rote memory or factual recall.

    b. Real-world, factual stimulus material (charts, graphs, tables, etc.) must cite the source used.

    c. Items requiring students to produce responses as pictures, diagrams, graphs, tables, etc., should provide workspace and/or templates where appropriate.

a. **Constructed Response (CR) Items (2 points)**

Constructed response items usually include a scenario and instructions on how to respond. The recommended time allotment for a student to respond is 5 minutes. A complete answer is worth 2 points, and a partial answer is worth 1 point. The constructed response holistic rubric and exemplar specific to each item are used for scoring as follows:

SCORING RUBRIC	
2	Work demonstrates a **clear and complete** understanding of the mathematical concepts and/or procedures required by the task. Appropriate strategy is shown with clear and complete explanations and interpretations.
1	Response demonstrates a **partial** understanding of the mathematical concepts and/or procedures. Appropriate strategy is shown, but explanation or interpretation has minor flaws. OR Response is incorrect because of calculation errors. Work and strategy indicate a **clear** understanding of the mathematical concepts and/or procedures required by the task.
0	Response is irrelevant, inappropriate, or not provided.

Exemplars: A specific exemplar should be developed for each constructed response item. Exemplars will be used as scoring guides and should be specific to the item, but not so specific as to discount multiple correct answers. Exemplars should include a clear and defensible description of the top score point, and contain straightforward language that is accurate, complete, and easy to interpret.

b. Extended Response (ER) Items (4 points)

Extended response items include a scenario and instructions on how to respond and are worth 4 points. However, ER items are usually more complex than SHR and 2-point CR items. The recommended time allotment for a student to respond is 10–15 minutes. The extended response holistic rubric and exemplar specific to each item are used for scoring as follows:

SCORING RUBRIC	
4	Work demonstrates a **clear and complete** understanding of the mathematical concepts and/or procedures required by the task. Appropriate strategy is shown with clear and complete explanations and interpretations.
3	Work demonstrates a **clear** understanding of the mathematical concepts and/or procedures but is not complete. Appropriate strategy is shown, but explanation or interpretation has minor flaws. OR Response is incorrect because of calculation errors. Work and strategy indicate a **clear** demonstration of the problem.
2	Response demonstrates a **partial** understanding of the mathematical concepts and/or procedures. Appropriate strategy is shown, but explanation or interpretation has minor flaws.
1	Response shows **minimal** understanding of the mathematical concepts and/or procedures or provides no explanation or interpretation for the solution or shows major flaws.
0	Response is irrelevant, inappropriate, or not provided.

Exemplars: A specific exemplar should be developed for each extended response item. Exemplars will be used as scoring guides and should be specific to the item, but not so specific as to discount multiple correct answers. Exemplars should include a clear and defensible description of the top score point, and contain straightforward language that is accurate, complete, and easy to interpret.

4. **Essay Response (ESR) Items (6 points)**

   The essay response item consists of asking a general question or providing a stimulus (such as an article or research paper on a relevant topic), and asking students to express their thoughts or provide facts about the topic using logic and reason. Essay response items encompass a higher level of thinking and a broader range of skills that includes CCSS literacy standards, which is critical to future success in higher education and the workforce.

   In most cases, essay responses will go beyond a single paragraph in length, with a distinct introduction, body, and conclusion. An essay response will be worth a total of 6 points, with a rubric structure similar to that of the 4-point extended response. Students should be given about 20 to 30 minutes to complete each item.

   Exemplars: A specific exemplar should be developed for each essay response item. Exemplars will be used as scoring guides and should be specific to the item, but not so specific as to discount multiple correct answers. Exemplars should include a clear and defensible description of the top score point, and contain straightforward language that is accurate, complete, and easy to interpret.

5. **Performance Tasks (PT) (1–10 points)**

   Performance tasks are used to measure students' ability to *demonstrate* knowledge and skills from one or more CCSS. Specifically, performance tasks may require students to create a product, demonstrate a process, or perform an activity that demonstrates proficiency in Mathematics. They are evaluated using customized scoring exemplars, and each task may be worth 1–10 points.

   Performance tasks may have the following characteristics:

   a. Performance tasks may cover a short time period or may cover an extended period.
   b. Performance tasks must contain clear and explicit directions for understanding and completing the required component tasks and producing the objective output.
   c. All tasks, skills, and/or behaviors required by the performance tasks must be objective, observable, and measurable.
   d. All necessary equipment, materials, and resources should be referenced within the text of the performance task.
   e. Performance tasks should elicit a range of score points.
   f. Performance tasks generally require students to organize, apply, analyze, synthesize, and/or evaluate concepts.
   g. Performance tasks may measure performance in authentic situations and outside the classroom, where appropriate and practical.
   h. Typical response formats include demonstrations, laboratory performance, oral presentations, exhibits, or other products.

i. Every performance task requires a companion exemplar to be used for scoring purposes. Exemplars should meet the following criteria.

i The exemplars and performance tasks should be developed in tandem to ensure compatibility.

ii Exemplars must be specific to the individual requirements of each performance task; generic rubrics are not acceptable.

iii The exemplar must allow for efficient and consistent scoring.

iv Each part of the performance task must have a clearly stated score point in the exemplar and when a part of the task is divided into sections or requirements, each of those must have a maximum score indicated.

v The exemplar descriptors consist of an ideal response exemplar and should allow for all foreseeable methods of correctly and thoroughly completing all requirements of the performance task.

F. Readability

Items must be written with readability in mind. In addition, vocabulary must be appropriate for the grade level being tested. The following sources provide information about the reading level of individual words:

Taylor, Stanford E. *EDL Core Vocabularies: Reading, Mathematics, Science, and Social Studies*. Austin, TX: Steck-Vaughn-EDL, 1989.

Mogilner, Alijandra. *Children's Writer's Word Book*. Cincinnati, OH: Writer's Digest Books, 1992.

G. Cognitive Complexity

1. Overview

Florida's adoption of the Common Core State Standards (CCSS) for Mathematics and English Language Arts & Literacy in History/Social Studies, Science, and Technical Subjects presents Florida with an opportunity to revise its current Depth of Knowledge (DOK) Model of Cognitive Complexity. More information about Florida's Depth of Knowledge levels is available online at *http://www.cpalms.org/cpalms/dok.aspx*.

2. Levels of Depth of Knowledge for Mathematics

Level 1 (Recall) includes the recall of information such as a fact, definition, term, or a simple procedure, as well as performing a simple algorithm or applying a formula. That is, in mathematics a one-step, well-defined, or straight algorithmic procedure should be included at this lowest level.

Some examples that represent but do not constitute all of Level 1 performance are:

- Count to 100 by ones and by tens.
- Fluently multiply and divide within 100, using strategies such as the relationship between multiplication and division (e.g., knowing that $8 \times 5 = 40$, one knows $40 \div 5 = 8$).

- Write simple expressions that record calculations with numbers, and interpret numerical expressions without evaluating them. For example, express the calculation "add 8 and 7 and then multiply by 2" as $2 \times (8 + 7)$. Recognize that $3 \times (18932 + 921)$ is three times as large as $18932 + 921$ without having to calculate the indicated sum or product.
- Enter measurement data into a data table.
- Identify the variables indicated in a two-dimensional graph.

Level 2 (Basic Application of Concepts & Skills) includes the engagement of some mental processing beyond a habitual response. A Level 2 standard or assessment item requires students to make some decisions as to how to approach the problem or activity, whereas Level 1 requires students to demonstrate a rote response, perform a well-known algorithm, follow a set procedure (like a recipe), or perform a clearly defined series of steps. For example, to compare data requires first identifying characteristics of the objects or phenomenon and then grouping or ordering the objects. Interpreting information from a simple graph, requiring reading information from the graph, also is a Level 2. Interpreting information from a complex graph that requires some decisions on what features of the graph need to be considered and how information from the graph can be aggregated is a Level 3. Caution is warranted in interpreting Level 2 as only skills because some reviewers will interpret skills very narrowly as primarily numerical skills, and such interpretation excludes from this level other skills such as visualization skills and probability skills, which may be more complex simply because they are less common and require more mental processing.

Some examples that represent but do not constitute all of Level 2 performance are:

- Understand that the last number name said tells the number of objects counted. The number of objects is the same regardless of their arrangement or the order in which they were counted.
- Express the length of an object as a whole number of length units by laying multiple copies of a shorter object (the length unit) end to end.
- Measure and estimate liquid volumes and masses of objects using standard units of grams (g), kilograms (kg), and liters (l).
- Apply properties of operations as strategies to add and subtract rational numbers.
- Measure and record data and produce graphs of relevant variables.
- Graph proportional relationships, interpreting the unit rate as the slope of the graph.

Level 3 (Strategic Thinking & Complex Reasoning) requires reasoning, planning, using evidence, and a higher level of thinking than the previous two levels. In most instances, requiring students to explain their thinking is a Level 3. Activities that require students to make conjectures are also at this level. The cognitive demands at Level 3 are complex and abstract. The complexity does not result from the fact that there are multiple answers, a possibility for both levels 1 and 2, but because the task requires more demanding reasoning. However, an activity that has more than one possible answer and requires students to justify the response they give would most likely be a Level 3.

Some examples that represent but do not constitute all of Level 3 performance are:

- Explain why addition and subtraction strategies work, using place value and the properties of operations.
- Interpret the rate of change and initial value of a linear function in terms of the situation it models, and in terms of its graph or a table of values.
- Given a real-world situation, formulate a problem.
- Organize, represent, and interpret data obtained through experiments or observations.
- Formulate a mathematical model to describe a complex phenomenon.
- Justify a solution to a problem.
- Analyze a deductive argument.

