

Middle Grades Mathematics

5–9

Section 25

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1 Knowledge of problem-solving and reasoning skills

1. Analyze realistic situations and identify the appropriate mathematical expression or equation.
2. Apply strategies to solve nonroutine problems with multiple steps.
3. Evaluate the reasonableness of results from the original problem.
4. Apply appropriate mathematical concepts and procedures to solve problems in various contexts.
5. Evaluate the validity of mathematical arguments (e.g., a justification that the sum of two odd numbers is always even).
6. Predict logical conclusions from given statements.
7. Identify appropriate instructional strategies to facilitate student understanding of problem solving.
8. Distinguish between deductive and inductive reasoning in a given situation.

2 Knowledge of mathematical manipulatives and models and instructional technology

1. Identify appropriate mathematical representations (e.g., verbal statements, manipulatives, pictures, graphs, algebraic expressions).
2. Interpret concepts with multiple representations (e.g., manipulatives, tables, graphs, symbolic expressions, technology).
3. Select appropriate manipulatives and technology for teaching specific mathematical concepts (e.g., graphing calculators, dynamic software, virtual and physical manipulatives).
4. Use appropriate manipulatives and technology for teaching diverse groups of students (e.g., varied learning styles and exceptionalities).

3 Knowledge of assessment in mathematics

1. Assess student learning through various methods (e.g., informal, formative, summative).
2. Analyze student work samples to assess and diagnose student learning needs.
3. Analyze student performance using technology (e.g., online resources, audience-response systems, instructor software).

4. Interpret student performance data to drive instruction.
5. Recognize cognitive complexity in various questioning strategies.
6. Evaluate appropriate alternative assessments (e.g., projects, portfolios) that utilize various cognitive complexity levels.

4 Knowledge of connections among mathematical concepts

1. Identify prerequisite skills for a given topic (e.g., ratio, slope).
2. Predict common misconceptions in mathematics (e.g., area and perimeter, box plot).
3. Connect interrelated mathematical concepts (e.g., scale factor and proportional reasoning).
4. Analyze mathematical errors (e.g., computational, algebraic, statistical, geometric).
5. Identify fundamental concepts that connect middle grades mathematics to high school and postsecondary mathematics (e.g., trigonometry, number theory, calculus).

5 Knowledge of number sense, operations, and proportionality

1. Compare the relative size of real numbers expressed in a variety of forms (e.g., fractions, decimals, percents, absolute value).
2. Apply mental computation and estimation strategies.
3. Apply prime factorization of composite numbers to other operations (e.g., cube roots, polynomials).
4. Compute fluently with rational numbers using the greatest common factor (GCF) and least common multiple (LCM).
5. Apply ratios and proportions to similar figures and to solve realistic problems.
6. Select the appropriate operation(s) to solve realistic problems that involve real numbers.

6 Knowledge of foundations of algebra

1. Predict missing terms in numerical, algebraic, and pictorial patterns.
2. Analyze relationships between tables, graphs, or equations.
3. Simplify rational and irrational expressions.
4. Simplify expressions involving radicals and rational exponents using the properties of exponents.

5. Solve equations or inequalities with one variable (e.g., number line).
6. Identify graphs of inequalities involving one variable on a number line.
7. Identify graphs of linear equations or inequalities involving two variables on the coordinate plane.
8. Identify and interpret the slope and intercepts using a graph, table, or an equation.
9. Determine the equation of a line.
10. Find and estimate square roots.
11. Apply properties of operations (e.g., commutative, associative, distributive) to generate equivalent expressions.

7 Knowledge of algebraic thinking

1. Determine the impact when changing values of given linear and nonlinear functions (e.g., change of y-intercept or coefficients).
2. Identify the equation of a line that is perpendicular or parallel to a given line.
3. Apply operations to analyze polynomials (e.g., finding zeros, factoring, arithmetic operations).
4. Solve systems of linear equations involving two variables using graphing, substitution, or elimination.
5. Determine the solution set of a system of linear inequalities involving two variables.
6. Use quadratic equations to solve abstract and realistic problems.
7. Identify the graph of quadratic functions.
8. Solve equations involving radicals, limited to square roots.
9. Apply the laws of exponents.

8 Knowledge of data analysis, statistics, and probability

1. Determine which measure of center (i.e., central tendency) is the most appropriate in a given situation.
2. Find and interpret the range and distribution of data.
3. Interpret information and patterns from various graphical representations using univariate (e.g., a line plot) and bivariate data (e.g., scatterplot).

4. Identify appropriate graphical representations for a given data set.
5. Identify an appropriate sample to draw inferences about a population.
6. Make predictions based on experimental or theoretical probabilities.

9 Knowledge of two-dimensional geometry

1. Identify precise definitions of symbols for lines, segments, rays, and distances based on point, line, and plane as undefined terms.
2. Identify and apply properties of the relationships of angles or pairs of angles.
3. Identify and apply properties of polygons to determine the measure(s) of interior angles and/or exterior angles.
4. Evaluate proofs and apply the properties of triangles (e.g., isosceles, scalene, equilateral).
5. Evaluate proofs and apply triangle inequality theorems (e.g., opposite the largest angle is the longest side, the sum of two sides is greater than the third side).
6. Use the SAS, ASA, and SSS postulates to show pairs of triangles congruent, including the case of overlapping triangles.
7. Apply theorems and postulates that apply to right triangles to solve mathematical and realistic problems (e.g., Pythagorean theorem, special right triangles).
8. Apply trigonometric ratios to solve right triangle problems.
9. Apply the specific properties of quadrilaterals (e.g., parallelograms, rectangles, rhombuses, squares, kites, trapezoids).
10. Apply the formulas for distance and midpoint on the coordinate plane.
11. Classify and apply the types of transformations of geometric figures including similar figures.
12. Apply properties and theorems about circles.

10 Knowledge of measurement and spatial sense

1. Convert units of measure within and between given measurement systems, including derived units.
2. Solve realistic and mathematical problems involving perimeter, circumference, area, surface area, and volume.
3. Determine how a change in dimensions (e.g., length, width, height, radius) affects other measurements (e.g., perimeter, area, surface area, volume).

4. Identify characteristics of three-dimensional figures (e.g., faces, edges, vertices).
5. Identify the net of a three-dimensional figure.
6. Identify the two-dimensional view of a three-dimensional object.