Level 4 (Extended Thinking & Complex Reasoning) in mathematics involves the application of Level 3 processes and skills over an extended period. This is likely to incorporate demands from other content areas (e.g., English language arts, science) in the development and support of mathematical arguments that describe some real-world phenomenon or situation.

Some examples that represent but do not constitute all of Level 4 performance are:

- Derive a mathematical model to explain a complex phenomenon or make a prediction.
- Complete a project requiring the formulation of questions, devising a plan, collecting data, analyzing the data, and preparing a written report describing the justification of the conclusions reached.

H. Item Difficulty

Item writers will not be expected to make a prediction of difficulty for each item created. However, item writers should develop items that reflect a range of difficulty.

I. Universal Design

The application of universal design principles helps develop assessments that are usable to the greatest number of students, including students with disabilities and nonnative speakers of English. To support the goal of providing access to all students, the items in the Florida Interim Assessment Item Bank maximize readability, legibility, and compatibility with accommodations, and item development includes a review for potential bias and sensitivity issues.

Items must allow for the widest possible range of student participation. Item writers must attend to the best practices suggested by universal design, including, but not limited to,

1. reduction in wordiness
2. avoidance of ambiguity
3. selection of reader-friendly construction and terminology
4. consistently applied concept names and graphic conventions

Universal design principles also inform decisions about item layout and design, including, but not limited to, type size, line length, spacing, and graphics.

J. Sample Items

Appendix A of this document contains a selection of sample items. The sample items represent a range of cognitive complexities and item types.

III. Review Procedures for Florida Interim Assessment Item Bank Items

Prior to being included in the Florida Interim Assessment Item Bank, all mathematics items must pass several levels of review as part of the item development process.

A. Review for Item Quality

Assessment items developed for the Florida Interim Assessment Item Bank will be reviewed by Florida educators, the FDOE, and the contractors to ensure the quality of the items, including grade-level/course appropriateness, alignment to the standard, accuracy, and other criteria for overall item quality.

B. Review for Bias and Sensitivity

Items are reviewed by groups of Florida educators generally representative of Florida's geographic regions and culturally diverse population. Items are reviewed for the following kinds of bias: gender, racial, ethnic, linguistic, religious, geographic, and socioeconomic. Item reviews also include consideration of issues related to individuals with disabilities.

This review is to ensure that the primary purpose of assessing student achievement is not undermined by inadvertently including in the item bank any material that students, parents, or other stakeholders may deem inappropriate. Reviewers are asked to consider the variety of cultural, regional, philosophical, political, and religious backgrounds throughout Florida and to determine whether the subject matter will be acceptable to Florida students, their parents, and other members of Florida communities.

IV. Guide to the Individual Standard Specifications

A. CCSS Mathematics Standards Classification System

The graphic below demonstrates the coding schema for the Common Core State Standards for Mathematics.

MACC.K.CC.1.1

Subject Grade Domain Cluster Standard

Using this schema:

Subject Code MACC: Mathematics Common Core

Grade: Kindergarten

Domain CC: Counting and Cardinality

Cluster 1: Know number names and the count sequence.

Standard 1: Count to 100 by ones and by tens.

MACC.912.A-APR.1.1

Subject   Grade   Category (Unit)   Domain Cluster   Standard

Using the schema, the bottom row refers to:

Subject Code MACC: Mathematics Common Core

Grade: High school Grades 9–12

Category A: Algebra

Domain APR: Arithmetic with Polynomials and Rational Expressions

Cluster 1: Perform arithmetic operations on polynomials.

Standard 1: Understand that polynomials form a system analogous to the integers, namely, they are closed under the operations of addition, subtraction, and multiplication; add, subtract, and multiply polynomials.

B. Definitions of Cluster and Standard Specifications

The *Item Specifications* identify how the standards in the CCSS are assessed by items in the Florida Interim Assessment Item Bank. For each assessed standard, the following information is provided in the Individual Standards Specifications section.

Domain	refers to larger groups of related standards. Standards from different domains may sometimes be closely related.
Cluster	refers to groups of related standards. Note that standards from different clusters may sometimes be closely related because mathematics is a connected subject.
Standards	define what students should understand and be able to do.
Standards Clarifications/ Content Limits	Standards clarifications, when needed as an explanation for some of the standards listed above, explain the type of behavior that the student should exhibit for mastery of the standard. The clarification statements explain what students are expected to do when responding to the question. Content limits define the range of content knowledge and degree of difficulty that should be assessed in the items for the standard. Content limits may be used to identify content beyond the scope of the targeted standard if the content is more appropriately assessed by another standard. These statements help to provide validity by ensuring the test items are clearly aligned to the targeted standard.

V. Individual Standards Specifications for Florida Interim Assessment Item Bank Mathematics Items

This section of the *Item Specifications* provides standard-specific guidance for assessment item development for the Florida Interim Assessment Item Bank based on the Common Core State Standards.

Each item developed for the Florida Interim Assessment Item Bank and Test Platform should assess one or more of the Mathematical Practice Standards listed in Appendix B.

A. Grade 3 Item Specifications

Domain	OPERATIONS AND ALGEBRAIC THINKING
Cluster	Represent and solve problems involving multiplication and division.
Standards	MACC.3.OA.1.1—Interpret products of whole numbers, e.g., interpret 5×7 as the total number of objects in 5 groups of 7 objects each. *For example, describe a context in which a total number of objects can be expressed as* 5×7. MACC.3.OA.1.2—Interpret whole-number quotients of whole numbers, e.g., interpret $56 \div 8$ as the number of objects in each share when 56 objects are partitioned equally into 8 shares, or as a number of shares when 56 objects are partitioned into equal shares of 8 objects each. *For example, describe a context in which a number of shares or a number of groups can be expressed as* $56 \div 8$. MACC.3.OA.1.3—Use multiplication and division within 100 to solve word problems in situations involving equal groups, arrays, and measurement quantities, e.g., by using drawings and equations with a symbol for the unknown number to represent the problem. MACC.3.OA.1.4—Determine the unknown whole number in a multiplication or division equation relating three whole numbers. *For example, determine the unknown number that makes the equation true in each of the equations* $8 \times ? = 48$, $5 = ? \div 3$, $6 \times 6 = ?$.
Standards Clarifications/ Content Limits	As an explanation for some of the standards listed above, in Grade 3, students will • interpret products and quotients of whole numbers as equal groups and arrays by using drawings and equations with an unknown • use and interpret products and dividends less than or equal to 100 • for MACC.1.OA.1.3, refer to Table 2 in the CCSS-M glossary

Domain	OPERATIONS AND ALGEBRAIC THINKING
Cluster	Understand properties of multiplication and the relationship between multiplication and division.
Standards	MACC.3.OA.2.5—Apply properties of operations as strategies to multiply and divide. *Examples: If $6 \times 4 = 24$ is known, then $4 \times 6 = 24$ is also known. (Commutative property of multiplication.) $3 \times 5 \times 2$ can be found by $3 \times 5 = 15$, then $15 \times 2 = 30$, or by $5 \times 2 = 10$, then $3 \times 10 = 30$. (Associative property of multiplication.) Knowing that $8 \times 5 = 40$ and $8 \times 2 = 16$, one can find 8×7 as $8 \times (5 + 2) = (8 \times 5) + (8 \times 2) = 40 + 16 = 56$. (Distributive property.)* MACC.3.OA.2.6—Understand division as an unknown-factor problem. *For example, find $32 \div 8$ by finding the number that makes 32 when multiplied by 8.*
Standards Clarifications/ Content Limits	As an explanation for some of the standards listed above, in Grade 3, students will • demonstrate an understanding of the inverse relationship of multiplication and division • identify related multiplication/division facts • use properties of operations as a means to determine a solution • not be expected to know formal operation property names

Domain	OPERATIONS AND ALGEBRAIC THINKING
Cluster	**Multiply and divide within 100.**
Standards	MACC.3.OA.3.7—Fluently multiply and divide within 100, using strategies such as the relationship between multiplication and division (e.g., knowing that $8 \times 5 = 40$, one knows $40 \div 5 = 8$) or properties of operations. By the end of Grade 3, know from memory all products of two 1-digit numbers.
Standards Clarifications/ Content Limits	In Grade 3, students will • demonstrate the ability to multiply and divide fluently using a range of strategies, including properties of operations, repeated addition/subtraction, drawings, algorithms, or arrays. Identifying a specific (instructionally sensitive) strategy should not be the explicit target as mastery of individual strategies and their names is not expected. *For example, an item may ask, "Which number sentence could be used to find 4×5?" with a repeated addition number sentence as the correct answer. But an item may not ask, "Which number sentence shows how to solve 4×5 using repeated addition?"* • determine the validity of a variety of proposed methods of solution

Domain	OPERATIONS AND ALGEBRAIC THINKING
Cluster	Solve problems involving the four operations, and identify and explain patterns in arithmetic.
Standards	MACC.3.OA.4.8—Solve two-step word problems using the four operations. Represent these problems using equations with a letter standing for the unknown quantity. Assess the reasonableness of answers using mental computation and estimation strategies including rounding. MACC.3.OA.4.9—Identify arithmetic patterns (including patterns in the addition table or multiplication table), and explain them using properties of operations. *For example, observe that 4 times a number is always even, and explain why 4 times a number can be decomposed into two equal addends.*
Standards Clarifications/ Content Limits	As an explanation for some of the standards listed above, in Grade 3, students will • demonstrate proficiency with addition and subtraction problem solving and the ability to integrate these skills correctly in two-step contextual word problems • interpret a representation of a problem with equations; judgment of the reasonableness of a solution should be the explicit target • perform operations in the conventional order without parentheses to specify a particular order For MACC.3.OA.4.8, refer to Table 2 in the CCSS-M glossary.

Domain	NUMBER AND OPERATIONS IN BASE TEN
Cluster	**Use place value understanding and properties of operations to perform multidigit arithmetic.**
Standards	MACC.3.NBT.1.1—Use place value understanding to round whole numbers to the nearest 10 or 100. MACC.3.NBT.1.2—Fluently add and subtract within 1,000 using strategies and algorithms based on place value, properties of operations, and/or the relationship between addition and subtraction. MACC.3.NBT.1.3—Multiply one-digit whole numbers by multiples of 10 in the range 10−90 (e.g., 9×80, 5×60) using strategies based on place value and properties of operations.
Standards Clarifications/ Content Limits	As an explanation for some of the standards listed above, in Grade 3, students will • demonstrate competency with place value concepts in the context of rounding, multiplication, addition, and subtraction • fluently compute sums and differences • reason about answers; this reasoning should extend beyond conceptually limited tricks such as “just add zeros” for standard MACC.3.NBT.1.3 • reason with or explain the results of rounding

Domain	NUMBER AND OPERATIONS−FRACTIONS
Cluster	Develop understanding of fractions as numbers.
Standards	MACC.3.NF.1.1—Understand a fraction $\frac{1}{b}$ as the quantity formed by 1 part when a whole is partitioned into *b* equal parts; understand a fraction $\frac{a}{b}$ as the quantity formed by *a* parts of size $\frac{1}{b}$. MACC.3.NF.1.2—Understand a fraction as a number on the number line; represent fractions on a number line diagram. MACC.3.NF.1.2.a—Represent a fraction $\frac{1}{b}$ on a number line diagram by defining the interval from 0 to 1 as the whole and partitioning it into *b* equal parts. Recognize that each part has size $\frac{1}{b}$ and that the endpoint of the part based at 0 locates the number $\frac{1}{b}$ on the number line. MACC.3.NF.1.2.b—Represent a fraction $\frac{a}{b}$ on a number line diagram by marking off *a* lengths $\frac{1}{b}$ from 0. Recognize that the resulting interval has size $\frac{a}{b}$ and that its endpoint locates the number $\frac{a}{b}$ on the number line. MACC.3.NF.1.3—Explain equivalence of fractions in special cases, and compare fractions by reasoning about their size. MACC.3.NF.1.3.a—Understand two fractions as equivalent (equal) if they are the same size, or the same point on a number line. MACC.3.NF.1.3.b—Recognize and generate simple equivalent fractions, e.g., $\frac{1}{2} = \frac{2}{4}$, $\frac{4}{6} = \frac{2}{3}$. Explain why the fractions are equivalent, e.g., by using a visual fraction model. MACC.3.NF.1.3.c—Express whole numbers as fractions, and recognize fractions that are equivalent to whole numbers. *Examples: Express 3 in the form* $3 = \frac{3}{1}$*; recognize that* $\frac{6}{1} = 6$*; locate* $\frac{4}{4}$ *and 1 at the same point of a number line diagram.* MACC.3.NF.1.3.d—Compare two fractions with the same numerator or the same denominator by reasoning about their size. Recognize that comparisons are valid only when the two fractions refer to the same whole. Record the results of comparisons with the symbols >, =, or <, and justify the conclusions, e.g., by using a visual fraction model.

Standards Clarifications/ Content Limits	As an explanation for some of the standards listed above, in Grade 3, students will • place fractions on a number line diagram • understand fractions as a component of the number system • understand, generate, and explain simple equivalent fractions • compare fractions by reasoning about their size • be expected to work with fractions with denominators of 2, 3, 4, 6, and 8

Domain	**MEASUREMENT AND DATA**
Cluster	**Solve problems involving measurement and estimation of intervals of time, liquid volumes, and masses of objects.**
Standards	MACC.3.MD.1.1—Tell and write time to the nearest minute and measure time intervals in minutes. Solve word problems involving addition and subtraction of time intervals in minutes, e.g., by representing the problem on a number line diagram. MACC.3.MD.1.2—Measure and estimate liquid volumes and masses of objects using standard units of grams (g), kilograms (kg), and liters (l). Add, subtract, multiply, or divide to solve one-step word problems involving masses or volumes that are given in the same units, e.g., by using drawings (such as a beaker with a measurement scale) to represent the problem.
Standards Clarifications/ Content Limits	As an explanation for some of the standards listed above, in Grade 3, students will • solve simple one-step contextual word problems using whole number measurement quantities • solve simple one-step contextual word problems using time intervals in minutes • not include compound units and geometric volume of shapes

Domain	MEASUREMENT AND DATA
Cluster	Represent and interpret data.
Standards	MACC.3.MD.2.3—Draw a scaled picture graph and a scaled bar graph to represent a data set with several categories. Solve one- and two-step "how many more" and "how many less" problems using information presented in scaled bar graphs. *For example, draw a bar graph in which each square in the bar graph might represent 5 pets.* MACC.3.MD.2.4—Generate measurement data by measuring lengths using rulers marked with halves and fourths of an inch. Show the data by making a line plot, where the horizontal scale is marked off in appropriate units—whole numbers, halves, or quarters.
Standards Clarifications/ Content Limits	As an explanation for some of the standards listed above, in Grade 3, students will • draw scaled picture and bar graphs and answer addition and subtraction problems using information presented in these types of graphs • measure lengths to the nearest half or quarter of an inch using a ruler • show measurement data on a line plot scaled by whole numbers, halves, or quarters

Domain	MEASUREMENT AND DATA
Cluster	Geometric measurement: understand concepts of area and relate area to multiplication and to addition.
Standards	MACC.3.MD.3.5—Recognize area as an attribute of plane figures and understand concepts of area measurement. MACC.3.MD.3.5.a—A square with side length 1 unit, called “a unit square,” is said to have “one square unit” of area, and can be used to measure area. MACC.3.MD.3.5.b—A plane figure which can be covered without gaps or overlaps by *n* unit squares is said to have an area of *n* square units. MACC.3.MD.3.6—Measure areas by counting unit squares (square cm, square m, square in, square ft, and improvised units). MACC.3.MD.3.7—Relate area to the operations of multiplication and addition. MACC.3.MD.3.7.a—Find the area of a rectangle with whole-number side lengths by tiling it, and show that the area is the same as would be found by multiplying the side lengths. MACC.3.MD.3.7.b—Multiply side lengths to find areas of rectangles with whole-number side lengths in the context of solving real world and mathematical problems, and represent whole-number products as rectangular areas in mathematical reasoning. MACC.3.MD.3.7.c—Use tiling to show in a concrete case that the area of a rectangle with whole-number side lengths a and $b + c$ is the sum of $a \times b$ and $a \times c$. Use area models to represent the distributive property in mathematical reasoning. MACC.3.MD.3.7.d—Recognize area as additive. Find areas of rectilinear figures by decomposing them into non-overlapping rectangles and adding the areas of the non-overlapping parts, applying this technique to solve real world problems.

Standards Clarifications/ Content Limits	As an explanation for some of the standards listed above, in Grade 3, students will • develop the concept of area as context for multiplication • for standard MACC.3.MD.3.6, use direct counting, not multiplication, to find the area and express in terms of standard units as well as improvised units such as squares • for standard MACC.3.MD.3.7.d, find areas of rectangular figures by decomposing them into non-overlapping rectangles

Domain	**MEASUREMENT AND DATA**
Cluster	**Geometric measurement: recognize perimeter as an attribute of plane figures and distinguish between linear and area measures.**
Standards	MACC.3.MD.4.8—Solve real world and mathematical problems involving perimeters of polygons, including finding the perimeter given the side lengths, finding an unknown side length, and exhibiting rectangles with the same perimeter and different areas or with the same area and different perimeters.
Standards Clarifications/ Content Limits	In Grade 3, students will • distinguish between linear and area measures by manipulating perimeter in real world and mathematical problems • compare rectangles with the same area but differing side lengths, such as a rectangle of 24 square units with side lengths of 6 by 4 or 8 by 3, as well as the same perimeter with different areas, such as 8 by 3 and 9 by 2.

Domain	GEOMETRY
Cluster	Reason with shapes and their attributes.
Standards	MACC.3.G.1.1—Understand that shapes in different categories (e.g., rhombuses, rectangles, and others) may share attributes (e.g., having four sides), and that the shared attributes can define a larger category (e.g., quadrilaterals). Recognize rhombuses, rectangles, and squares as examples of quadrilaterals, and draw examples of quadrilaterals that do not belong to any of these subcategories. MACC.3.G.1.2—Partition shapes into parts with equal areas. Express the area of each part as a unit fraction of the whole. *For example, partition a shape into 4 parts with equal areas, and describe the area of each part as* $\frac{1}{4}$ *of the area of the shape.*
Standards Clarifications/ Content Limits	As an explanation for some of the standards listed above, in Grade 3, students will • demonstrate an understanding of the hierarchy of quadrilaterals

B. Grade 4 Item Specifications

Domain	OPERATIONS AND ALGEBRAIC THINKING
Cluster	**Use the four operations with whole numbers to solve problems.**
Standards	MACC.4.OA.1.1—Interpret a multiplication equation as a comparison, e.g., interpret 35 = 5 × 7 as a statement that 35 is 5 times as many as 7 and 7 times as many as 5. Represent verbal statements of multiplicative comparisons as multiplication equations. MACC.4.OA.1.2—Multiply or divide to solve word problems involving multiplicative comparison, e.g., by using drawings and equations with a symbol for the unknown number to represent the problem, distinguishing multiplicative comparison from additive comparison. MACC.4.OA.1.3—Solve multistep word problems posed with whole numbers and having whole-number answers using the four operations, including problems in which remainders must be interpreted. Represent these problems using equations with a letter standing for the unknown quantity. Assess the reasonableness of answers using mental computation and estimation strategies including rounding.
Standards Clarifications/ Content Limits	As an explanation for some of the standards listed above, in Grade 4, students will • use the four operations with whole numbers to solve problems • interpret multiplication as comparison • multiply or divide to solve word problems. • for standard MACC.4.OA.1.2, will address problems of a wide variety of types, as summarized in Table 2 of the CCSS-M glossary • solve multistep word problems using all four operations, including interpreting remainders from division problems expressed as whole numbers

Domain	OPERATIONS AND ALGEBRAIC THINKING
Cluster	**Gain familiarity with factors and multiples.**
Standards	MACC.4.OA.2.4—Find all factor pairs for a whole number in the range 1−100. Recognize that a whole number is a multiple of each of its factors. Determine whether a given whole number in the range 1−100 is a multiple of a given one-digit number. Determine whether a given whole number in the range 1−100 is prime or composite.
Standards Clarifications/ Content Limits	In Grade 4, students will • identify factor pairs for whole numbers • determine whether a given whole number is a multiple of a given one-digit number • determine whether a given whole number is prime or composite

Domain	OPERATIONS AND ALGEBRAIC THINKING
Cluster	**Generate and analyze patterns.**
Standards	MACC.4.OA.3.5—Generate a number or shape pattern that follows a given rule. Identify apparent features of the pattern that were not explicit in the rule itself. *For example, given the rule “Add 3” and the starting number 1, generate terms in the resulting sequence and observe that the terms appear to alternate between odd and even numbers. Explain informally why the numbers will continue to alternate in this way.*
Standards Clarifications/ Content Limits	In Grade 4, students will • generate number or shape patterns that follow a rule • identify features of the pattern that were not explicit in the rule itself

Domain	NUMBER AND OPERATIONS IN BASE TEN
Cluster	**Generalize place value understanding for multidigit whole numbers.**
Standards	MACC.4.NBT.1.1—Recognize that in a multidigit whole number, a digit in one place represents ten times what it represents in the place to its right. *For example, recognize that 700 ÷ 70 = 10 by applying concepts of place value and division.* MACC.4.NBT.1.2—Read and write multidigit whole numbers using base-ten numerals, number names, and expanded form. Compare two multidigit numbers based on meanings of the digits in each place, using >, =, and < symbols to record the results of comparisons. MACC.4.NBT.1.3—Use place value understanding to round multidigit whole numbers to any place.
Standards Clarifications/ Content Limits	As an explanation for some of the standards listed above, in Grade 4, students will • show place value understanding for multidigit whole numbers • compare the value of digits in a multidigit whole number • read and write multidigit whole numbers • compare two multidigit numbers • use place value understanding to round multidigit numbers to any place

Domain	NUMBER AND OPERATIONS IN BASE TEN
Cluster	Use place value understanding and properties of operations to perform multidigit arithmetic.
Standards	MACC.4.NBT.2.4—Fluently add and subtract multidigit whole numbers using the standard algorithm. MACC.4.NBT.2.5—Multiply a whole number of up to four digits by a one-digit whole number, and multiply two two-digit numbers, using strategies based on place value and the properties of operations. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models. MACC.4.NBT.2.6—Find whole-number quotients and remainders with up to four-digit dividends and one-digit divisors, using strategies based on place value, the properties of operations, and/or the relationship between multiplication and division. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models.
Standards Clarifications/ Content Limits	As an explanation for some of the standards listed above, in Grade 4, students will • add and subtract multidigit whole numbers using the standard algorithm • multiply two multidigit whole numbers and illustrate and explain the calculations • find whole number quotients and remainders and explain the calculations of up to four-digit dividends that may be divided by one-digit divisors

Domain	NUMBER AND OPERATIONS–FRACTIONS
Cluster	**Extend understanding of fraction equivalence and ordering.**
Standards	MACC.4.NF.1.1—Explain why a fraction $\frac{a}{b}$ is equivalent to a fraction $\frac{(n \times a)}{(n \times b)}$ by using visual fraction models, with attention to how the number and size of the parts differ even though the two fractions themselves are the same size. Use this principle to recognize and generate equivalent fractions. MACC.4.NF.1.2—Compare two fractions with different numerators and different denominators, e.g., by creating common denominators or numerators or by comparing to a benchmark fraction such as $\frac{1}{2}$. Recognize that comparisons are valid only when the two fractions refer to the same whole. Record the results of comparisons with symbols >, =, or <, and justify the conclusions, e.g., by using a visual fraction model.
Standards Clarifications/ Content Limits	As an explanation for some of the standards listed above, in Grade 4, students will • use visual fraction models to explain why a fraction $\frac{a}{b}$ is equivalent to a fraction $\frac{(n \times a)}{(n \times b)}$ • compare two fractions with different numerators and different denominators

Domain	NUMBER AND OPERATIONS–FRACTIONS
Cluster	**Build fractions from unit fractions by applying and extending previous understandings of operations on whole numbers.**
Standards	MACC.4.NF.2.3—Understand a fraction $\frac{a}{b}$ with $a > 1$ as a sum of fractions $\frac{1}{b}$. MACC.4.NF.2.3.a—Understand addition and subtraction of fractions as joining and separating parts referring to the same whole. MACC.4.NF.2.3.b—Decompose a fraction into a sum of fractions with the same denominator in more than one way, recording each decomposition by an equation. Justify decompositions, e.g., by using a visual fraction model. *Examples:* $\frac{3}{8} = \frac{1}{8} + \frac{1}{8} + \frac{1}{8}$; $\frac{3}{8} = \frac{1}{8} + \frac{2}{8}$; $2\frac{1}{8} = 1 + 1 + \frac{1}{8} = \frac{8}{8} + \frac{8}{8} + \frac{1}{8}$. MACC.4.NF.2.3.c—Add and subtract mixed numbers with like denominators, e.g., by replacing each mixed number with an equivalent fraction, and/or by using properties of operations and the relationship between addition and subtraction. MACC.4.NF.2.3.d—Solve word problems involving addition and subtraction of fractions referring to the same whole and having like denominators, e.g., by using visual fraction models and equations to represent the problem. MACC.4.NF.2.4—Apply and extend previous understandings of multiplication to multiply a fraction by a whole number. MACC.4.NF.2.4.a—Understand a fraction $\frac{a}{b}$ as a multiple of $\frac{1}{b}$. *For example, use a visual fraction model to represent* $\frac{5}{4}$ *as the product* $5 \times (\frac{1}{4})$, *recording the conclusion by the equation* $\frac{5}{4} = 5 \times (\frac{1}{4})$. MACC.4.NF.2.4.b—Understand a multiple of $\frac{a}{b}$ as a multiple of $\frac{1}{b}$, and use this understanding to multiply a fraction by a whole number. *For example, use a visual fraction model to express* $3 \times (\frac{2}{5})$ *as* $6 \times (\frac{1}{5})$, *recognizing this product as* $\frac{6}{5}$. *(In general,* $n \times (\frac{a}{b}) = \frac{(n \times a)}{b}$.*)* MACC.4.NF.2.4.c—Solve word problems involving multiplication of a fraction by a whole number, e.g., by using visual fraction models and equations to represent the problem. *For example, if each person at a party will eat* $\frac{3}{8}$ *of a pound of roast beef, and there will be 5 people at the party, how many pounds of roast beef will be needed? Between what two whole numbers does your answer lie?*

Standards Clarifications/ Content Limits	As an explanation for some of the standards listed above, in Grade 4, students will • understand a fraction with $\frac{a}{b}$ with $a > 1$ as a sum of fractions $\frac{1}{b}$. This includes decomposing a fraction into a sum of fractions with the same denominator, adding and subtracting mixed numbers with like denominators, and solving word problems involving addition and subtraction of fractions having the same denominator. • extend previous understandings of multiplication to multiply a fraction by a whole number. This includes understanding a fraction $\frac{a}{b}$ as a multiple of $\frac{1}{b}$ and using this understanding to multiply a fraction by a whole number, as well as solving word problems involving multiplication of a fraction by a whole number.

Domain	NUMBER AND OPERATIONS–FRACTIONS
Cluster	Understand decimal notation for fractions, and compare decimal fractions.
Standards	MACC.4.NF.3.5—Express a fraction with denominator 10 as an equivalent fraction with denominator 100, and use this technique to add two fractions with respective denominators 10 and 100. *For example, express* $\frac{3}{10}$ *as* $\frac{30}{100}$, *and add* $\frac{3}{10} + \frac{4}{100} = \frac{34}{100}$. MACC.4.NF.3.6—Use decimal notation for fractions with denominators 10 or 100. *For example, rewrite 0.62 as* $\frac{62}{100}$; *describe a length as 0.62 meters; locate 0.62 on a number line diagram.* MACC.4.NF.3.7—Compare two decimals to hundredths by reasoning about their size. Recognize that comparisons are valid only when the two decimals refer to the same whole. Record the results of comparisons with the symbols >, =, or <, and justify the conclusions, e.g., by using a visual model.
Standards Clarifications/ Content Limits	As an explanation for some of the standards listed above, in Grade 4, students will • add two fractions with denominators 10 and 100 after rewriting with like denominators of 100 • rewrite decimal fractions as decimals • compare two decimals to hundredths when they refer to the same whole

Domain	MEASUREMENT AND DATA
Cluster	Solve problems involving measurement and conversion of measurements from a larger unit to a smaller unit.
Standards	MACC.4.MD.1.1—Know relative sizes of measurement units within one system of units including km, m, cm; kg, g; lb, oz.; l, ml; hr, min, sec. Within a single system of measurement, express measurements in a larger unit in terms of a smaller unit. Record measurement equivalents in a two-column table. *For example, know that 1 ft is 12 times as long as 1 in. Express the length of a 4 ft snake as 48 in. Generate a conversion table for feet and inches listing the number pairs (1, 12), (2, 24), (3, 36).* MACC.4.MD.1.2—Use the four operations to solve word problems involving distances, intervals of time, liquid volumes, masses of objects, and money, including problems involving simple fractions or decimals, and problems that require expressing measurements given in a larger unit in terms of a smaller unit. Represent measurement quantities using diagrams such as number line diagrams that feature a measurement scale. MACC.4.MD.1.3—Apply the area and perimeter formulas for rectangles in real world and mathematical problems. *For example, find the width of a rectangular room given the area of the flooring and the length by viewing the area formula as a multiplication equation with an unknown factor.*
Standards Clarifications/ Content Limits	As an explanation for some of the standards listed above, in Grade 4, students will • use the four operations to solve problems involving measurements in real world and mathematical problems • apply the area and perimeter formulas to problems involving rectangles • convert units in problems that are limited to kilometers, meters, centimeters, kilograms, grams, pounds, ounces, liters, milliliters, hours, minutes, and seconds • express larger units in terms of smaller units within the same system

Domain	MEASUREMENT AND DATA
Cluster	**Represent and interpret data.**
Standards	MACC.4.MD.2.4—Make a line plot to display a data set of measurements in fractions of a unit ($\frac{1}{2}$, $\frac{1}{4}$, $\frac{1}{8}$). Solve problems involving addition and subtraction of fractions by using information presented in line plots. *For example, from a line plot find and interpret the difference in length between the longest and shortest specimens in an insect collection.*
Standards Clarifications/ Content Limits	In Grade 4, students will • make a line plot to display data in fractions of a unit • use this plot to solve real world problems involving addition and subtraction of fractions

Domain	MEASUREMENT AND DATA
Cluster	**Geometric measurement: understand concepts of angle and measure angles.**
Standards	MACC.4.MD.3.5—Recognize angles as geometric shapes that are formed wherever two rays share a common endpoint, and understand concepts of angle measurement: MACC.4.MD.3.5.a—An angle is measured with reference to a circle with its center at the common endpoint of the rays, by considering the fraction of the circular arc between the points where the two rays intersect the circle. An angle that turns through $\frac{1}{360}$ of a circle is called a "one-degree angle," and can be used to measure angles. MACC.4.MD.3.5.b—An angle that turns through *n* one-degree angles is said to have an angle measure of *n* degrees. MACC.4.MD.3.6—Measure angles in whole-number degrees using a protractor. Sketch angles of specified measure. MACC.4.MD.3.7—Recognize angle measure as additive. When an angle is decomposed into non-overlapping parts, the angle measure of the whole is the sum of the angle measures of the parts. Solve addition and subtraction problems to find unknown angles on a diagram in real world and mathematical problems, e.g., by using an equation with a symbol for the unknown angle measure.

Standards Clarifications/ Content Limits	As an explanation for some of the standards listed above, in Grade 4, students will • measure angles in whole-number degrees using a protractor • solve addition and subtraction problems to find unknown angles on a diagram

Domain	**GEOMETRY**
Cluster	**Draw and identify lines and angles and classify shapes by properties of their lines and angles.**
Standards	MACC.4.G.1.1—Draw points, lines, line segments, rays, angles (right, acute, obtuse), and perpendicular and parallel lines. Identify these in two-dimensional figures. MACC.4.G.1.2—Classify two-dimensional figures based on the presence or absence of parallel or perpendicular lines, or the presence or absence of angles of a specified size. Recognize right triangles as a category, and identify right triangles. MACC.4.G.1.3—Recognize a line of symmetry for a two-dimensional figure as a line across the figure such that the figure can be folded along the line into matching parts. Identify line-symmetric figures and draw lines of symmetry.
Standards Clarifications/ Content Limits	As an explanation for some of the standards listed above, in Grade 4, students will • identify types of lines and angles in two-dimensional figures • classify two-dimensional figures based on the types of lines and angles they have • identify lines of symmetry and line-symmetric figures

C. Grade 5 Item Specifications

Domain	**OPERATIONS AND ALGEBRAIC THINKING**
Cluster	**Write and interpret numerical expressions.**
Standards	MACC.5.OA.1.1—Use parentheses, brackets, or braces in numerical expressions, and evaluate expressions with these symbols. MACC.5.OA.1.2—Write simple expressions that record calculations with numbers, and interpret numerical expressions without evaluating them. *For example, express the calculation "add 8 and 7, then multiply by 2" as* $2 \times (8 + 7)$. *Recognize that* $3 \times (18932 + 921)$ *is three times as large as* $18932 + 921$, *without having to calculate the indicated sum or product.*
Standards Clarifications/ Content Limits	As an explanation for some of the standards listed above, in Grade 5, students will • simplify and write numerical expressions that may include parentheses, brackets, and/or braces • write and interpret simple numerical expressions without evaluating them

Domain	**OPERATIONS AND ALGEBRAIC THINKING**
Cluster	**Analyze patterns and relationships.**
Standards	MACC.5.OA.2.3—Generate two numerical patterns using two given rules. Identify apparent relationships between corresponding terms. Form ordered pairs consisting of corresponding terms from the two patterns, and graph the ordered pairs on a coordinate plane. *For example, given the rule "Add 3" and the starting number 0, and given the rule "Add 6" and the starting number 0, generate terms in the resulting sequences, and observe that the terms in one sequence are twice the corresponding terms in the other sequence. Explain informally why this is so.*
Standards Clarifications/ Content Limits	In Grade 5, students will • generate number patterns using two given rules • identify apparent relationships between corresponding terms from the two patterns • form ordered pairs from the patterns and graph these ordered pairs on the coordinate plane

Domain	NUMBER AND OPERATIONS IN BASE TEN
Cluster	Understand the place value system.
Standards	MACC.5.NBT.1.1—Recognize that in a multidigit number, a digit in one place represents 10 times as much as it represents in the place to its right and $\frac{1}{10}$ of what it represents in the place to its left. MACC.5.NBT.1.2—Explain patterns in the number of zeros of the product when multiplying a number by powers of 10, and explain patterns in the placement of the decimal point when a decimal is multiplied or divided by a power of 10. Use whole-number exponents to denote powers of 10. MACC.5.NBT.1.3—Read, write, and compare decimals to thousandths. MACC.5.NBT.1.3.a—Read and write decimals to thousandths using base-ten numerals, number names, and expanded form, e.g., $347.392 = 3 \times 100 + 4 \times 10 + 7 \times 1 + 3 \times (\frac{1}{10}) + 9 \times (\frac{1}{100}) + 2 \times (\frac{1}{1000})$. MACC.5.NBT.1.3.b—Compare two decimals to thousandths based on meanings of the digits in each place, using >, =, and < symbols to record the results of comparisons. MACC.5.NBT.1.4—Use place value understanding to round decimals to any place.
Standards Clarifications/ Content Limits	As an explanation for some of the standards listed above, in Grade 5, students will • demonstrate competency with place value concepts in the context of rounding and explaining patterns involving powers of 10 • be able to round decimals to emphasize conceptual understanding • use the meaning of the digits in each place to compare two decimals and to read and write decimals

Domain	NUMBER AND OPERATIONS IN BASE TEN
Cluster	Perform operations with multidigit whole numbers and with decimals to hundredths.
Standards	MACC.5.NBT.2.5—Fluently multiply multidigit whole numbers using the standard algorithm. MACC.5.NBT.2.6—Find whole-number quotients of whole numbers with up to four-digit dividends and two-digit divisors, using strategies based on place value, the properties of operations, and/or the relationship between multiplication and division. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models. MACC.5.NBT.2.7—Add, subtract, multiply, and divide decimals to hundredths, using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; relate the strategy to a written method and explain the reasoning used.
Standards Clarifications/ Content Limits	As an explanation for some of the standards listed above, in Grade 5, students will • multiply multidigit whole numbers using the standard algorithm • find whole-number quotients with up to four-digit dividends and two-digit divisors and illustrate and explain the calculations • add, subtract, multiply, and divide decimals to hundredths, using concrete models and drawings as well as the properties of operations and their understanding of place value

Domain	NUMBER AND OPERATIONS–FRACTIONS
Cluster	Use equivalent fractions as a strategy to add and subtract fractions.
Standards	MACC.5.NF.1.1—Add and subtract fractions with unlike denominators (including mixed numbers) by replacing given fractions with equivalent fractions in such a way as to produce an equivalent sum or difference of fractions with like denominators. *For example,* $\frac{2}{3} + \frac{5}{4} = \frac{8}{12} + \frac{15}{12} = \frac{23}{12}$. *(In general,* $\frac{a}{b} + \frac{c}{d} = \frac{(ad + bc)}{bd}$.*)* MACC.5.NF.1.2—Solve word problems involving addition and subtraction of fractions referring to the same whole, including cases of unlike denominators, e.g., by using visual fraction models or equations to represent the problem. Use benchmark fractions and number sense of fractions to estimate mentally and assess the reasonableness of answers. *For example, recognize an incorrect result* $\frac{2}{5} + \frac{1}{2} = \frac{3}{7}$, *by observing that* $\frac{3}{7} < \frac{1}{2}$.
Standards Clarifications/ Content Limits	As an explanation for some of the standards listed above, in Grade 5, students will • add and subtract fractions with unlike denominators • solve word problems involving fractions by using visual models or equations • use benchmark fractions and number sense of fractions to estimate and assess reasonableness of answers • Subtrahends may not be greater than the minuends and cannot be a negative number.

Domain	NUMBER AND OPERATIONS–FRACTIONS
Cluster	**Apply and extend previous understandings of multiplication and division to multiply and divide fractions.**
Standards	MACC.5.NF.2.3—Interpret a fraction as division of the numerator by the denominator ($\frac{a}{b} = a \div b$). Solve word problems involving division of whole numbers leading to answers in the form of fractions or mixed numbers, e.g., by using visual fraction models or equations to represent the problem. *For example, interpret $\frac{3}{4}$ as the result of dividing 3 by 4, noting that $\frac{3}{4}$ multiplied by 4 equals 3, and that when 3 wholes are shared equally among 4 people each person has a share of size $\frac{3}{4}$. If 9 people want to share a 50-pound sack of rice equally by weight, how many pounds of rice should each person get? Between what two whole numbers does your answer lie?* MACC.5.NF.2.4—Apply and extend previous understandings of multiplication to multiply a fraction or whole number by a fraction. MACC.5.NF.2.4.a—Interpret the product $(\frac{a}{b}) \times q$ as a parts of a partition of q into b equal parts; equivalently, as the result of a sequence of operations $a \times q \div b$. *For example, use a visual fraction model to show $(\frac{2}{3}) \times 4 = \frac{8}{3}$, and create a story context for this equation. Do the same with $(\frac{2}{3}) \times (\frac{4}{5}) = \frac{8}{15}$. (In general, $(\frac{a}{b}) \times (\frac{c}{d}) = \frac{ac}{bd}$.)* MACC.5.NF.2.4.b—Find the area of a rectangle with fractional side lengths by tiling it with unit squares of the appropriate unit fraction side lengths, and show that the area is the same as would be found by multiplying the side lengths. Multiply fractional side lengths to find areas of rectangles, and represent fraction products as rectangular areas. MACC.5.NF.2.5—Interpret multiplication as scaling (resizing), by: MACC.5.NF.2.5.a—Comparing the size of a product to the size of one factor on the basis of the size of the other factor, without performing the indicated multiplication.

	MACC.5.NF.2.5.b—Explaining why multiplying a given number by a fraction greater than 1 results in a product greater than the given number (recognizing multiplication by whole numbers greater than 1 as a familiar case); explaining why multiplying a given number by a fraction less than 1 results in a product smaller than the given number; and relating the principle of fraction equivalence $\frac{a}{b} = \frac{(n \times a)}{(n \times b)}$ to the effect of multiplying $\frac{a}{b}$ by 1. MACC.5.NF.2.6—Solve real world problems involving multiplication of fractions and mixed numbers, e.g., by using visual fraction models or equations to represent the problem. MACC.5.NF.2.7—Apply and extend previous understandings of division to divide unit fractions by whole numbers and whole numbers by unit fractions. MACC.5.NF.2.7.a—Interpret division of a unit fraction by a non-zero whole number, and compute such quotients. *For example, create a story context for* $(\frac{1}{3}) \div 4$*, and use a visual fraction model to show the quotient. Use the relationship between multiplication and division to explain that* $(\frac{1}{3}) \div 4 = \frac{1}{12}$ *because* $(\frac{1}{12}) \times 4 = \frac{1}{3}$. MACC.5.NF.2.7.b—Interpret division of a whole number by a unit fraction, and compute such quotients. *For example, create a story context for* $4 \div (\frac{1}{5})$*, and use a visual fraction model to show the quotient. Use the relationship between multiplication and division to explain that* $4 \div (\frac{1}{5}) = 20$ *because* $20 \times (\frac{1}{5}) = 4$. MACC.5.NF.2.7.c—Solve real world problems involving division of unit fractions by non-zero whole numbers and division of whole numbers by unit fractions, e.g., by using visual fraction models and equations to represent the problem. *For example, how much chocolate will each person get if 3 people share* $\frac{1}{2}$ *lb of chocolate equally? How many* $\frac{1}{3}$*-cup servings are in 2 cups of raisins?*

Standards Clarifications/ Content Limits	As an explanation for some of the standards listed above, in Grade 5, students will • apply and extend previous understandings of multiplication and division to problems involving fractions • use fraction models and equations to solve and interpret real world problems involving fractions • find the area of rectangles with fractional side lengths and interpret multiplying by a fraction in the context of scaling (resizing) the product

Domain	**MEASUREMENT AND DATA**
Cluster	**Convert like measurement units within a given measurement system.**
Standards	MACC.5.MD.1.1—Convert among different-sized standard measurement units within a given measurement system (e.g., convert 5 cm to 0.05 m), and use these conversions in solving multi-step, real world problems.
Standards Clarifications/ Content Limits	In Grade 5, students will • become fluent with metric and customary units within a system measuring linear, mass, weight, and time

Domain	**MEASUREMENT AND DATA**
Cluster	**Represent and interpret data.**
Standards	MACC.5.MD.2.2—Make a line plot to display a data set of measurements in fractions of a unit ($\frac{1}{2}$, $\frac{1}{4}$, $\frac{1}{8}$). Use operations on fractions for this grade to solve problems involving information presented in line plots. *For example, given different measurements of liquid in identical beakers, find the amount of liquid each beaker would contain if the total amount in all the beakers were redistributed equally.*
Standards Clarifications/ Content Limits	In Grade 5, students will • be able to make a line plot to display a data set of measurements • create a scale for the line plot using simple fractional units, such as $\frac{1}{2}$, $\frac{1}{4}$, or $\frac{1}{8}$ • be able to use addition and subtraction of fractions to solve problems involving information presented in these line plots

Domain	MEASUREMENT AND DATA
Cluster	Geometric measurement: understand concepts of volume and relate volume to multiplication and to addition.
Standards	MACC.5.MD.3.3—Recognize volume as an attribute of solid figures and understand concepts of volume measurement. MACC.5.MD.3.3.a—A cube with side length 1 unit, called a "unit cube," is said to have "one cubic unit" of volume, and can be used to measure volume. MACC.5.MD.3.3.b—A solid figure which can be packed without gaps or overlaps using *n* unit cubes is said to have a volume of *n* cubic units. MACC.5.MD.3.4—Measure volumes by counting unit cubes, using cubic cm, cubic in, cubic ft, and improvised units. MACC.5.MD.3.5—Relate volume to the operations of multiplication and addition and solve real world and mathematical problems involving volume. MACC.5.MD.3.5.a—Find the volume of a right rectangular prism with whole-number side lengths by packing it with unit cubes, and show that the volume is the same as would be found by multiplying the edge lengths, equivalently by multiplying the height by the area of the base. Represent threefold whole-number products as volumes, e.g., to represent the associative property of multiplication. MACC.5.MD.3.5.b—Apply the formulas $V = l \times w \times h$ and $V = b \times h$ for rectangular prisms to find volumes of right rectangular prisms with whole-number edge lengths in the context of solving real world and mathematical problems. MACC.5.MD.3.5.c—Recognize volume as additive. Find volumes of solid figures composed of two non-overlapping right rectangular prisms by adding the volumes of the non-overlapping parts, applying this technique to solve real world problems.
Standards Clarifications/ Content Limits	As an explanation for some of the standards listed above, in Grade 5, students will • be able to solve various real world and mathematical volume problems, including using both models and formulas to find the volume of simple solid figures

Domain	GEOMETRY
Cluster	**Graph points on the coordinate plane to solve real-world and mathematical problems.**
Standards	MACC.5.G.1.1—Use a pair of perpendicular number lines, called axes, to define a coordinate system, with the intersection of the lines (the origin) arranged to coincide with the 0 on each line and a given point in the plane located by using an ordered pair of numbers, called its coordinates. Understand that the first number indicates how far to travel from the origin in the direction of one axis, and the second number indicates how far to travel in the direction of the second axis, with the convention that the names of the two axes and the coordinates correspond (e.g., x-axis and x-coordinate, y-axis and y-coordinate). MACC.5.G.1.2—Represent real world and mathematical problems by graphing points in the first quadrant of the coordinate plane, and interpret coordinate values of points in the context of the situation.
Standards Clarifications/ Content Limits	As an explanation for some of the standards listed above, in Grade 5, students will • understand and use the components of the coordinate system, graphing points in the first quadrant and interpreting coordinate values of points in the context of the situation

Domain	GEOMETRY
Cluster	**Classify two-dimensional figures into categories based on their properties.**
Standards	MACC.5.G.2.3—Understand that attributes belonging to a category of two-dimensional figures also belong to all subcategories of that category. *For example, all rectangles have four right angles and squares are rectangles, so all squares have four right angles.* MACC.5.G.2.4—Classify two-dimensional figures in a hierarchy based on properties.
Standards Clarifications/ Content Limits	As an explanation for some of the standards listed above, in Grade 5, students will • understand the attributes of different categories and subcategories of two-dimensional figures • classify these figures by their attributes

Appendices

Appendix A

Sample Items

Grade: 5

Item Type: Selected Response

Correct Answer: C

Possible Points: 1

DOK: 3

Calculator Usage: No

CCSS Standard:

MACC.5.G.2.4—Classify two-dimensional figures in a hierarchy based on properties.

Standards for Mathematical Practice:

1. Make sense of problems and persevere in solving them.

Luke and Riley are drawing shapes. Luke's shape has 4 sides of equal length and 4 right angles. Riley's shape has 4 right angles and 2 pairs of parallel sides. The lengths of the 4 sides in Riley's shape are not congruent, but the opposite sides are congruent. Which statement about the boys' shapes is true?

A. They are both pentagons.

B. They are both rhombuses.

* **C.** They are both rectangles.

D. They are both squares.

Distractor Rationales

A. Pentagons have five sides instead of four.

B. Rhombuses have four congruent sides.

C. Correct answer

D. All squares are rectangles, but not all rectangles are squares.

Grade: 4

Item Type: Gridded Response

Correct Answer: 912

Possible Points: 1

DOK: 2

Calculator Usage: No

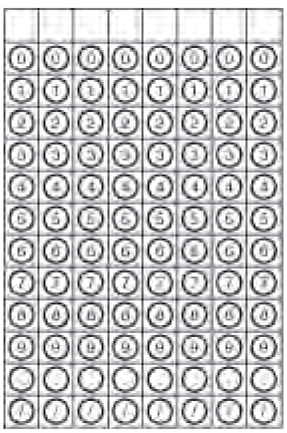
CCSS Standard:

MACC.4.NBT.2.5—Multiply a whole number of up to four digits by a one-digit whole number, and multiply two two-digit numbers, using strategies based on place value and the properties of operations. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models.

Standard for Mathematical Practice:

1. Make sense of problems and persevere in solving them.

The owner of a shoe store placed 6 stacks of boxes on each of her shelves. Each stack contained 4 boxes. She continued stacking boxes in the same way until 38 shelves were full. How many boxes of shoes did the owner stack?



Grade: 3

Item Type: Short Answer

Correct Answer: $6 \times 7 = 42$ or equivalent equation

Possible Points: 1

DOK: 2

Calculator Usage: No

CCSS Standard:

MACC.3.OA.1.3—Use multiplication and division within 100 to solve word problems in situations involving equal groups, arrays, and measurement quantities, e.g., by using drawings and equations with a symbol for the unknown number to represent the problem.

Standards for Mathematical Practice:

2. Reason abstractly and quantitatively.

Kayla has 6 bags of basketballs. Each bag has 7 basketballs in it. Write a multiplication or division equation to show how many basketballs she has in all.

____

Grade: 4

Item Type: Constructed Response

Correct Answer: See Scoring Exemplar

Possible Points: 2

DOK: 3

Calculator Usage: No

CCSS Standard:

MACC.4.MD.1.2—Use the four operations to solve word problems involving distances, intervals of time, liquid volumes, masses of objects, and money, including problems involving simple fractions or decimals, and problems that require expressing measurements given in a larger unit in terms of a smaller unit. Represent measurement quantities using diagrams such as number line diagrams that feature a measurement scale.

Standards for Mathematical Practice:

1. Make sense of problems and persevere in solving them.

A sign at Ted's Repair Shop lists the cost of a hammer at \$4.50, a box of nails at \$2.35, and a screwdriver at \$3.55. Pam used all of the money shown below to buy two of the items.

Part A. Which two items did Pam buy if she used all of her money?

Part B. Show or explain how you know that is what she bought.

Use words, numbers, and/or pictures to show your work.

SCORING RUBRIC	
2	Work demonstrates a **clear and complete** understanding of the mathematical concepts and/or procedures required by the task. Appropriate strategy is shown with clear and complete explanations and interpretations.
1	Response demonstrates a **partial** understanding of the mathematical concepts and/or procedures. Appropriate strategy is shown, but explanation or interpretation has minor flaws. OR Response is incorrect because of calculation errors. Work and strategy indicate a **clear** understanding of the mathematical concepts and/or procedures required by the task.
0	Response is irrelevant, inappropriate, or not provided.

SCORING EXEMPLAR
Maximum Points—2 **Part A—1 point** a hammer and a screwdriver **Part B—1 point** Pam can pay for the hammer with four 1-dollar bills and two quarters. Then she can use two 1-dollar bills and four quarters to make \$3.00 and the rest of the change to make \$0.55. or equivalent work

Grade: 3

Item Type: Extended Response

Correct Answer: See Scoring Exemplar

Possible Points: 4

DOK: 3

Calculator Usage: No

CCSS Standard:

MACC.3.NF.1.3—Explain equivalence of fractions in special cases, and compare fractions by reasoning about their size.

MACC.3.NF.1.3d—Compare two fractions with the same numerator or the same denominator by reasoning about their size. Recognize that comparisons are valid only when the two fractions refer to the same whole. Record the results of comparisons with the symbols >, =, or <, and justify the conclusions, e.g., by using a visual fraction model.

Standards for Mathematical Practice:

2. Reason abstractly and quantitatively.

4. Model with mathematics.

Henry, Oliver, and Jack were painting a fence. They had two buckets that were the same size. Henry filled $\frac{3}{4}$ of one bucket with paint. Oliver filled $\frac{3}{3}$ of the other bucket with paint.

Part A. Draw a model to compare $\frac{3}{4}$ and $\frac{3}{3}$.

Part B. Write a number sentence using <, >, or = to compare $\frac{3}{4}$ and $\frac{3}{3}$.

The boys found another bucket. Jack filled $\frac{3}{4}$ of this bucket with paint. Jack had more paint in his bucket than Henry had in his.

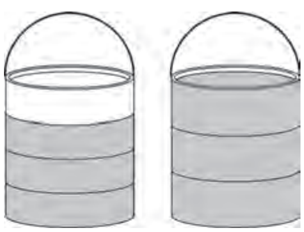
Part C. Explain how this could be true.

SCORING RUBRIC	
4	Work demonstrates a **clear and complete** understanding of the mathematical concepts and/or procedures required by the task. Appropriate strategy is shown with clear and complete explanations and interpretations.
3	Work demonstrates a **clear** understanding of the mathematical concepts and/or procedures but is not complete. Appropriate strategy is shown, but explanation or interpretation has minor flaws. OR Response is incorrect because of calculation errors. Work and strategy indicate a **clear** demonstration of the problem.
2	Response demonstrates a **partial** understanding of the mathematical concepts and/or procedures. Appropriate strategy is shown, but explanation or interpretation has minor flaws.
1	Response shows **minimal** understanding of the mathematical concepts and/or procedures or provides no explanation or interpretation for the solution or shows major flaws.
0	Response is irrelevant, inappropriate, or not provided.

SCORING EXEMPLAR

Maximum Points—4

Part A—1 point



Part B—1 point

$$\frac{3}{4} < \frac{3}{3}$$

Part C—2 points

Jack's bucket must be bigger than Henry's. $\frac{3}{4}$ is equal to $\frac{3}{4}$, but only if the two wholes are the same size.

or equivalent work

Grade: 5

Item Type: Performance Task

Correct Answer: See Scoring Exemplar

Possible Points: 6

DOK: 4

Calculator Usage: No

CCS Standards:

MACC.5.OA.2.3—Generate two numerical patterns using two given rules. Identify apparent relationships between corresponding terms. Form ordered pairs consisting of corresponding terms from the two patterns, and graph the ordered pairs on a coordinate plane. *For example, given the rule "Add 3" and the starting number 0, and given the rule "Add 6" and the starting number 0, generate terms in the resulting sequences, and observe that the terms in one sequence are twice the corresponding terms in the other sequence. Explain informally why this is so.*

MACC.5.NBT.2.7—Add, subtract, multiply, and divide decimals to hundredths, using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; relate the strategy to a written method and explain the reasoning used.

Standards for Mathematical Practice:

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.

Copy Center

Teacher Directions:

Before administration, discuss various types of graphical representations that students might use to display mathematical relationships.

Instruct students to use words, numbers, and/or pictures to show their work.

Allow 30 to 40 minutes for this task.

This task can be modified by using different multiples of each independent variable (prices of copies) as long as the relationship between both variables remains the same.

Make all necessary materials available.

Suggested Materials: Paper, graphing paper, pencils

TASK:

A copy center offers black-and-white and color copies. Each black-and-white copy costs $0.06, and each color copy costs $0.18.

Part A. Create a graphic representation showing costs of copies made for both the black-and-white and color copies starting from 1 copy to 8 copies. What is the relationship between the black-and-white and color copy costs?

Part B. Cathy needs the same number of color copies and black-and-white copies. The color copies will cost $1.20 more than the black-and-white copies. How many of each kind of copy is Cathy having made? Use words, numbers, and/or pictures to show your work.

Part C. Sean has $1.50 to spend making copies at the copy center. He needs more black-and-white copies than colored copies. What is a combination of black-and-white and color copies that will total exactly $1.50? Write an equation to show how you found your answer. Use words, numbers, and/or pictures to show your work.

SCORING EXEMPLAR

Maximum Points—6

Part A—2 points

- A table or any other type of graphical representation is used to correctly show the costs of copies made for both the black-and-white and color copies, starting from 1 copy to 8 copies. Student indicates the relationship between color copies and black and white is that color copies cost 3 times as much as black-and-white copies. The table should include headers and title.

  For example:

COPY CENTER COSTS

Number of Copies	Black and White Add $0.06	Color Add $0.18
1	$0.06	$0.18
2	$0.12	$0.36
3	$0.18	$0.54
4	$0.24	$0.72
5	$0.30	$0.90
6	$0.36	$1.08
7	$0.42	$1.26
8	$0.48	$1.44

Other appropriate equations or strategies may earn full points.

Part B—2 points

- The response indicates that 10 copies of each are made, and an appropriate explanation is given. Explanations might include the following:

  "I know that at 5 copies each, the difference between $0.90 and $0.30 is $0.60. And $0.60 is half of $1.20, so I multiplied 5 and 2 to get 10 copies of each."

OR

"Using the information in the table, at 8 copies each: $1.44 – $0.48 = $0.96, which is close to $1.20, I continued the table until I found 10 copies of each make a difference of $1.20."

Part C—2 points

- The response indicates that 10 black-and-white copies and 5 color copies are made and an appropriate explanation is given. Explanations might include the following.

  "(10 x $0.06) + (5 x $0.18) = $1.50. I know that 5 color copies cost $0.90 using the table. I subtracted $0.90 from $1.50 and had $0.60 left over. I know that the cost of 5 black-and-white copies is $0.30, which is half of $0.60, so I doubled the 5 black-and-white copies to 10 copies to make the total amount."

OR

"16 black-and-white copies and 3 color copies. (16 x 0.06) + (3 x 0.54) = $1.50. I subtracted 0.54 (the amount for 3 color copies) from 1.50 and got a difference of 0.96. I divided 0.96 by 0.06 to see whether it was evenly divisible. It was divisible by 16."

Any other appropriate strategy is acceptable with pertinent analysis and/or justification.

Appendix B

Standards for Mathematical Practice

The Standards for Mathematical Practice describe varieties of expertise that mathematics educators at all levels should seek to develop in their students. These practices rest on important "processes and proficiencies" with longstanding importance in mathematics education. The first of these are the NCTM process standards of problem solving, reasoning and proof, communication, representation, and connections. The second are the strands of mathematical proficiency specified in the National Research Council's report *Adding It Up*: adaptive reasoning, strategic competence, conceptual understanding (comprehension of mathematical concepts, operations and relations), procedural fluency (skill in carrying out procedures flexibly, accurately, efficiently and appropriately), and productive disposition (habitual inclination to see mathematics as sensible, useful, and worthwhile, coupled with a belief in diligence and one's own efficacy).[1]

MACC.K12.MP.1.1 Make sense of problems and persevere in solving them.

Mathematically proficient students start by explaining to themselves the meaning of a problem and looking for entry points to its solution. They analyze givens, constraints, relationships, and goals. They make conjectures about the form and meaning of the solution and plan a solution pathway rather than simply jumping into a solution attempt. They consider analogous problems, and try special cases and simpler forms of the original problem in order to gain insight into its solution. They monitor and evaluate their progress and change course if necessary. Older students might, depending on the context of the problem, transform algebraic expressions or change the viewing window on their graphing calculator to get the information they need. Mathematically proficient students can explain correspondences between equations, verbal descriptions, tables, and graphs or draw diagrams of important features and relationships, graph data, and search for regularity or trends. Younger students might rely on using concrete objects or pictures to help conceptualize and solve a problem. Mathematically proficient students check their answers to problems using a different method, and they continually ask themselves, "Does this make sense?" They can understand the approaches of others to solving complex problems and identify correspondences between different approaches.

MACC.K12.MP.2.1 Reason abstractly and quantitatively.

Mathematically proficient students make sense of quantities and their relationships in problem situations. They bring two complementary abilities to bear on problems involving quantitative relationships: the ability to *decontextualize*—to abstract a given situation and represent it symbolically and manipulate the representing symbols as if they have a life of their own, without necessarily attending to their referents—and the ability to *contextualize*, to pause as needed during the manipulation process in order to probe into the referents for the symbols involved. Quantitative reasoning entails habits of creating a coherent representation of the problem at hand; considering the units involved; attending to the meaning of quantities, not just how to compute them; and knowing and flexibly using different properties of operations and objects.

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[1] Common Core State Standards Initiative (CCSSI), 2010, Common Core State Standards for Mathematics. Washington, DC: National Governors Association Center for Best Practices and the Council of Chief State School Officers. *http://www.corestandards.org/assets/CCSSI_Math%20Standards.pdf*

MACC.K12.MP.3.1 Construct viable arguments and critique the reasoning of others.

Mathematically proficient students understand and use stated assumptions, definitions, and previously established results in constructing arguments. They make conjectures and build a logical progression of statements to explore the truth of their conjectures. They are able to analyze situations by breaking them into cases, and can recognize and use counterexamples. They justify their conclusions, communicate them to others, and respond to the arguments of others. They reason inductively about data, making plausible arguments that take into account the context from which the data arose. Mathematically proficient students are also able to compare the effectiveness of two plausible arguments, distinguish correct logic or reasoning from that which is flawed, and—if there is a flaw in an argument—explain what it is. Elementary students can construct arguments using concrete referents such as objects, drawings, diagrams, and actions. Such arguments can make sense and be correct, even though they are not generalized or made formal until later grades. Later, students learn to determine domains to which an argument applies. Students at all grades can listen or read the arguments of others, decide whether they make sense, and ask useful questions to clarify or improve the arguments.

MACC.K12.MP.4.1 Model with mathematics.

Mathematically proficient students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. In early grades, this might be as simple as writing an addition equation to describe a situation. In middle grades, a student might apply proportional reasoning to plan a school event or analyze a problem in the community. By high school, a student might use geometry to solve a design problem or use a function to describe how one quantity of interest depends on another. Mathematically proficient students who can apply what they know are comfortable making assumptions and approximations to simplify a complicated situation, realizing that these may need revision later. They are able to identify important quantities in a practical situation and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts and formulas. They can analyze those relationships mathematically to draw conclusions. They routinely interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.

MACC.K12.MP.5.1 Use appropriate tools strategically.

Mathematically proficient students consider the available tools when solving a mathematical problem. These tools might include pencil and paper, concrete models, a ruler, a protractor, a calculator, a spreadsheet, a computer algebra system, a statistical package, or dynamic geometry software. Proficient students are sufficiently familiar with tools appropriate for their grade or course to make sound decisions about when each of these tools might be helpful, recognizing both the insight to be gained and their limitations. For example, mathematically proficient high school students analyze graphs of functions and solutions generated using a graphing calculator. They detect possible errors by strategically using estimation and other mathematical knowledge. When making mathematical models, they know that technology can enable them to visualize the results of varying assumptions, explore consequences, and compare predictions with data. Mathematically proficient students at various grade levels are able to identify relevant external mathematical resources, such as digital content located on a website, and use them to pose or solve problems. They are able to use technological tools to explore and deepen their understanding of concepts.

MACC.K12.MP.6.1 Attend to precision.

Mathematically proficient students try to communicate precisely to others. They try to use clear definitions in discussion with others and in their own reasoning. They state the meaning of the symbols they choose, including using the equal sign consistently and appropriately. They are careful about specifying units of measure, and labeling axes to clarify the correspondence with quantities in a problem. They calculate accurately and efficiently, express numerical answers with a degree of precision appropriate for the problem context. In the elementary grades, students give carefully formulated explanations to each other. By the time they reach high school they have learned to examine claims and make explicit use of definitions.

MACC.K12.MP.7.1 Look for and make use of structure.

Mathematically proficient students look closely to discern a pattern or structure. Young students, for example, might notice that three and seven more is the same amount as seven and three more, or they may sort a collection of shapes according to how many sides the shapes have. Later, students will see 7×8 equals the well remembered $7 \times 5 + 7 \times 3$, in preparation for learning about the distributive property. In the expression $x^2 + 9x + 14$, older students can see the 14 as 2×7 and the 9 as $2 + 7$. They recognize the significance of an existing line in a geometric figure and can use the strategy of drawing an auxiliary line for solving problems. They also can step back for an overview and shift perspective. They can see complicated things, such as some algebraic expressions, as single objects or as being composed of several objects. For example, they can see $5 - 3(x - y)^2$ as 5 minus a positive number times a square and use that to realize that its value cannot be more than 5 for any real numbers x and y.

MACC.K12.MP.8.1 Look for and express regularity in repeated reasoning.

Mathematically proficient students notice if calculations are repeated, and look both for general methods and for shortcuts. Upper elementary students might notice when dividing 25 by 11 that they are repeating the same calculations over and over again, and conclude they have a repeating decimal. By paying attention to the calculation of slope as they repeatedly check whether points are on the line through (1, 2) with slope 3, middle school students might abstract the equation $(y - 2)/(x - 1) = 3$. Noticing the regularity in the way terms cancel when expanding $(x - 1)(x + 1)$, $(x - 1)(x^2 + x + 1)$, and $(x - 1)(x^3 + x^2 + x + 1)$ might lead them to the general formula for the sum of a geometric series. As they work to solve a problem, mathematically proficient students maintain oversight of the process, while attending to the details. They continually evaluate the reasonableness of their intermediate results.